The Waisman Laboratory for Brain Imaging and Behavior

University of Wisconsin SCHOOL OF MEDICINE AND PUBLIC HEALTH

Persistence Diagram: Topological Characterization of Noise in Images

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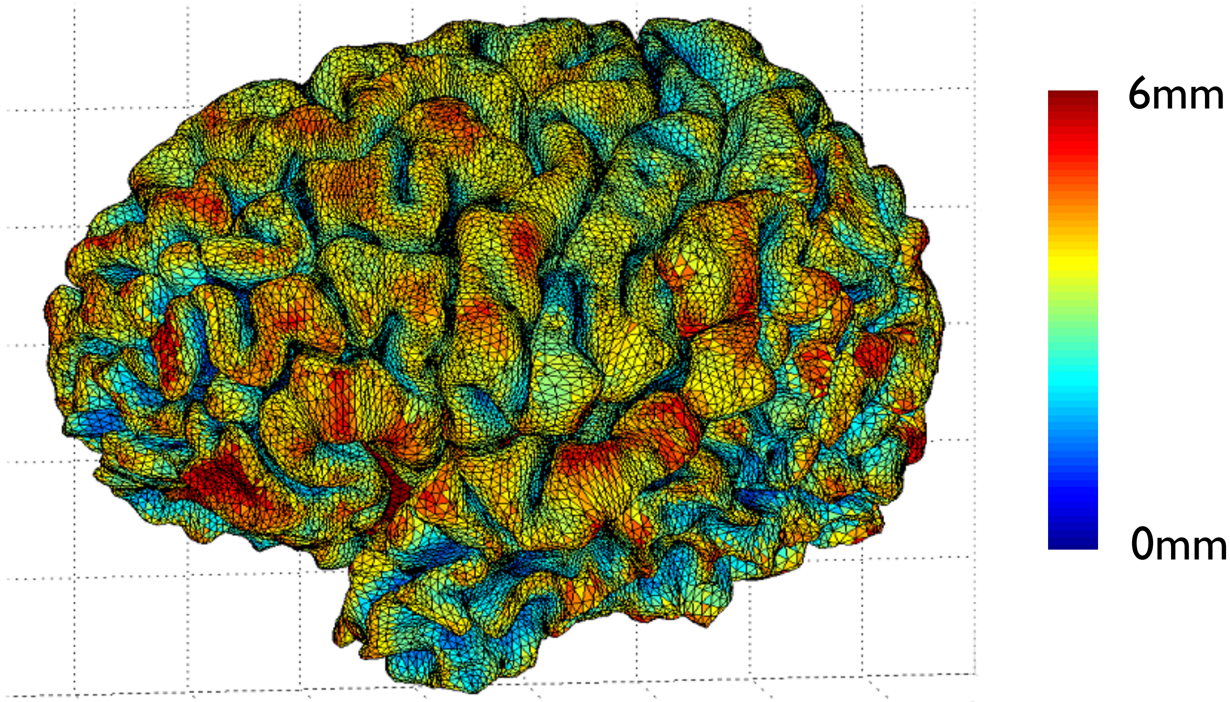
CANADA

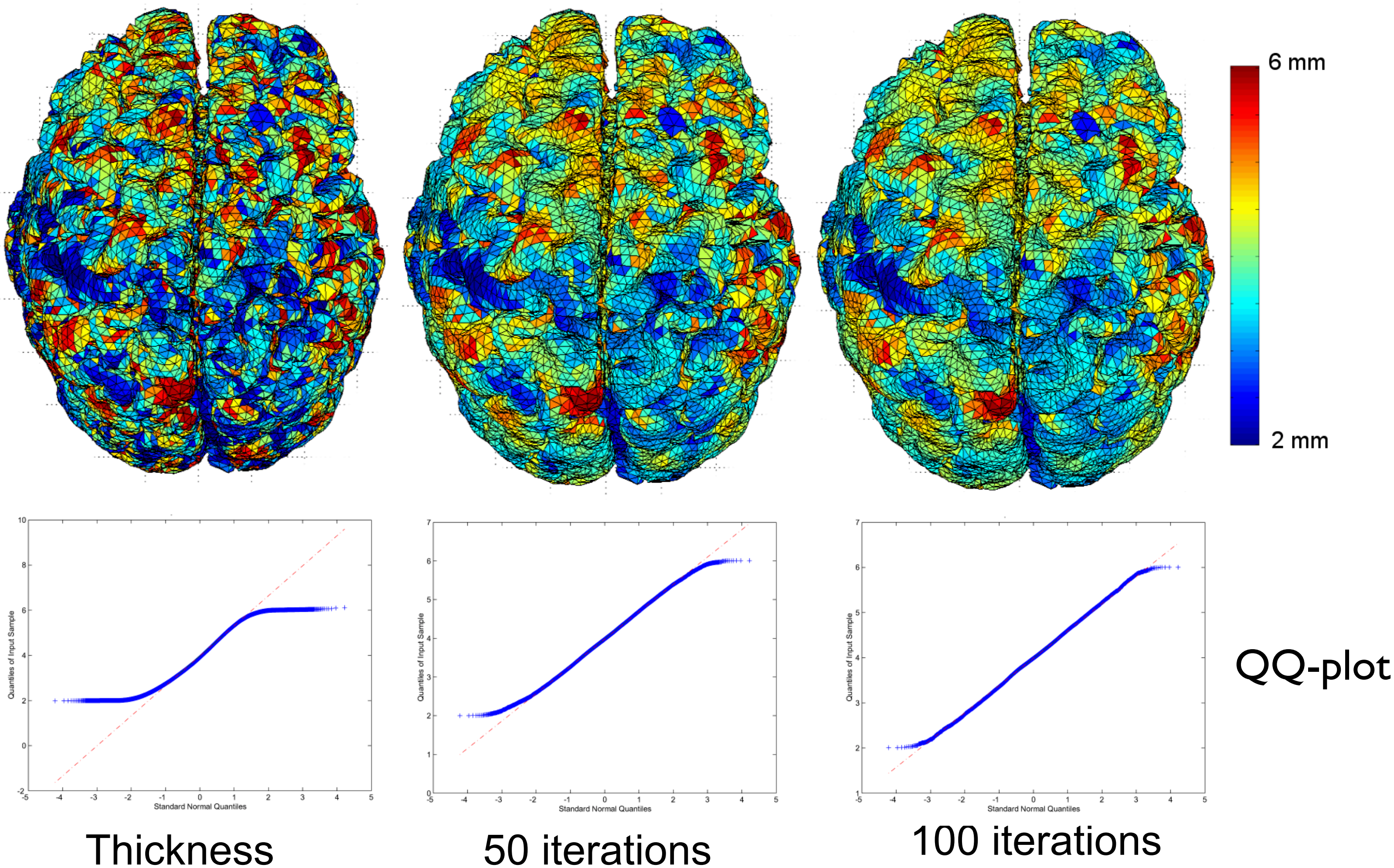
Standard model on cortical thickness



$$Y(p) = \theta(p) + \epsilon(p), p \in \partial\Omega$$

Gaussian → GLM

Heat kernel smoothing makes data more Gaussian – central limit theorem



Chung *et al.*, 2005. NeuroImage

Heat kernel smoothing
widely used cortical data smoothing technique

cortical curvatures (Luders, 2006; Gaser, 2006)
cortical thickness (Luders, 2006; Bernal-Rusiel, 2008)
Hippocampus (Shen, 2006; Zhu, 2007)
Magnetoencephalography (MEG) (Han, 2007)
functional-MRI (Hagler, 2006: Jo, 2007)

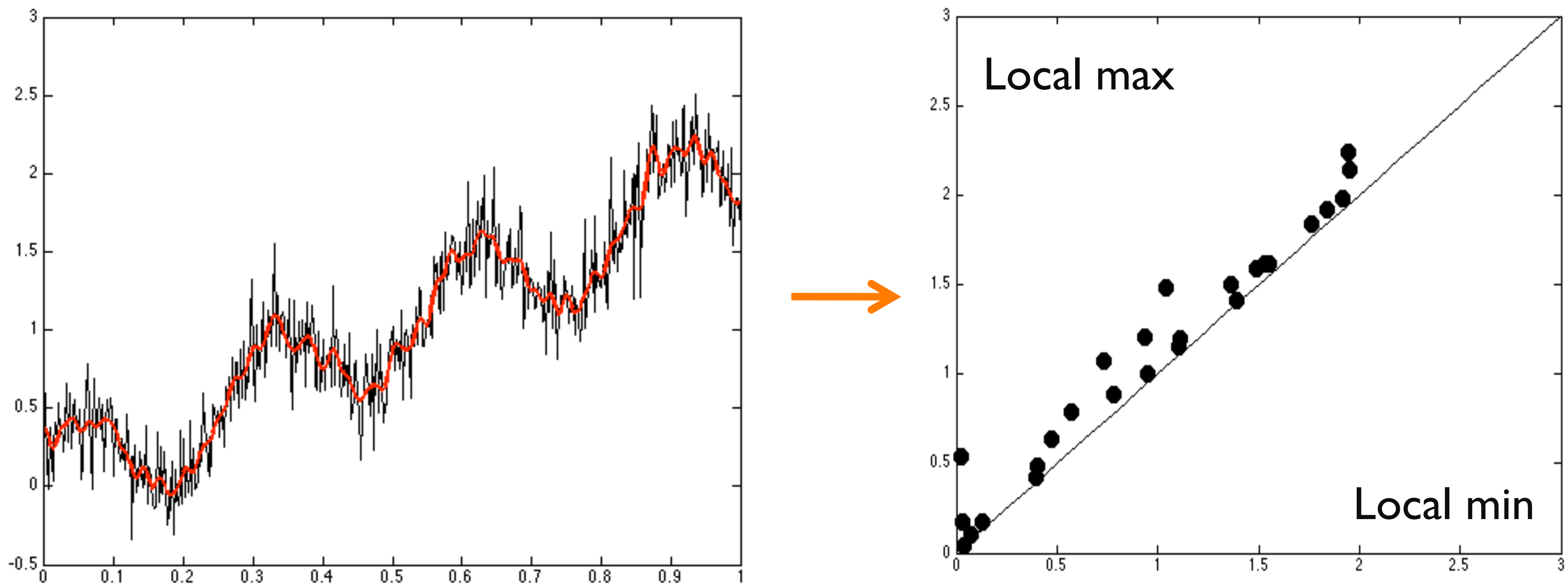
↓

General linear model (GLM) + random field theory

Can we do data analysis in a really crazy way?

Why? We may be able to detect some features that can't be detected using the traditional approach.

Persistence diagram



A way to pair local min to local max in a nonlinear fashion.

Persistence diagram

Sublevel set $R(x) = f^{-1}(-\infty, x]$

Number of connected components $\#R(x)$

Local min: $g_1, \cdots, g_n$

Birth: $\#R(g_i - \epsilon) = \#R(g_i) + 1$

Local max: $h_1, \cdots, h_n$

Death: $\#R(h_i - \epsilon) = \#R(h_i) - 1$

Pair the time of death with the time of the closest earlier birth

PD-algorithm for pairing

Set of local max $H = \{h_1, \cdots, h_m\}$

Ordered local min $g_{(1)} < \cdots < g_{(n)}$

For *i* from *n* to 1,

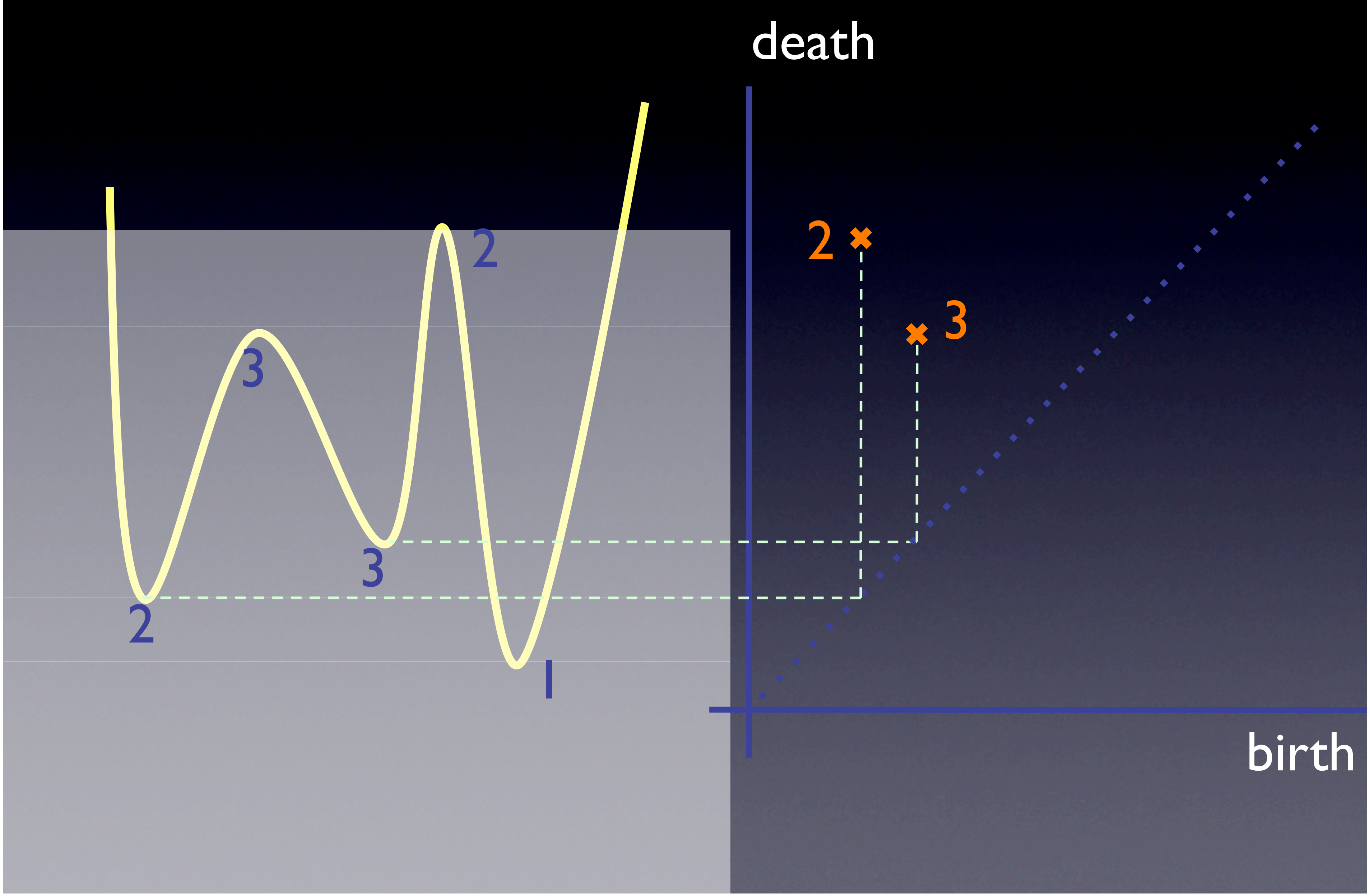
iterate

let h_i^* be the smallest of two adjacent local max

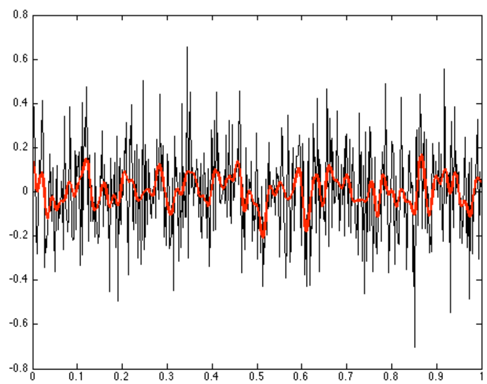
pair $(g_{(i)}, h_i^*)$

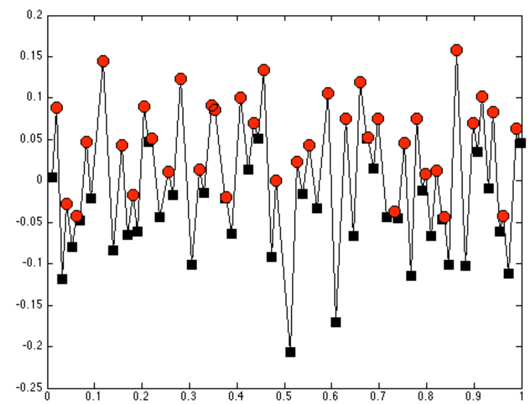
delete $H \leftarrow H - h_j^*$

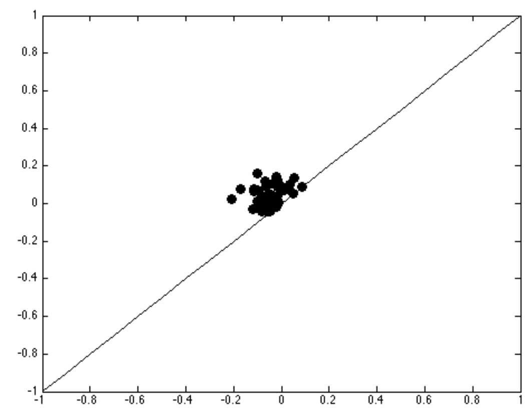
Rule for pairing local minimum to local maximum



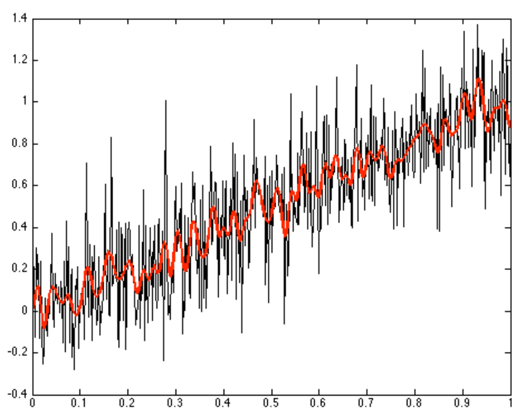
Persistence diagrams will show signal pattern

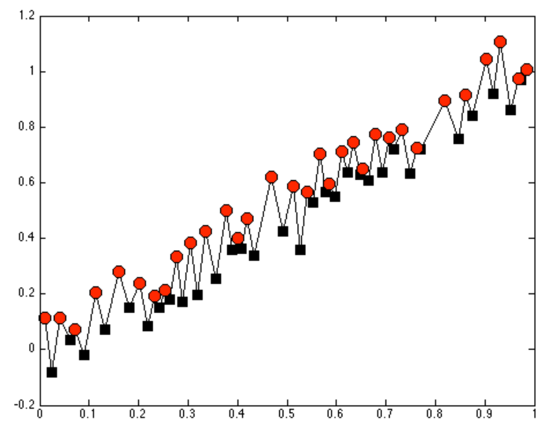


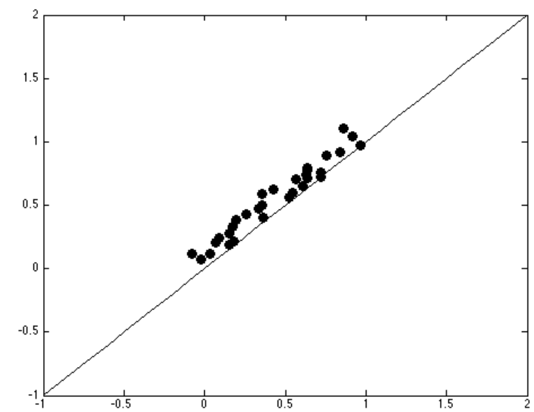




$$f(t) = e(t)$$







$$f(t) = t + e(t)$$

Orthonormal basis in [0,1]

$$\Delta f + \lambda f = 0$$

$$f(t) = f(-t)$$

Produces sine and cosine basis

$$f(t+2) = f(t)$$

Removes sine basis

↓ $\lambda_l = -l^2\pi^2$

$$\psi_0 = 1, \psi_l = \sqrt{2}\cos(l\pi t)$$

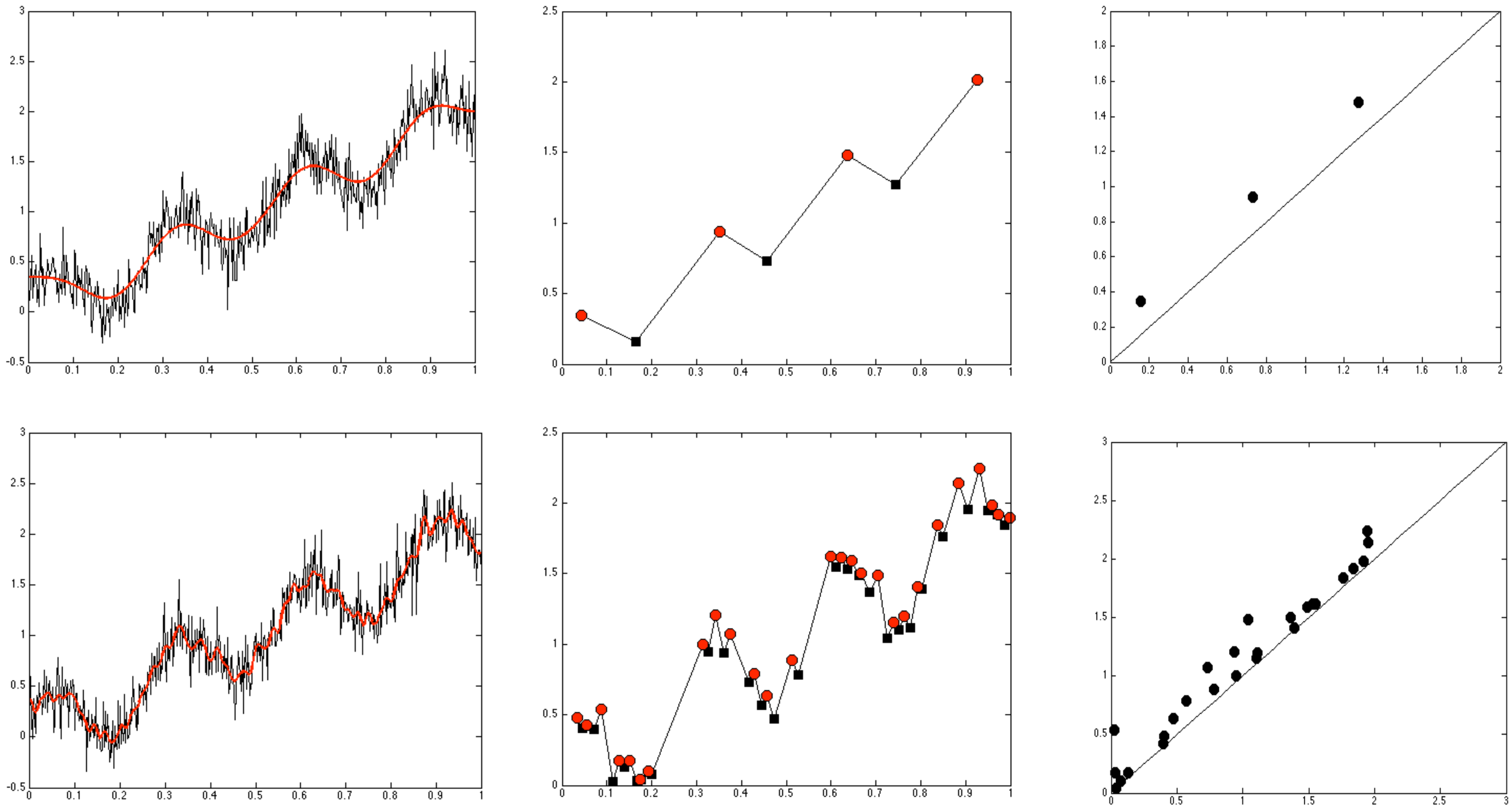
High order derivatives are computed analytically without using finite difference
→ more stable computing

Measurement = *f* + noise

$$\widehat{f} = \sum_{l=0}^{k} e^{-l^2\pi^2\sigma} f_l \psi_l(t)$$

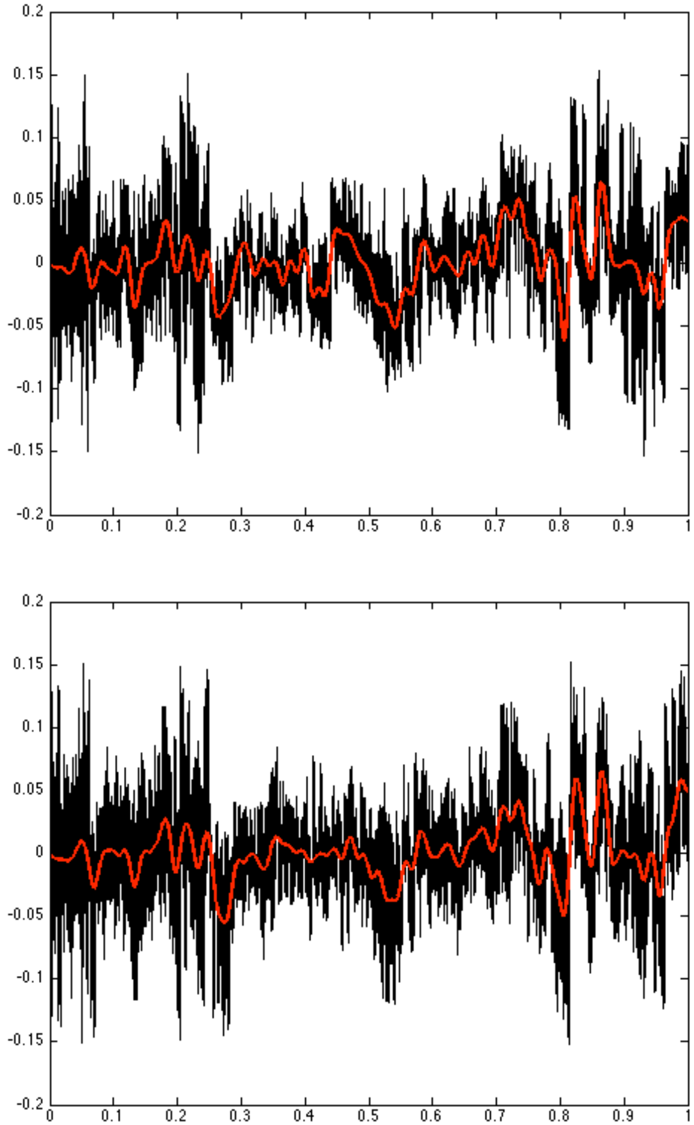
$$\widehat{f}'(t) = -\sum_{l=0}^{k} \sqrt{2} l\pi e^{-l^2\pi^2\sigma} f_l \sin(l\pi t)$$

Persistence diagram



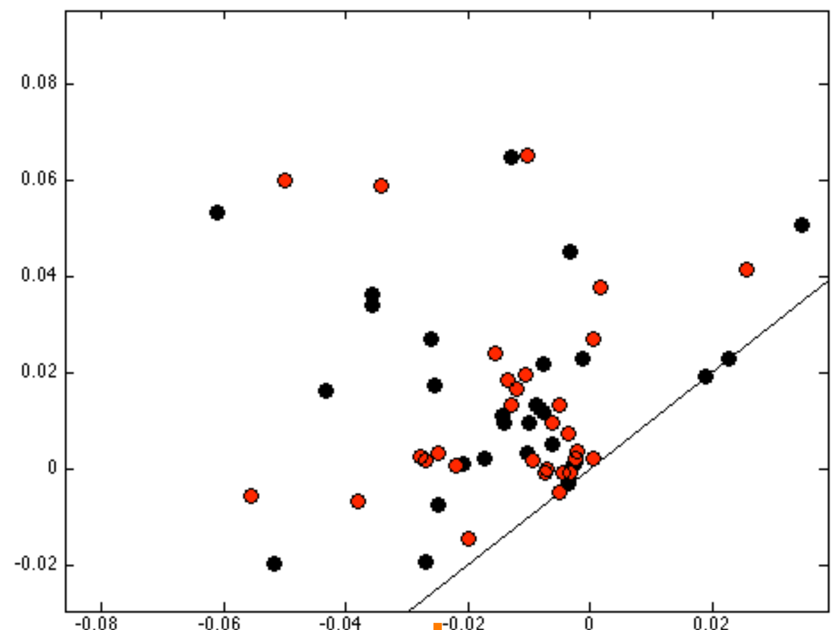
Red: local max
Black: local min

More complicated example

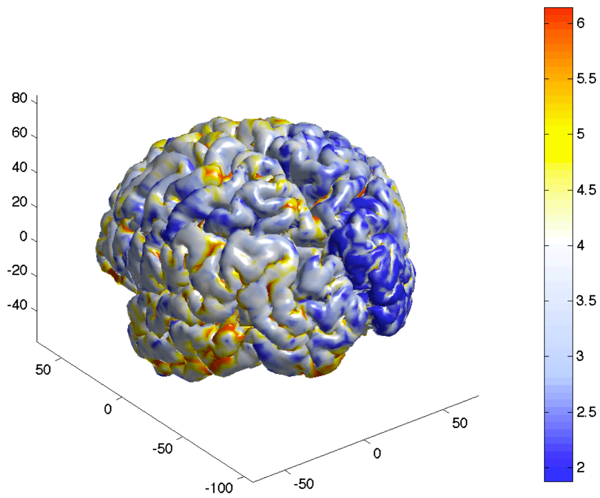


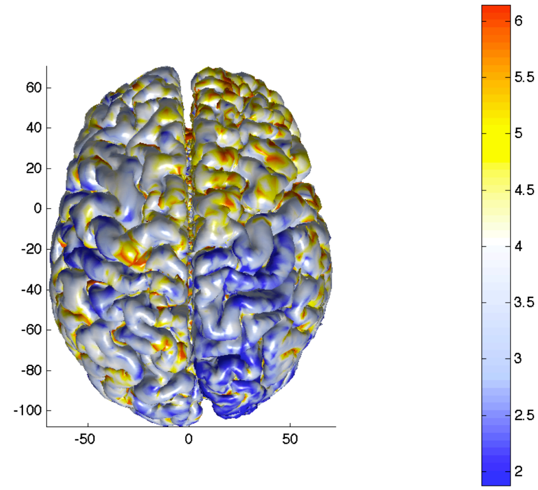
black = top

red = bottom

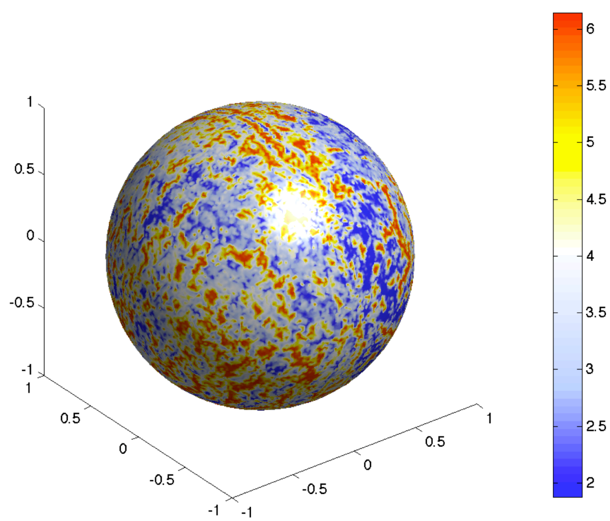


Statistical analysis?

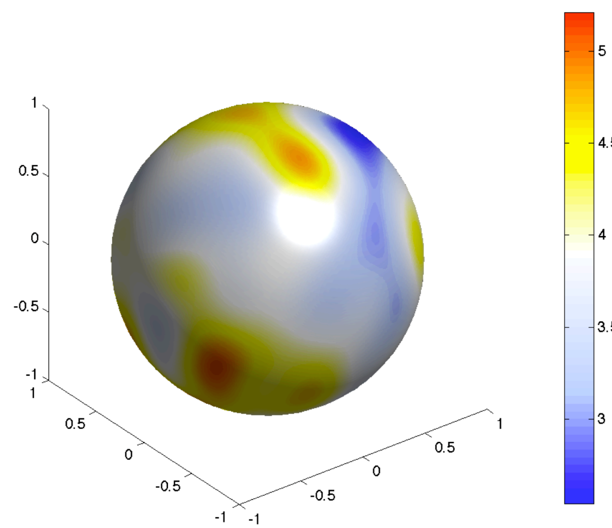




Cortical thickness

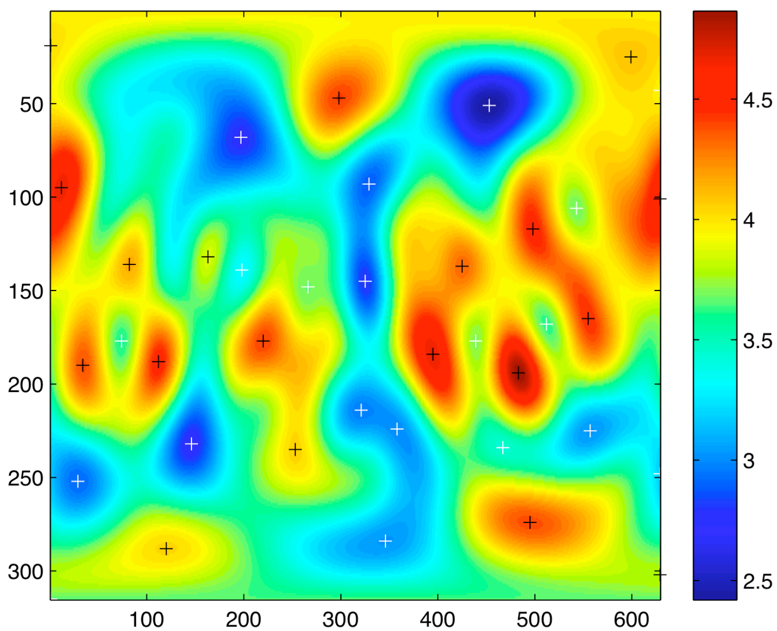


Flattening

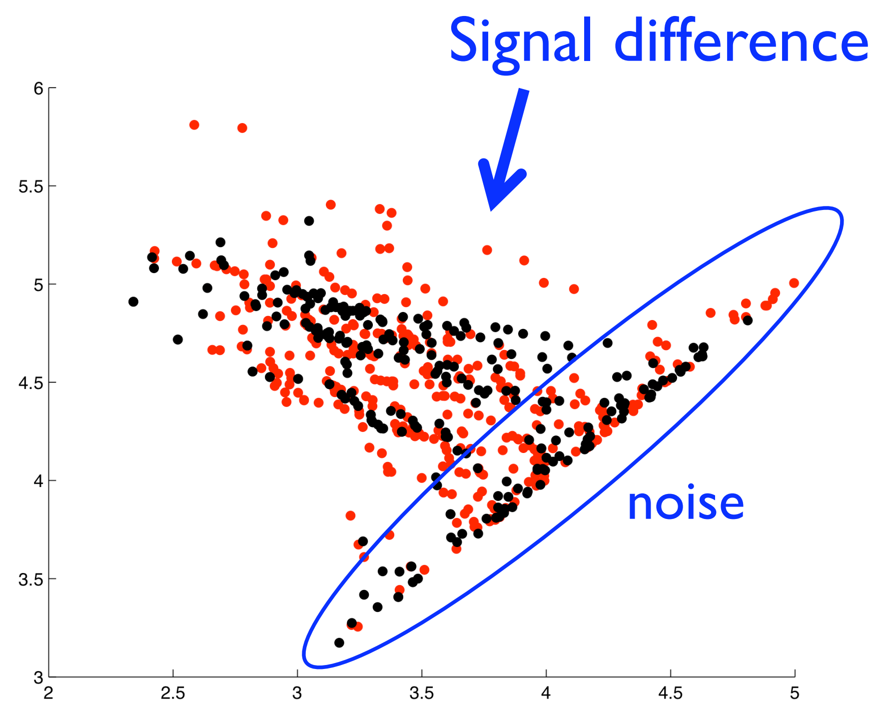


Weighted-SPHARM

Chung et al., 2007, IEEE-TMI



local min and max computation



red=autism
black=control

New inference & classification framework under development

Thank you

If you want to analyze your data this way, please talk to me ☺