The Waisman Laboratory for Brain Imaging and Behavior

University of Wisconsin
SCHOOL OF MEDICINE AND PUBLIC HEALTH

Heat kernel smoothing on manifolds and its application to longitudinal brain substructure modeling

Moo K. Chung

Department of Biostatistics and Medical Informatics
Waisman Laboratory for Brain Imaging and Behavior
University of Wisconsin-Madison

www.stat.wisc.edu/~mchung

Acknowledgments

Jamie L. Hanson, Richard J. Davidson, Seth D. Pollak

University of Wisconsin-Madison

SeongHo Seo

Department of Brain and Cognitive Sciences

Seoul National University

Abstract

We present a novel kernel smoothing framework on an arbitrary manifold using the Laplace-Beltrami eigenfunctions. The Green's function of an isotropic diffusion equation on a manifold is analytically represented first using the eigenfunctions of the Laplace-Beltraimi operator. The Green's function is then used in constructing heat kernel smoothing analytically. Unlike many previous diffusion based smoothing approaches developed for manifolds data, diffusion is analytically solved avoiding various numerical instability and inaccuracy issues. Our method is compared to a widely used iterative kernel smoothing technique in brain imaging to show significant improvement in numerical accuracy. The proposed framework is illustrated with longitudinally collected hippocampus surfaces.

Effect of family income on hippocampus growth

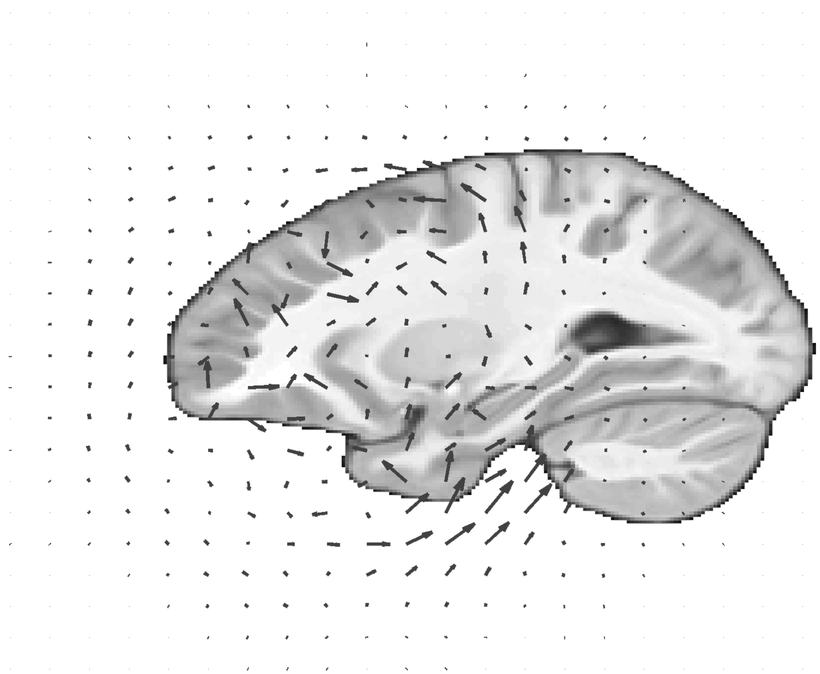
86 teens from high income family (> $75000)
mean age = 12+/- 4 years

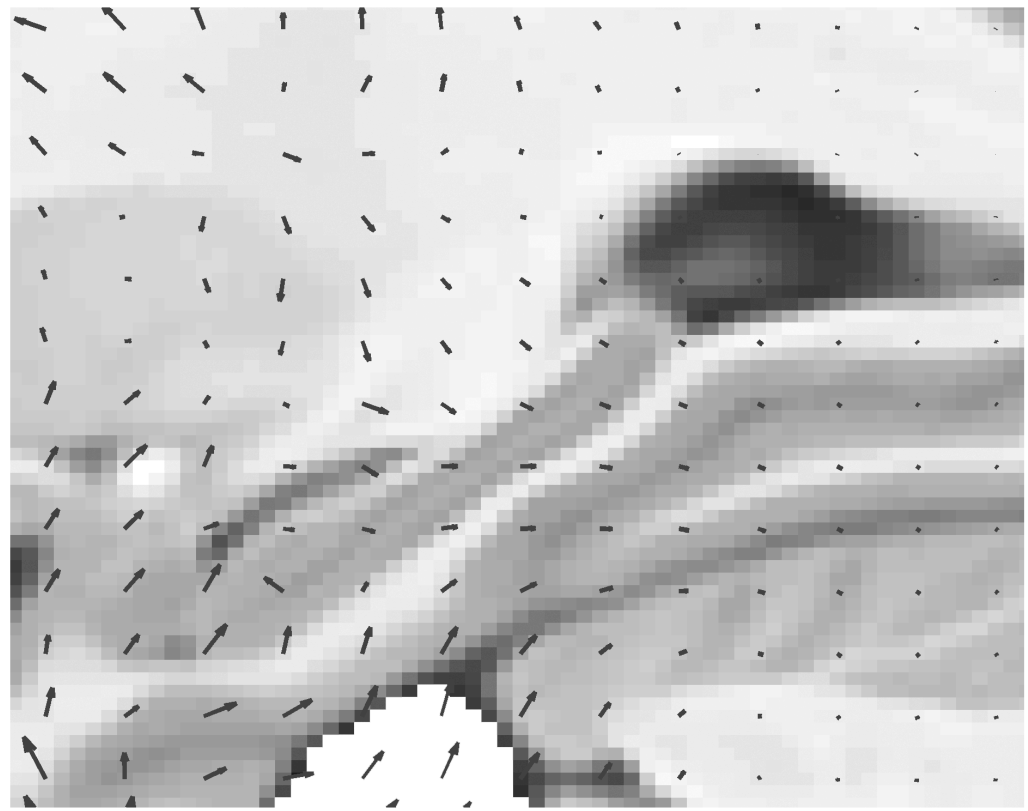
38 teens from low income family (< $35000)
mean age= 12 +/- 4 years old

Each subject has multiple scans (1-2 scans).

Image Processing

Image registration
Deformation from the template to a subject.

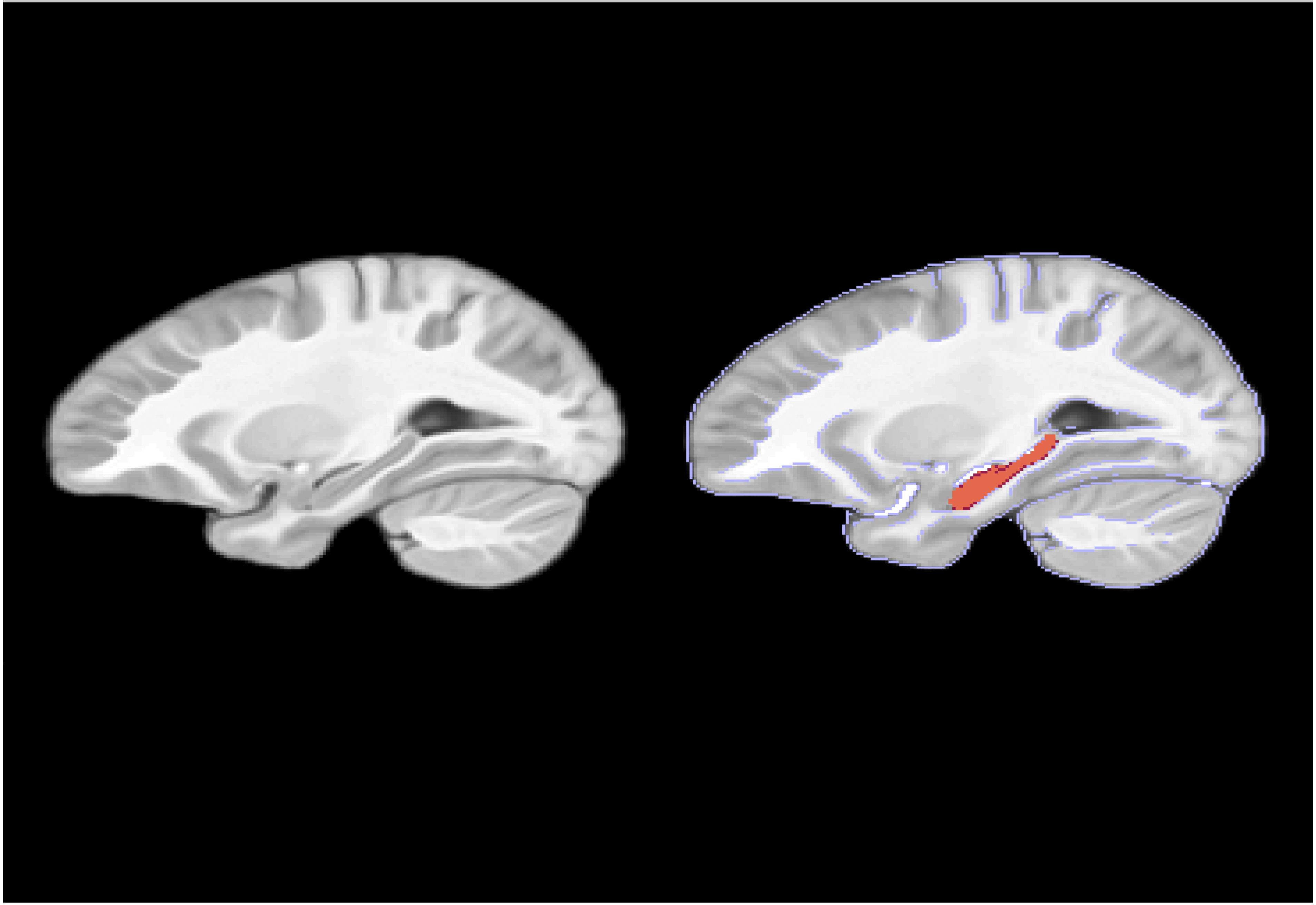




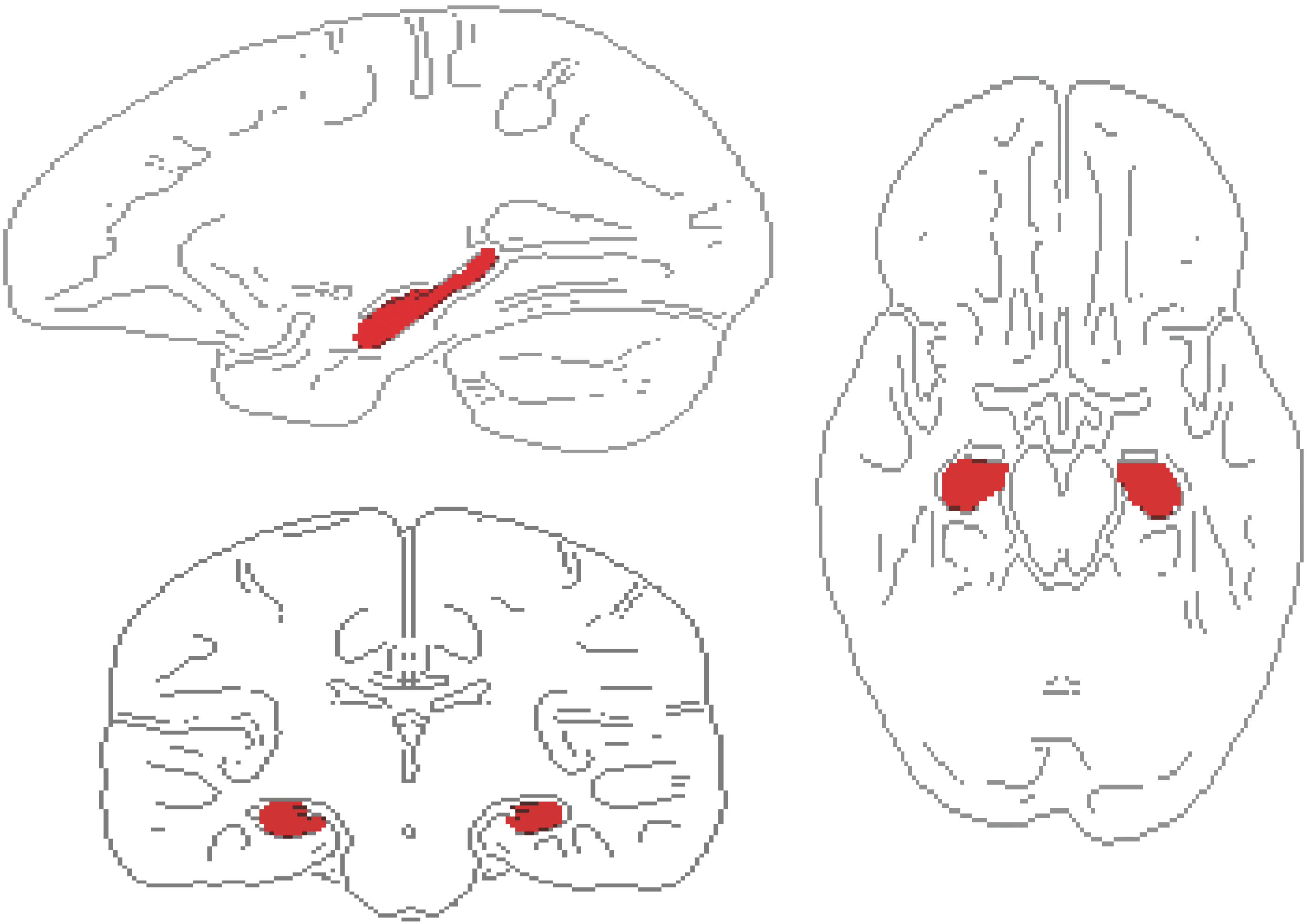
Longitudinal image processing

Deformation form the template to Scan2 is given by warp1 + warp2.

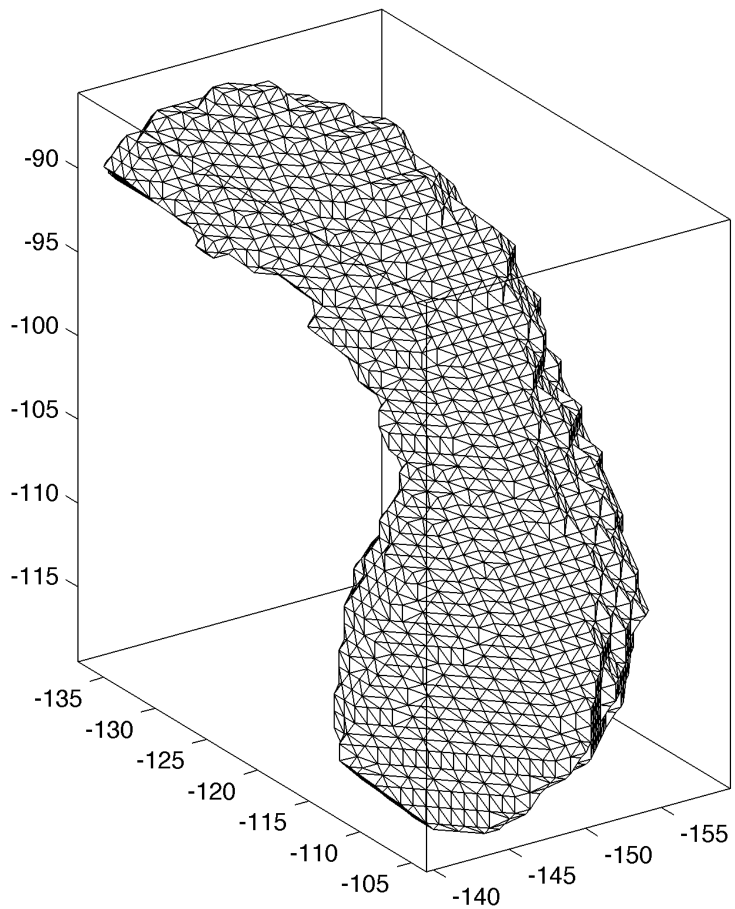
Manual hippocampus segmentation on MRI template

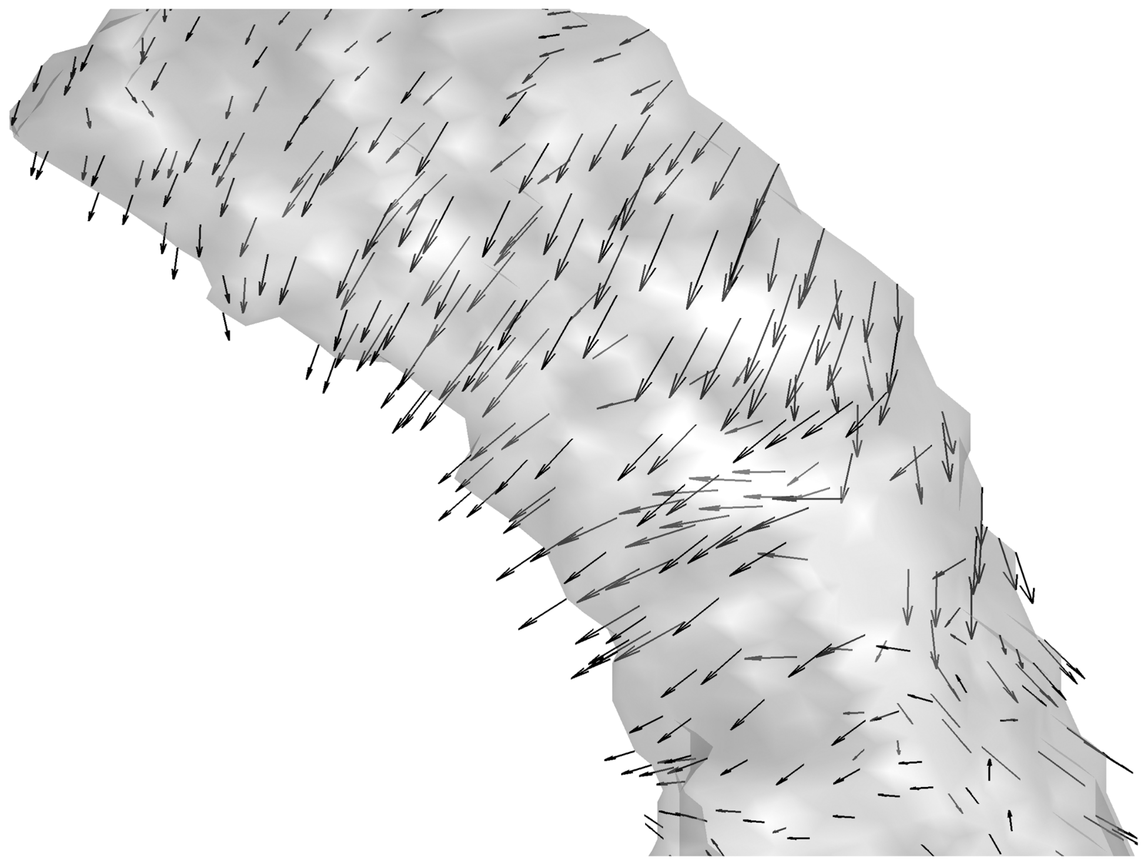


Subcortical structures: hippocampus



surface template & deformation on template





Deformation field
of warping the template
to a subject

Heat kernel smoothing on manifolds

Original idea given in Chung et al. 2005 (NeuroImage).

The most widely used cortical data smoothing technique in brain imaging.

Similar iterative smoothing methods are now implemented in FreeSurfer, AFNI, SurfStat.

Approximating heat kernel with Gaussian kernel

Parametrix expansion (*Rosenberg, 1997*):

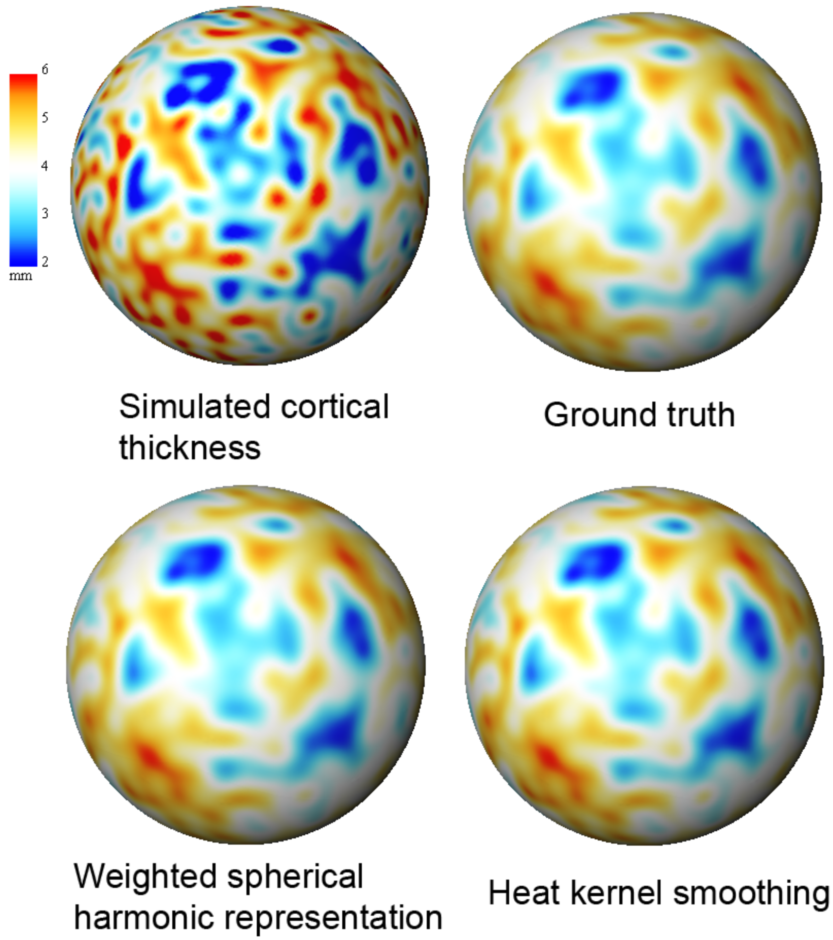
$$K_\sigma(p,q) \quad = \quad \frac{1}{(4\pi\sigma)^{1/2}} e^{-\frac{d^2(p,q)}{4\sigma}} [1 + O(\sigma^2)]$$

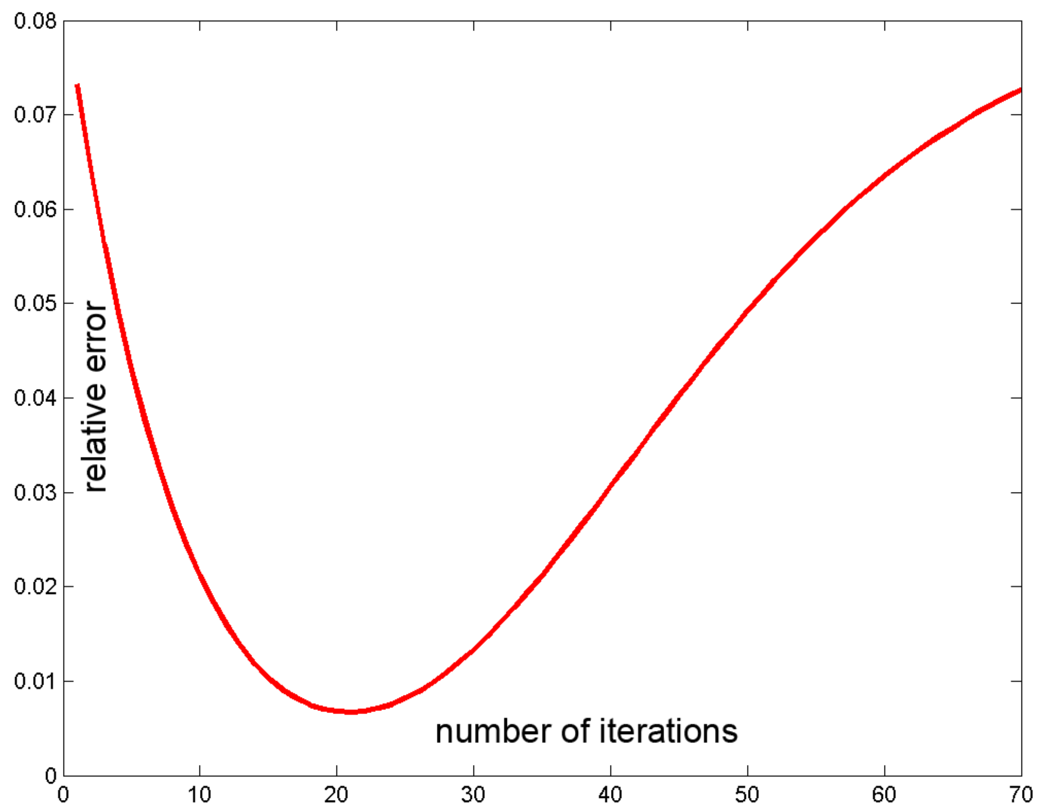
for small bandwdith

$$K_{k\sigma} * f = \underbrace{K_\sigma * \cdots * K_\sigma}_{k \text{ times}} * f$$

Validation against the ground truth

Chung et al., 2008. Statistica Sinica



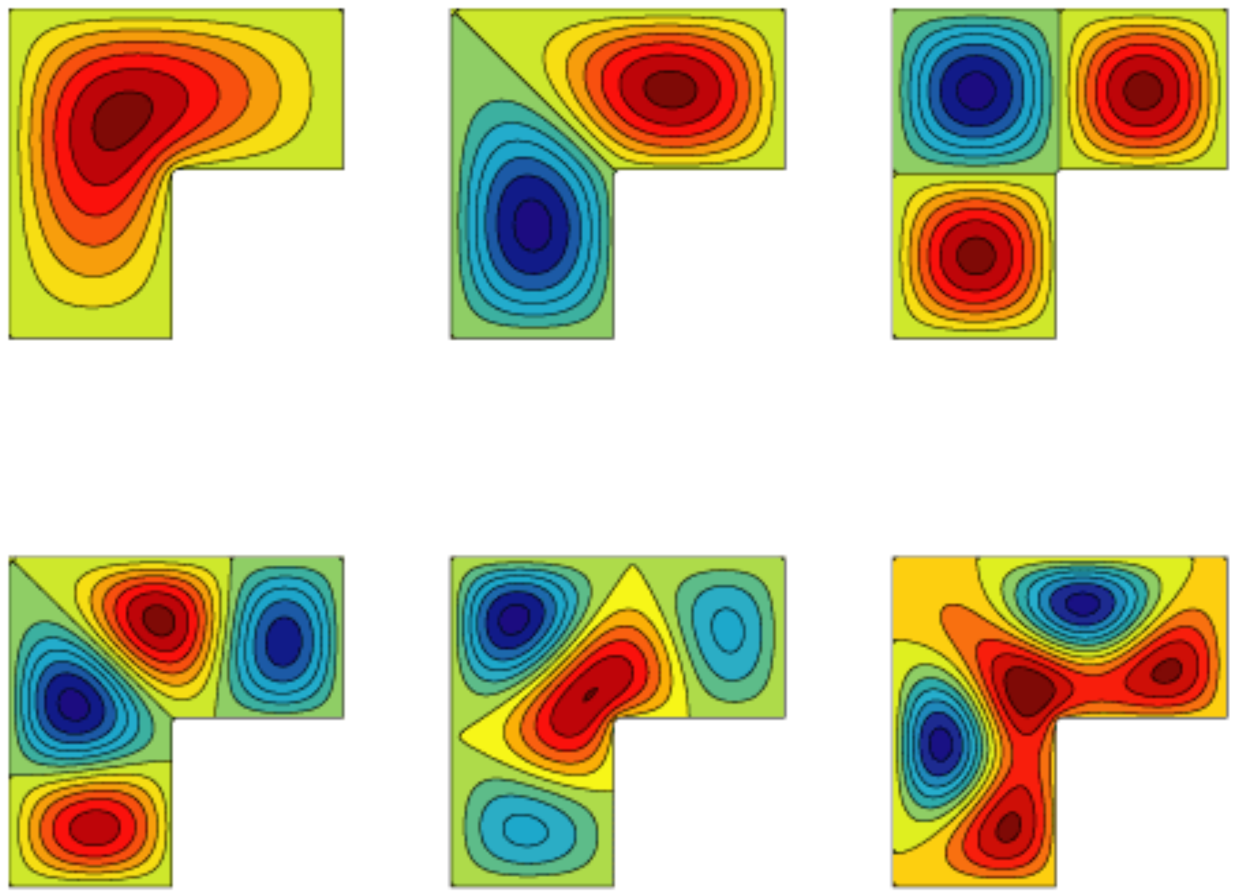


Can we do better?

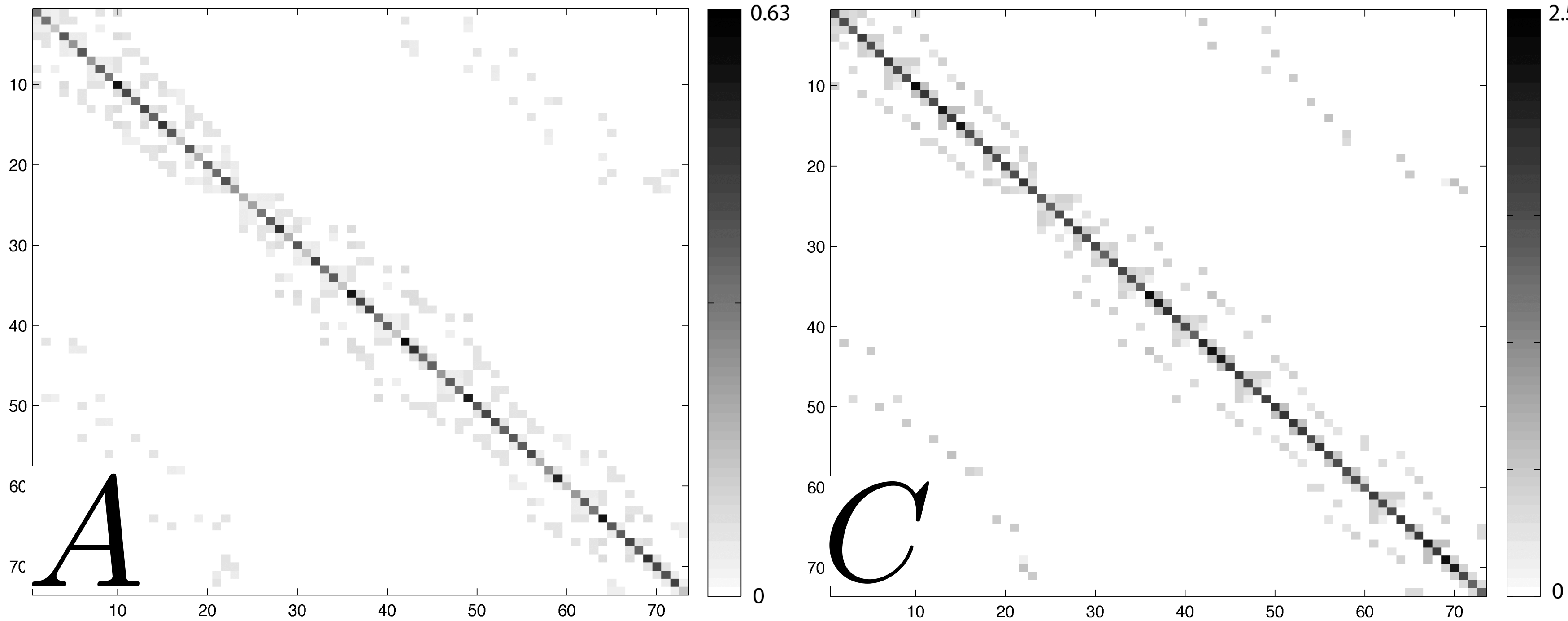
Laplace-Beltrami eigenfunctions

Eigenfunctions of Laplace-Beltrami operator

Helmholtz equation $$\Delta\psi_j = \lambda_j\psi_j$$



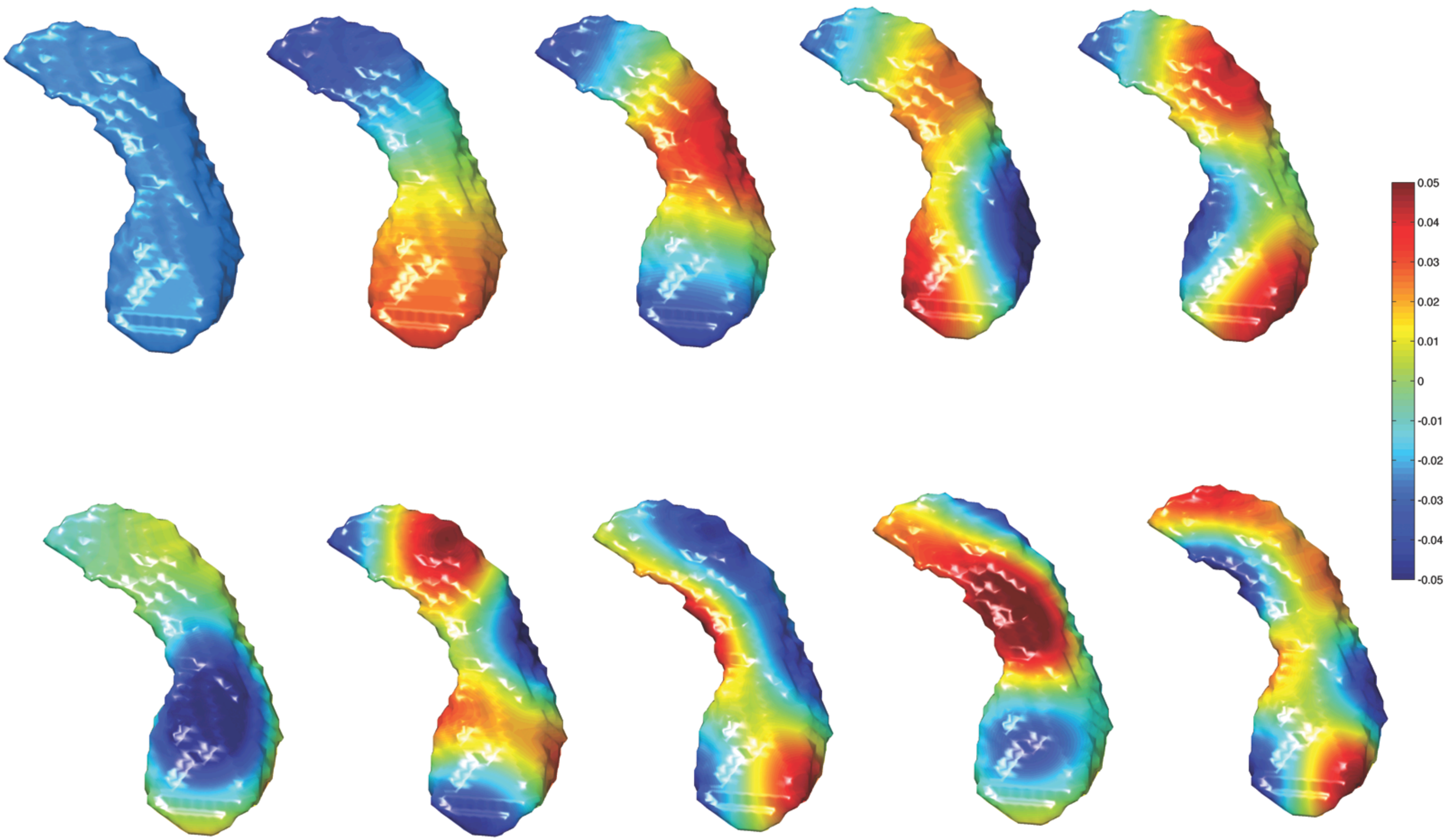
Generalized eigenvalue problem via FEM



$$\Delta f = \lambda f \longrightarrow C\psi = \lambda A \psi$$

Qiu et al. IEEE TMI 2006
Seo et al. MICCAI 2010

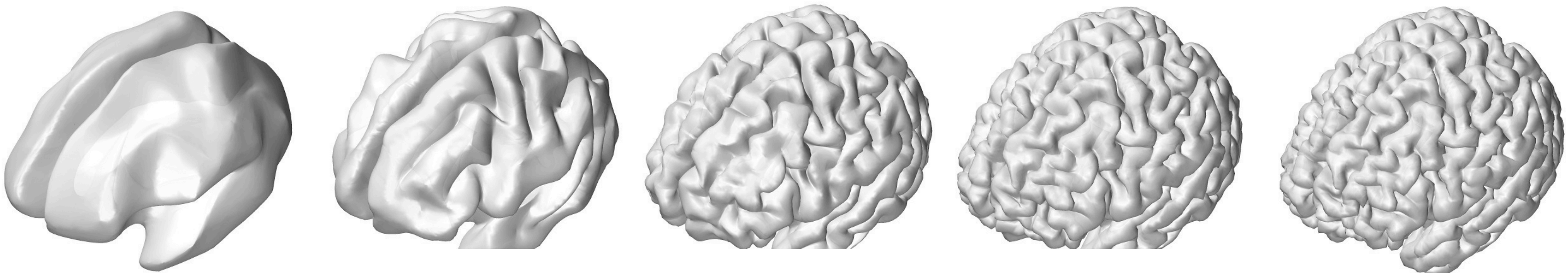
First 10 orthonormal basis on left hippocampus



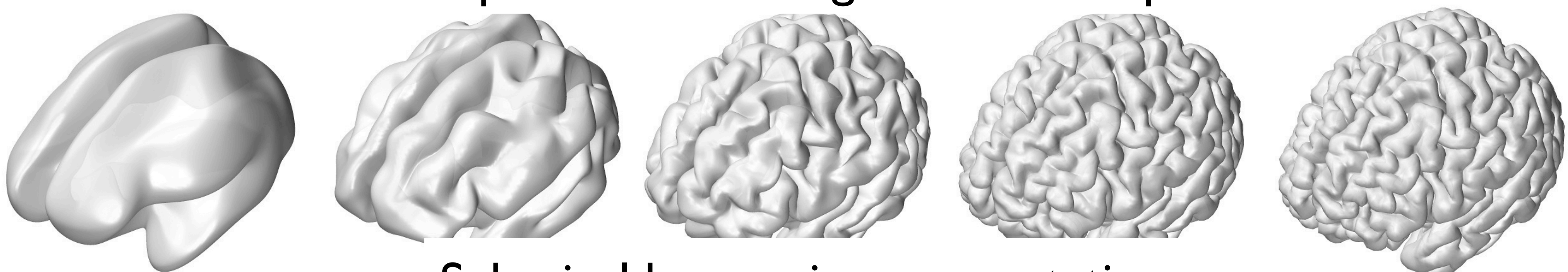
MATLAB code available from my website.

Laplace-Beltrami eigenfunction expansion

$$x_i(p) = \sum_{j=0}^{k} f_{ij} \psi_j(p)$$



Laplace-Beltrami eigenfunction expansion

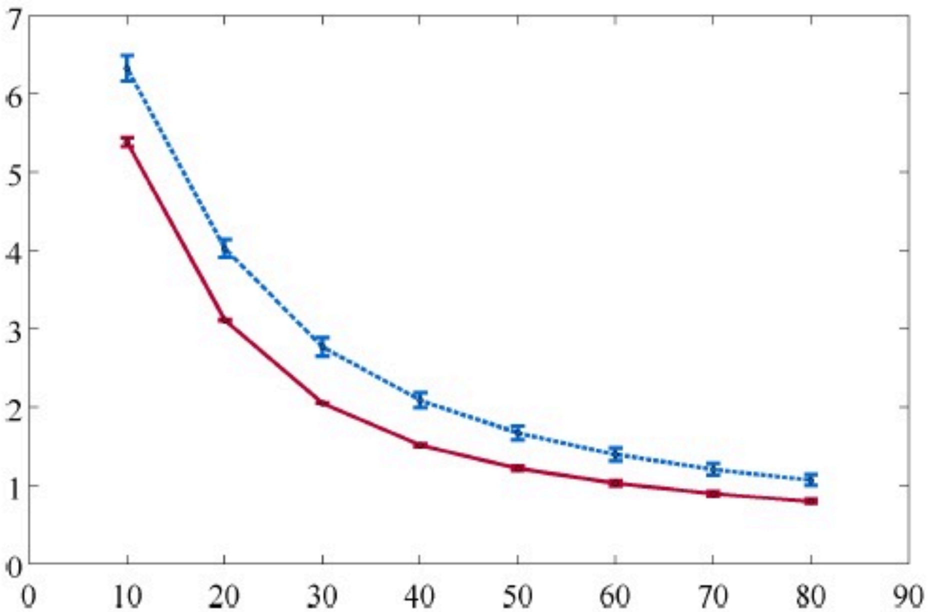


Spherical harmonic representation

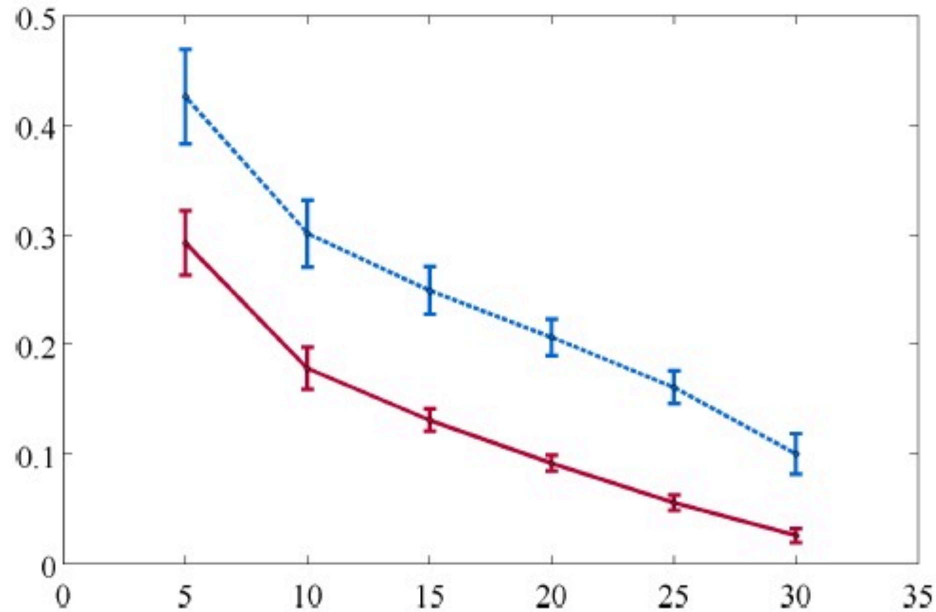
Reconstruction error

Red= LB-expansion
Blue= SPHARM

mm



Cortical surfaces



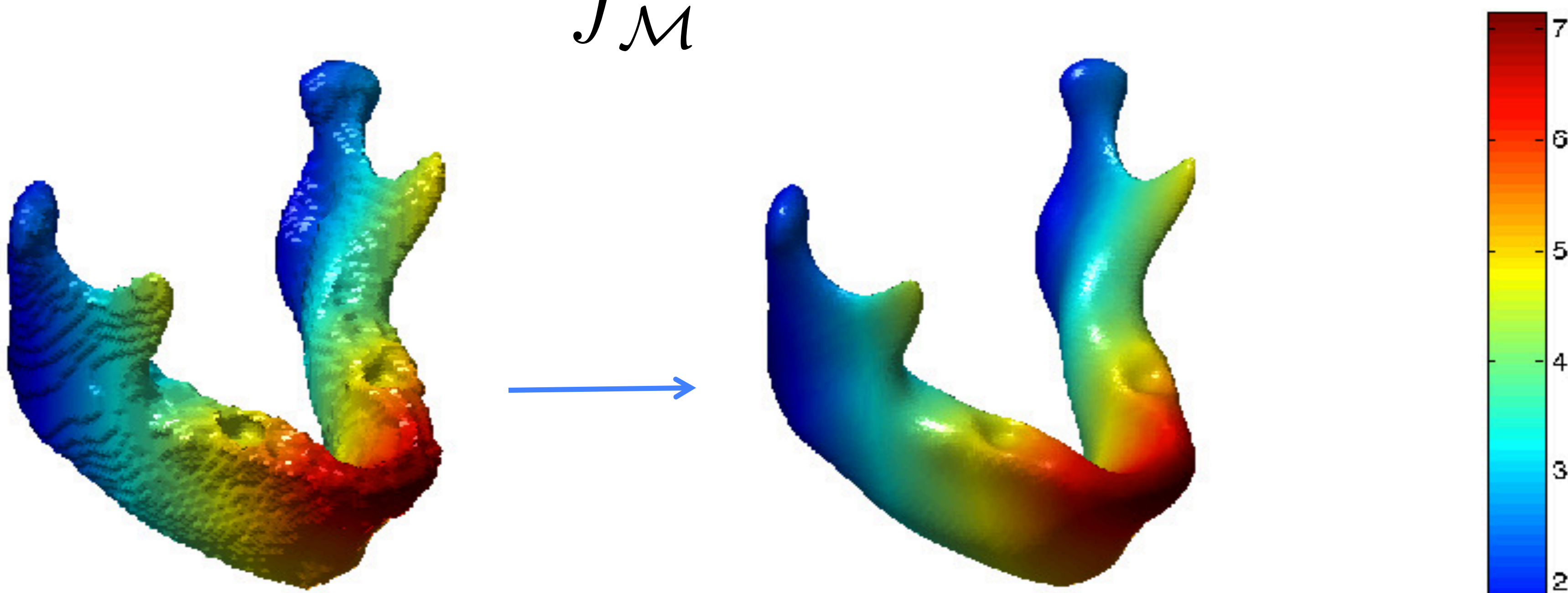
Amygdala surfaces

Seo at al. 2010. ISIB

Heat kernel smoothing on manifold

Heat kernel: $$K_t(p,q) = \sum_{i=0}^{\infty} e^{-\lambda_i t}\psi_i(p)\psi_i(q)$$

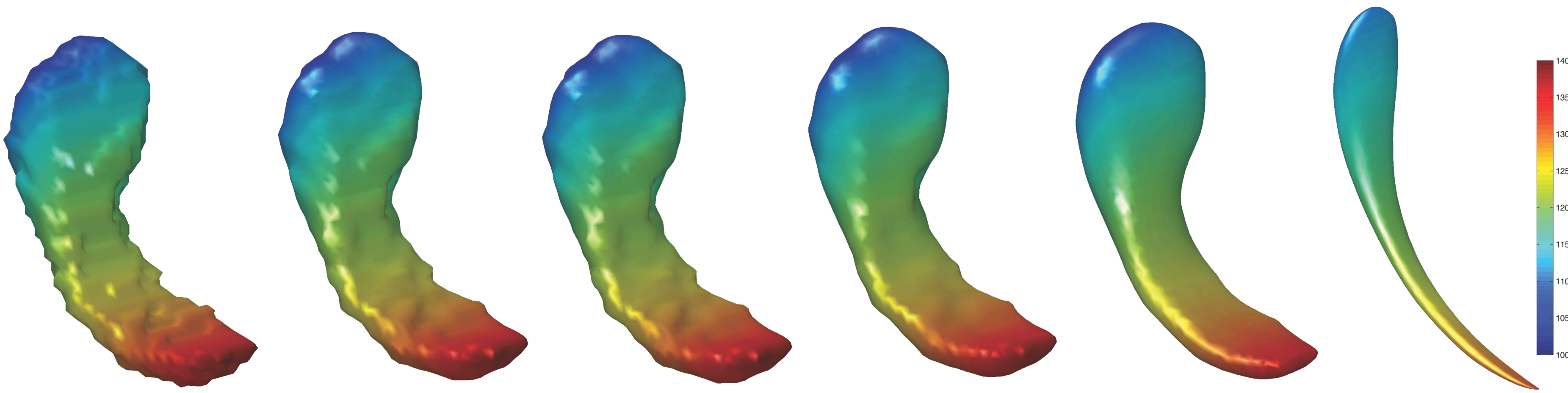
$$K_t * f = \int_{\mathcal{M}} K_t(p,q) f(q)\, dq$$



x-coordinate on mandible surface

smoothed with bandwidth 10 and 1269 eigenfunctions

Heat kernel smoothing of hippocampus

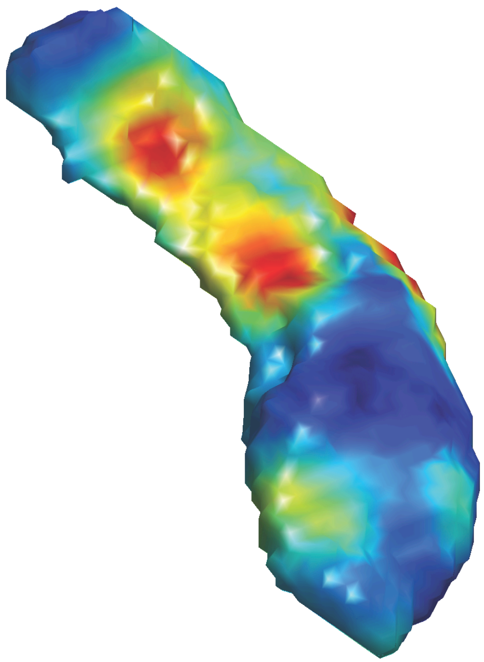


Original 0.01 0.1 1 10 100

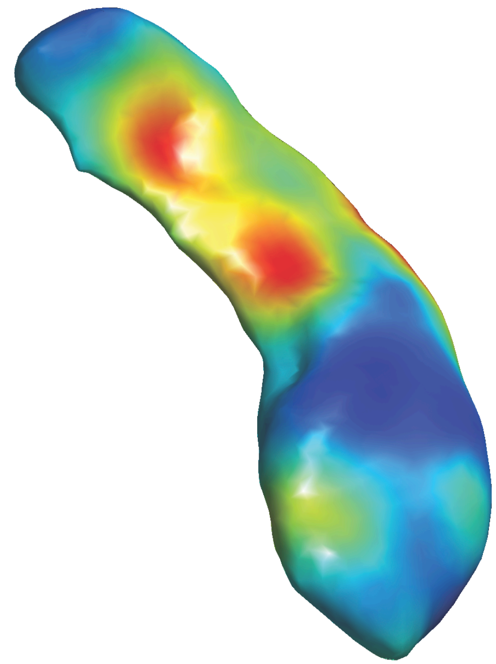
↑

Smoothing scale
used in the study

Heat kernel smoothing on hippocampus surface



Original
deformation field



Heat kernel
smoothing with
bandwidth = 1

Mixed Effect Modeling

y_{ij} *i*-th subject, *j*-th scan (*j*=1,2)

Fixed effect model:

$$y_{ij} = \beta_0 + \beta_1 age_{ij} + \epsilon_{ij}$$

Mixed effect model:

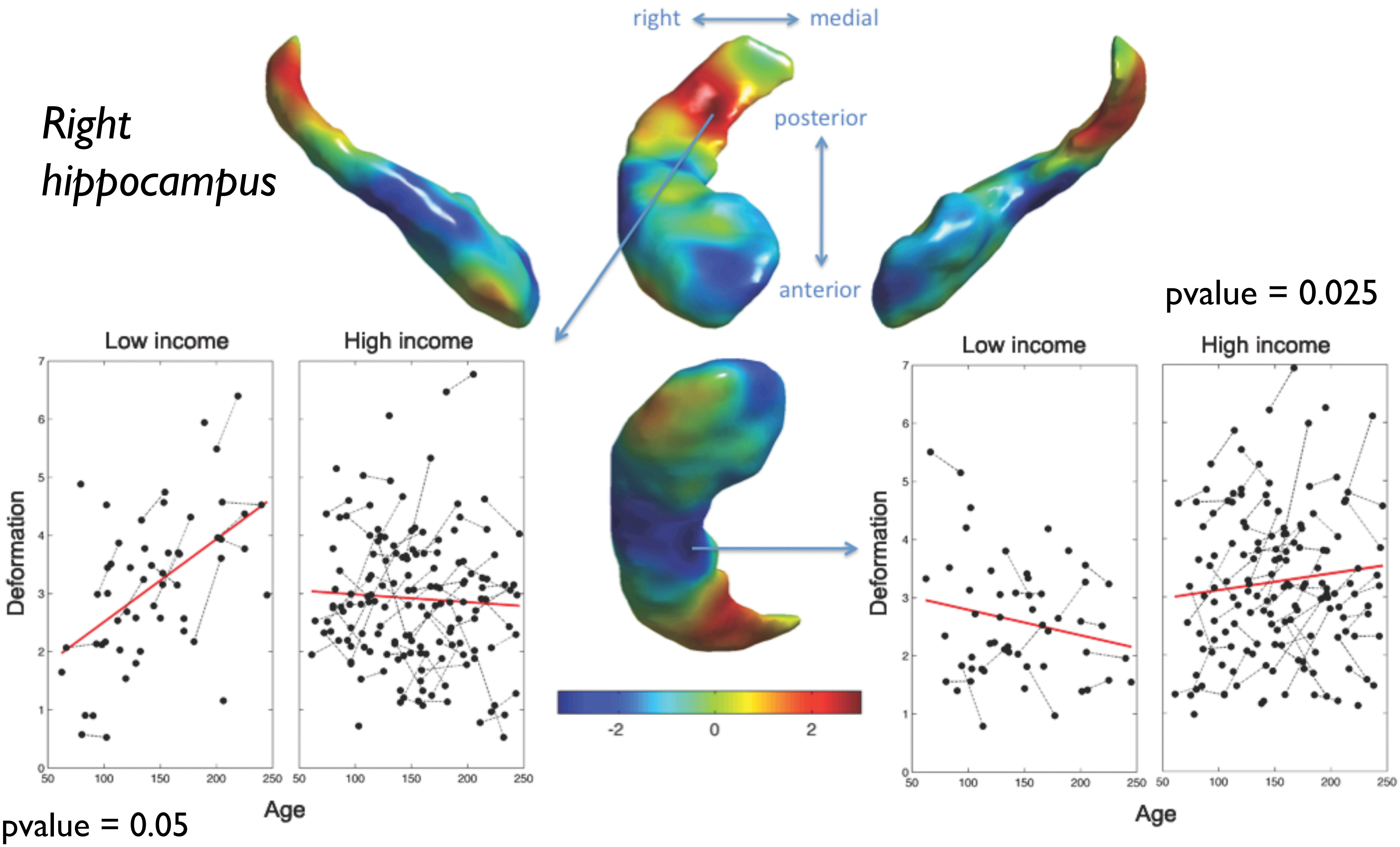
$$y_{ij} = \beta_0 + \gamma_{i0} + (\beta_1 + \gamma_{i1}) age_{ij} + \epsilon_{ij}$$

Each subject has its own growth intercept and slope.

Results

Effect of family income on hippocampus growth

Right hippocampus



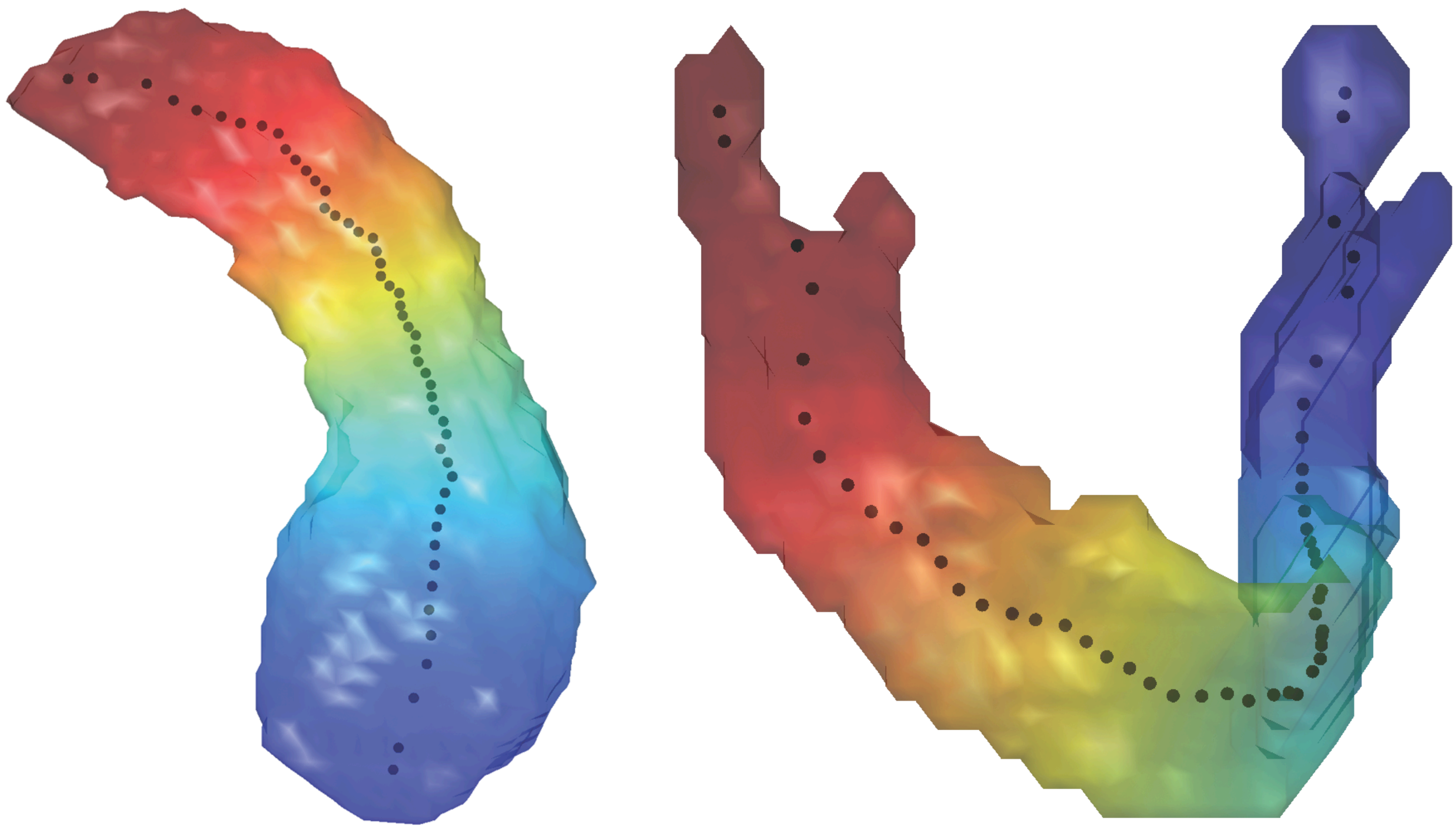
Chung et al. HBM 2011

Conclusion

Low income level
→ Adverse environment
→ Stress
→ Abnormal hippocampus growth

Next Project

Second eigenfunction of elongated objects



Hot spot conjecture in differential geometry:
Min and max of the second eigenfunction always occur at the extreme end points or boundary

Rauch's hot spot conjecture unsolved since 1974:

$$K_\sigma * f(p) = \frac{\int_{\mathcal{M}} f(p)\, d\mu(p)}{\mu(\mathcal{M})} + f_1 e^{-\lambda_1 \sigma} \psi_1(p) + R(\sigma, p)$$

Mean signal
over manifold

Error term

The heat kernel smoothing asymptotically behaves like the second eigenfunction.

Thank you

MATLAB codes can be downloaded from
www.stat.wisc.edu/~mchung