Computational Methods in NeuroImage Analysis

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Lecture 8
Geometric computation

October 29, 2010

NOTICE

Final Exam: December 3 9:00-12:00am (35%)

Topics: Covers everything discussed in lectures (lecture notes + required reading)

Open book exam: you can bring laptops, calculators, books or even your puppy.

Previous exam in the directory /exam/

NOTICE

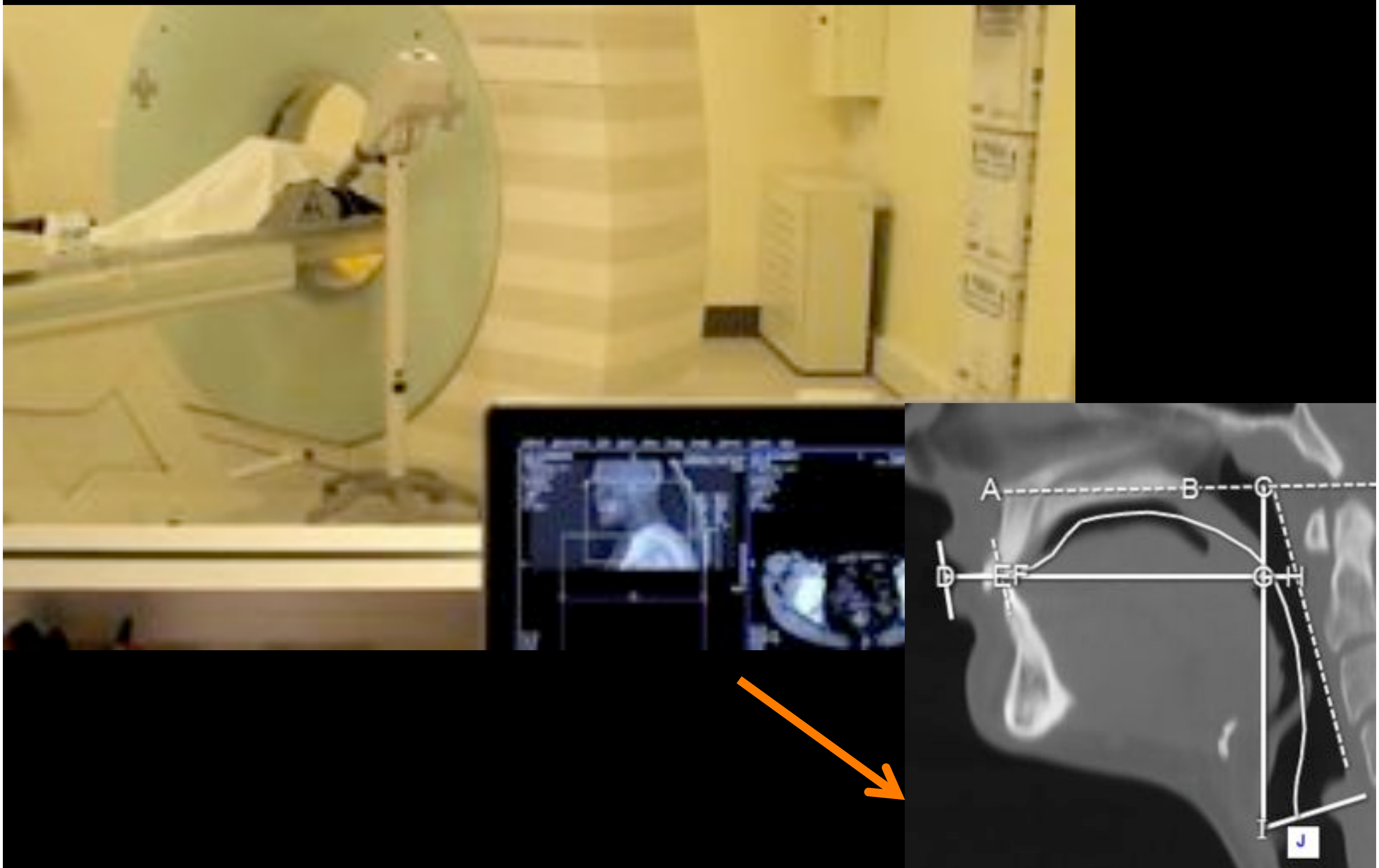
Oral presentation (15%)
December 10 9:00-12:00am .
Each student will present 20min.

Final report submission (40%)
Deadline: December 17 9:00am.
10% penalty/day after 9:00am.

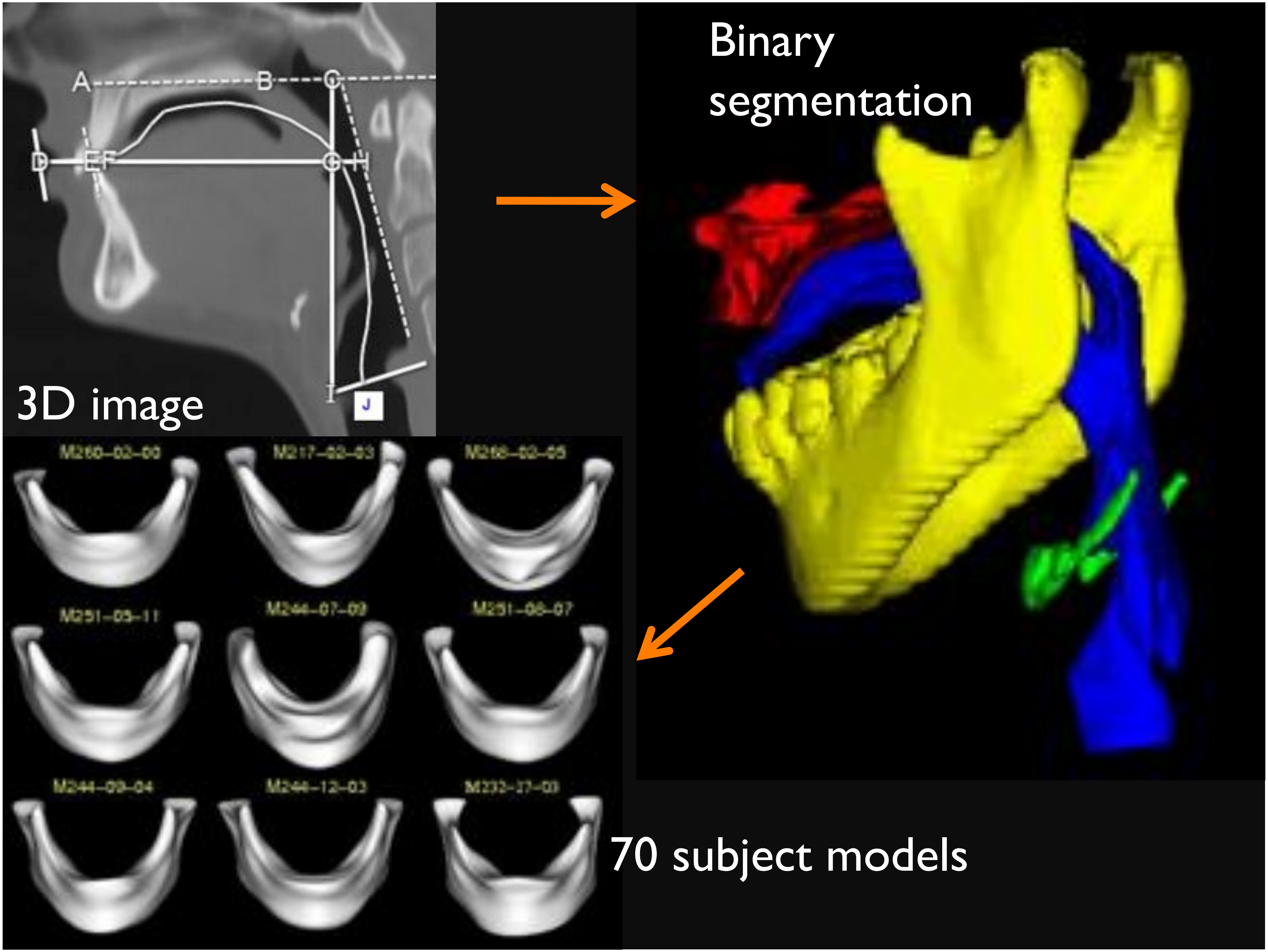
Sample final reports in the directory /projects

3D images to surface models

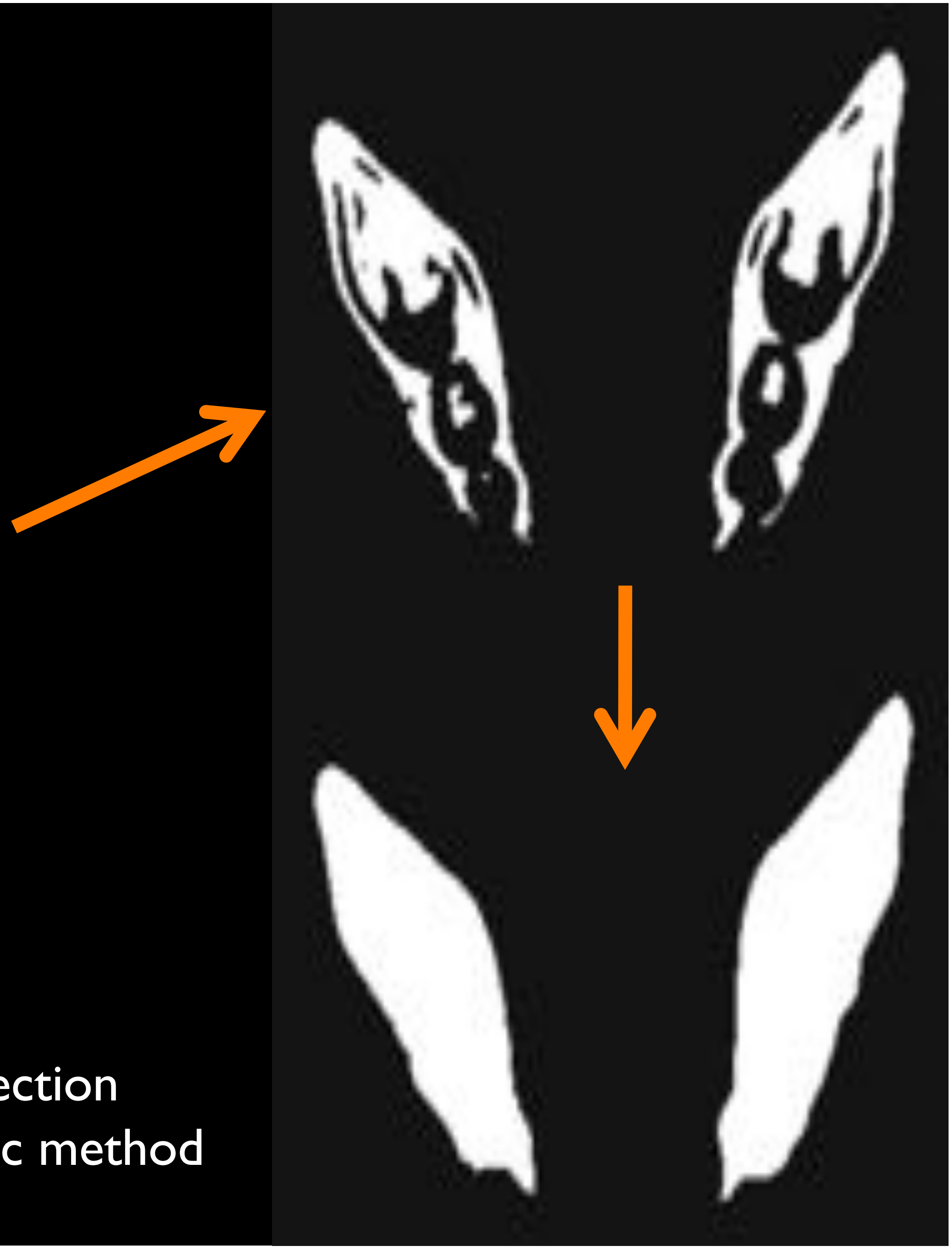
Computed Tomograpy (CT) scanner



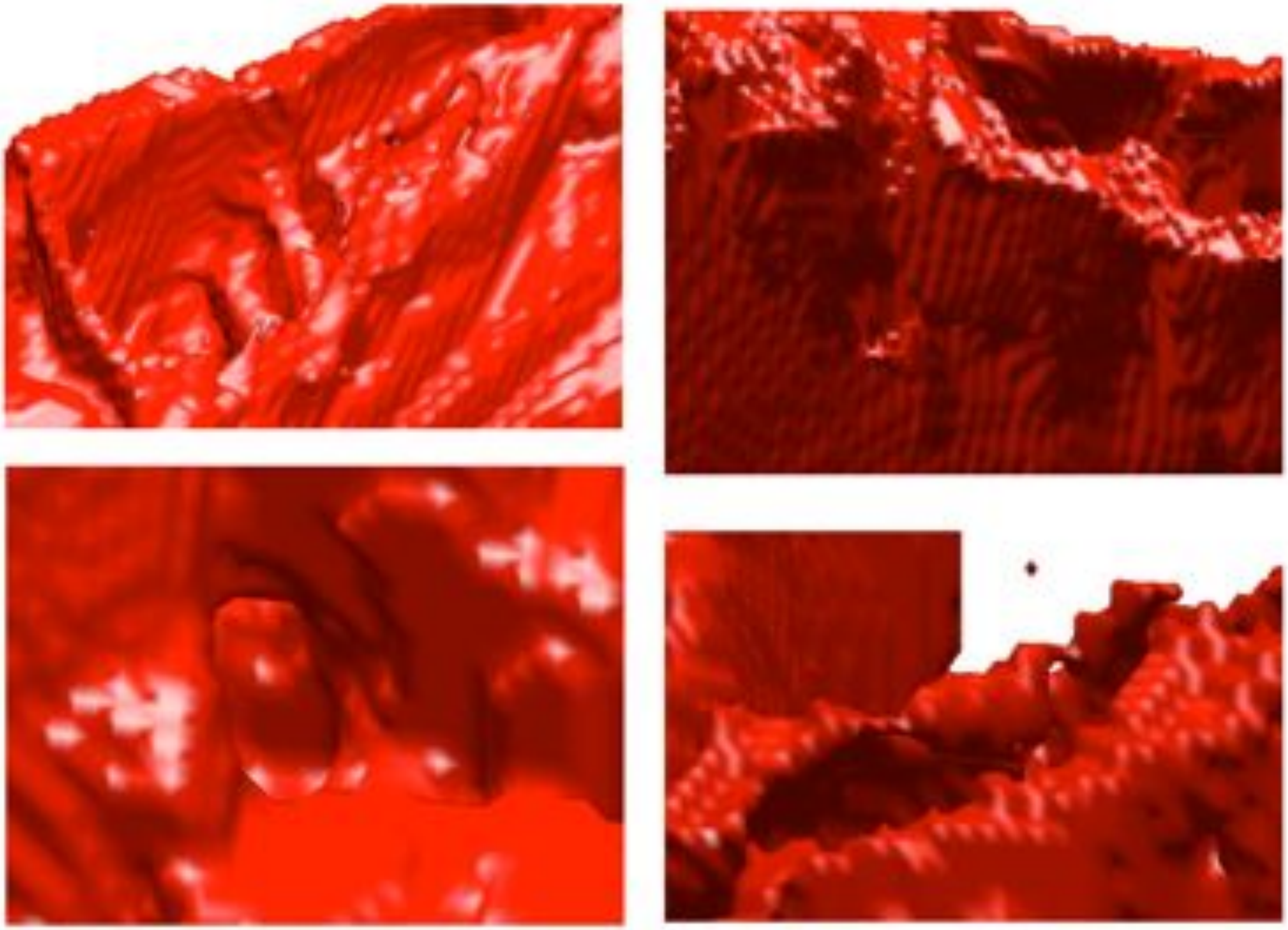
3D image


3D image
Binary segmentation
70 subject models

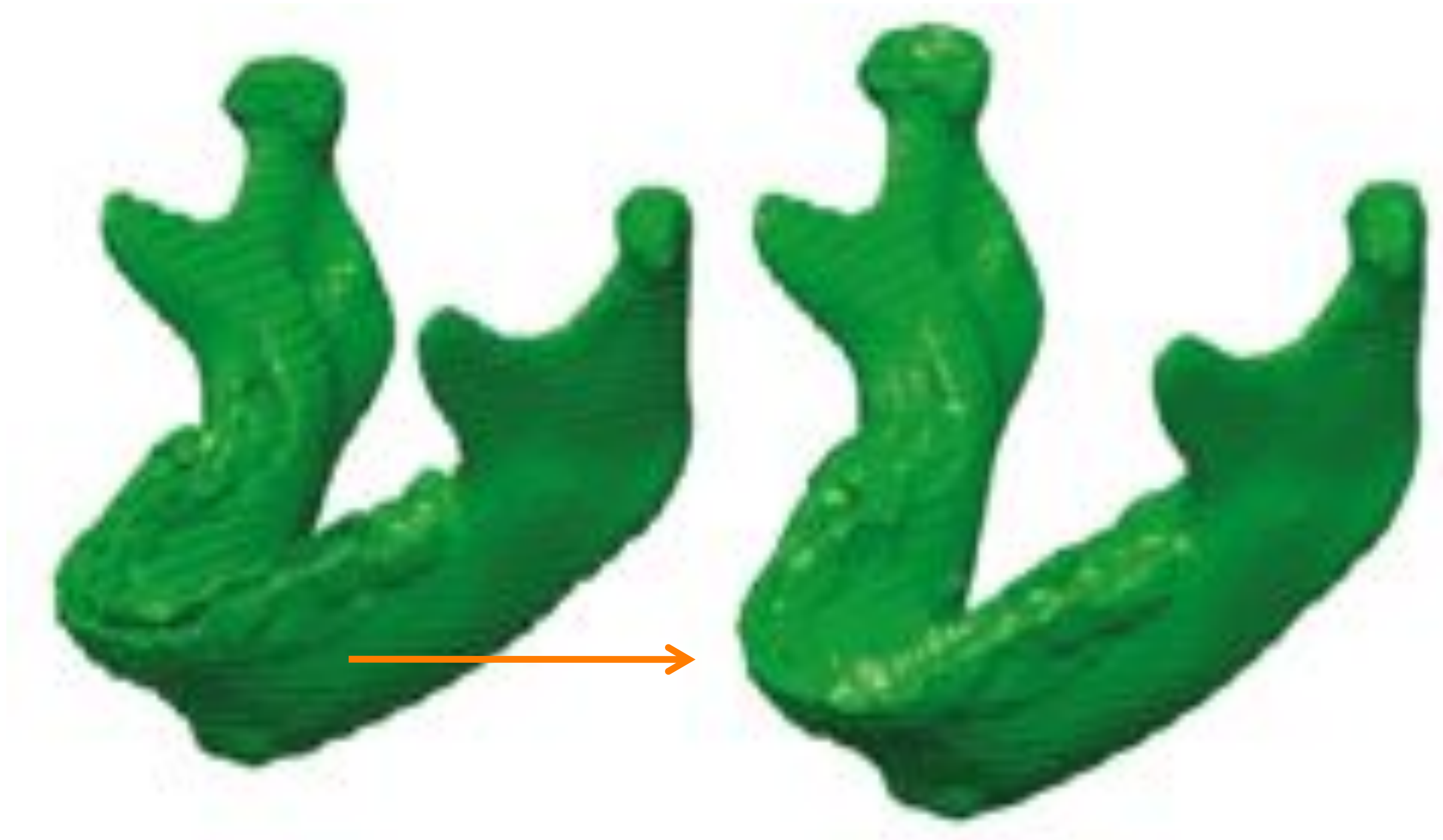
Topological defects



Automatic topological correction using the Euler characteristic method

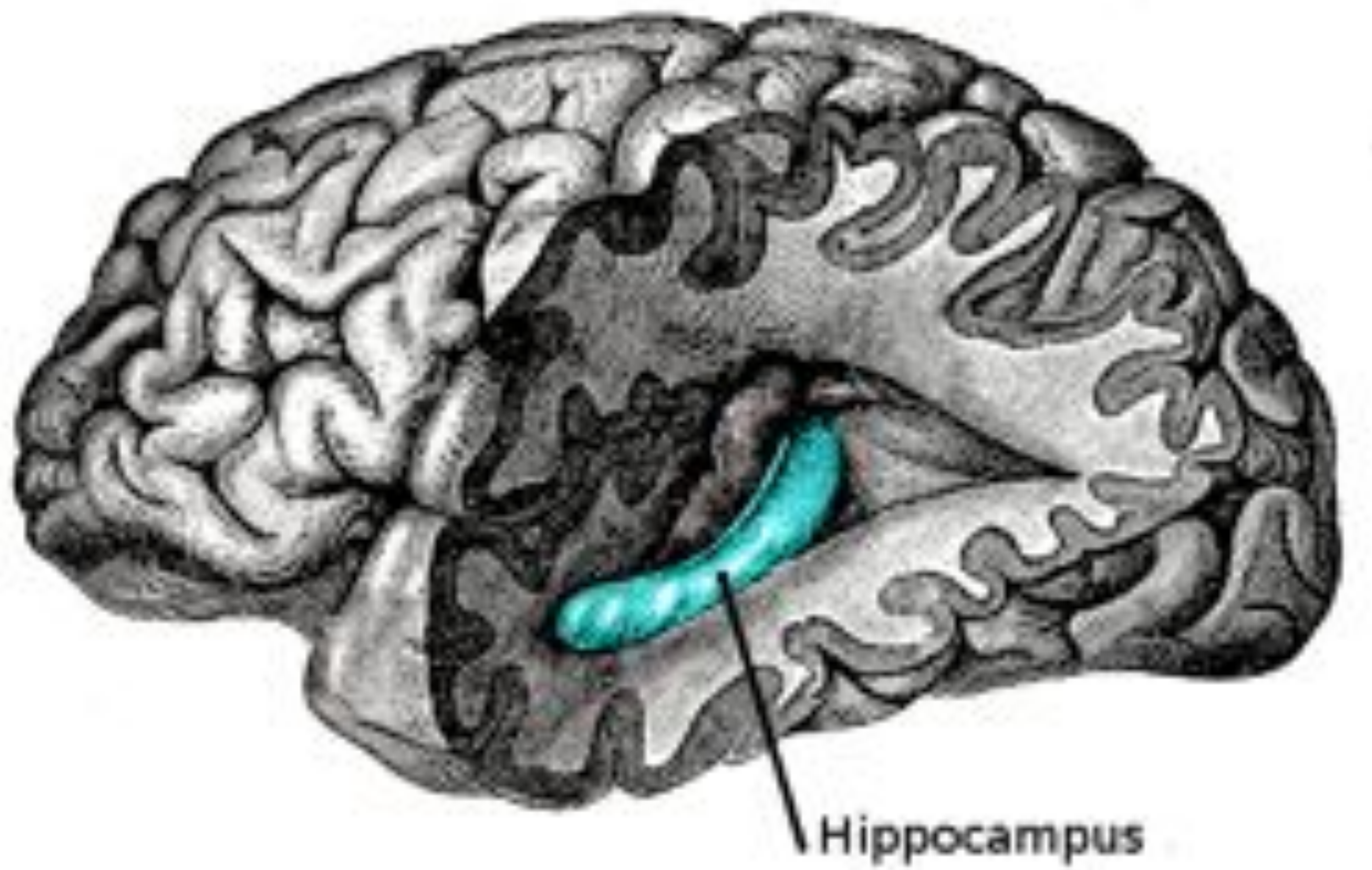


Holes and handles in teeth regions need to be fixed for subsequent image processing and analysis.



Additional morphological closing operation was done to patch up the space that was occupied by teeth. Without this morphological operation, the final statistical result will be highly biased in teeth regions.

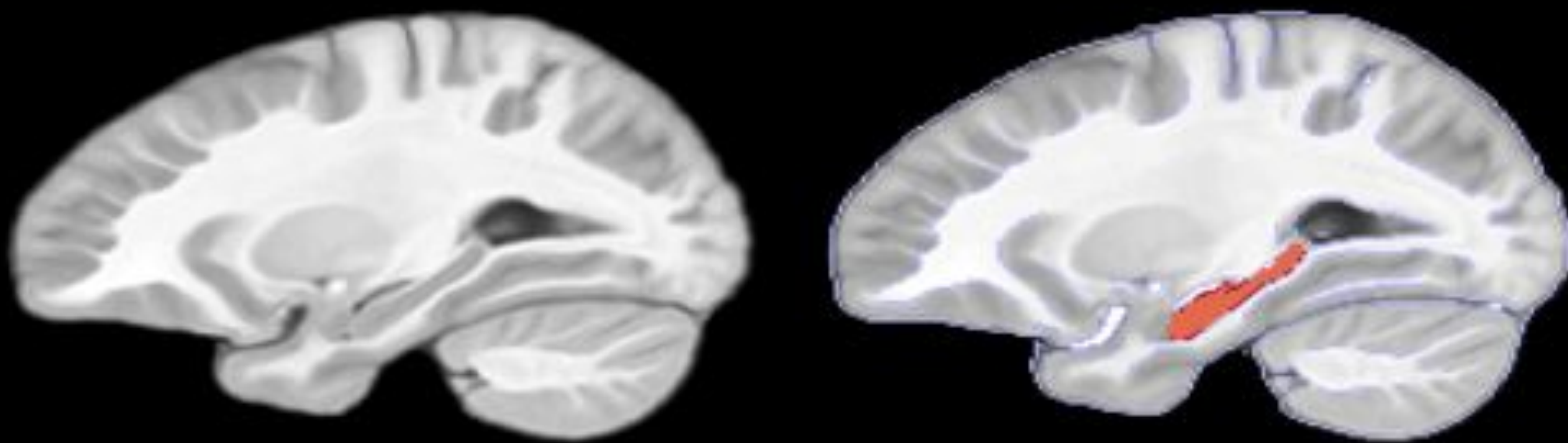
Limbic system associated with long-term memory and spatial navigation

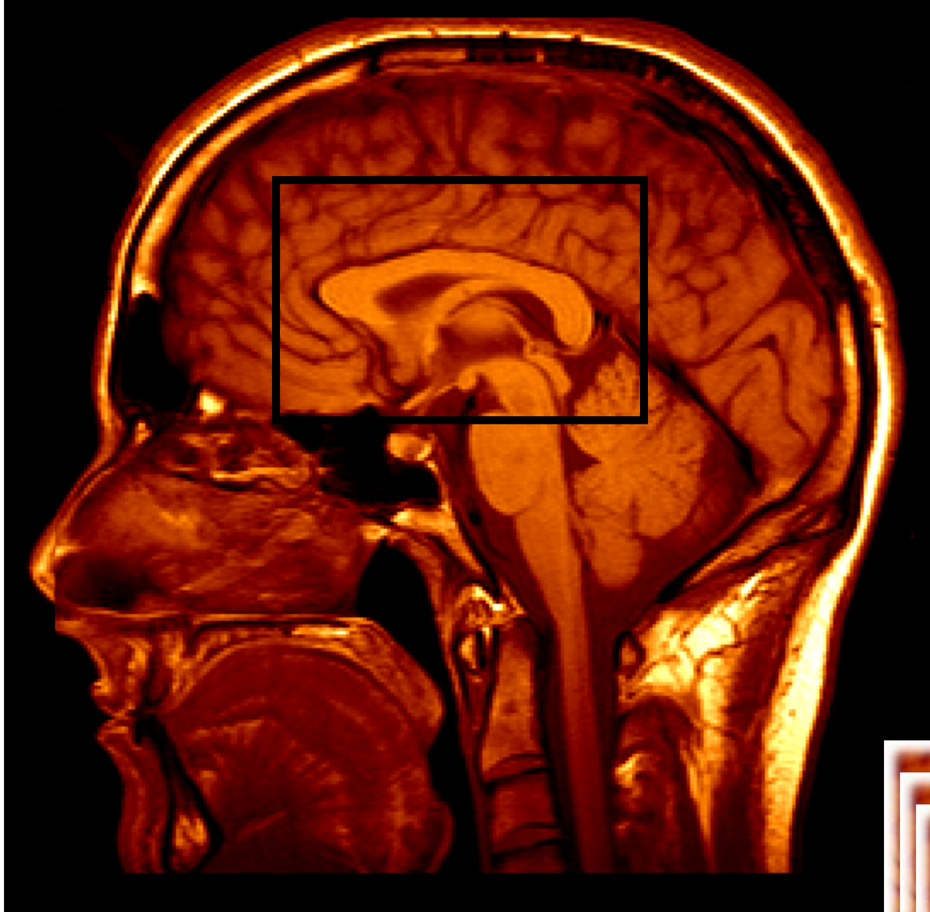


3.0 Tesla GE Scanner

3D image

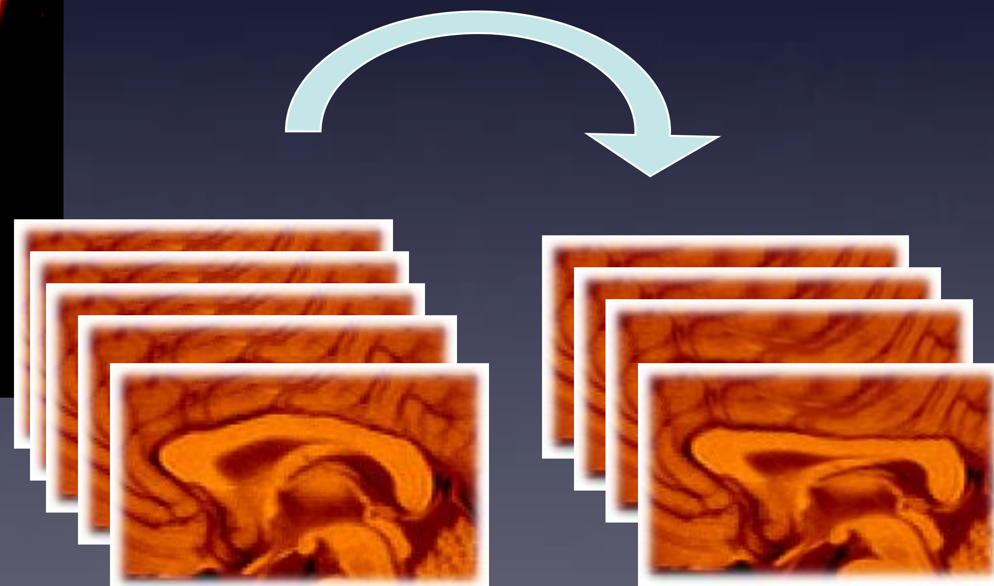
Manual hippocampus segmentation on MRI template





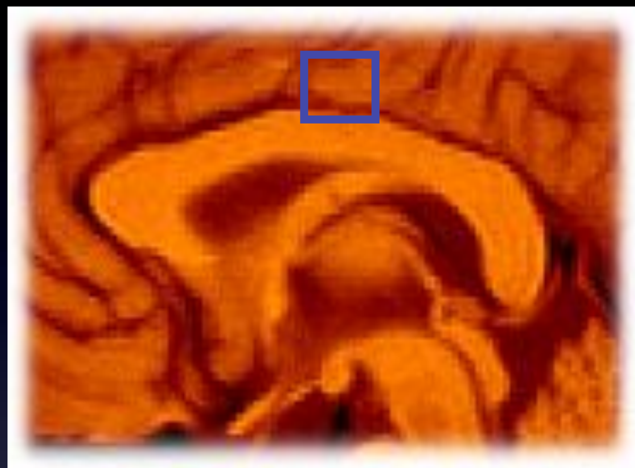
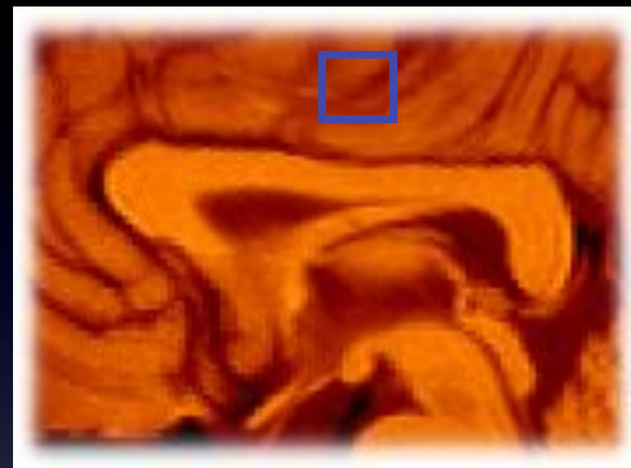
Each subject has different brain shape. So how do we compare across subjects?

Group comparison ?



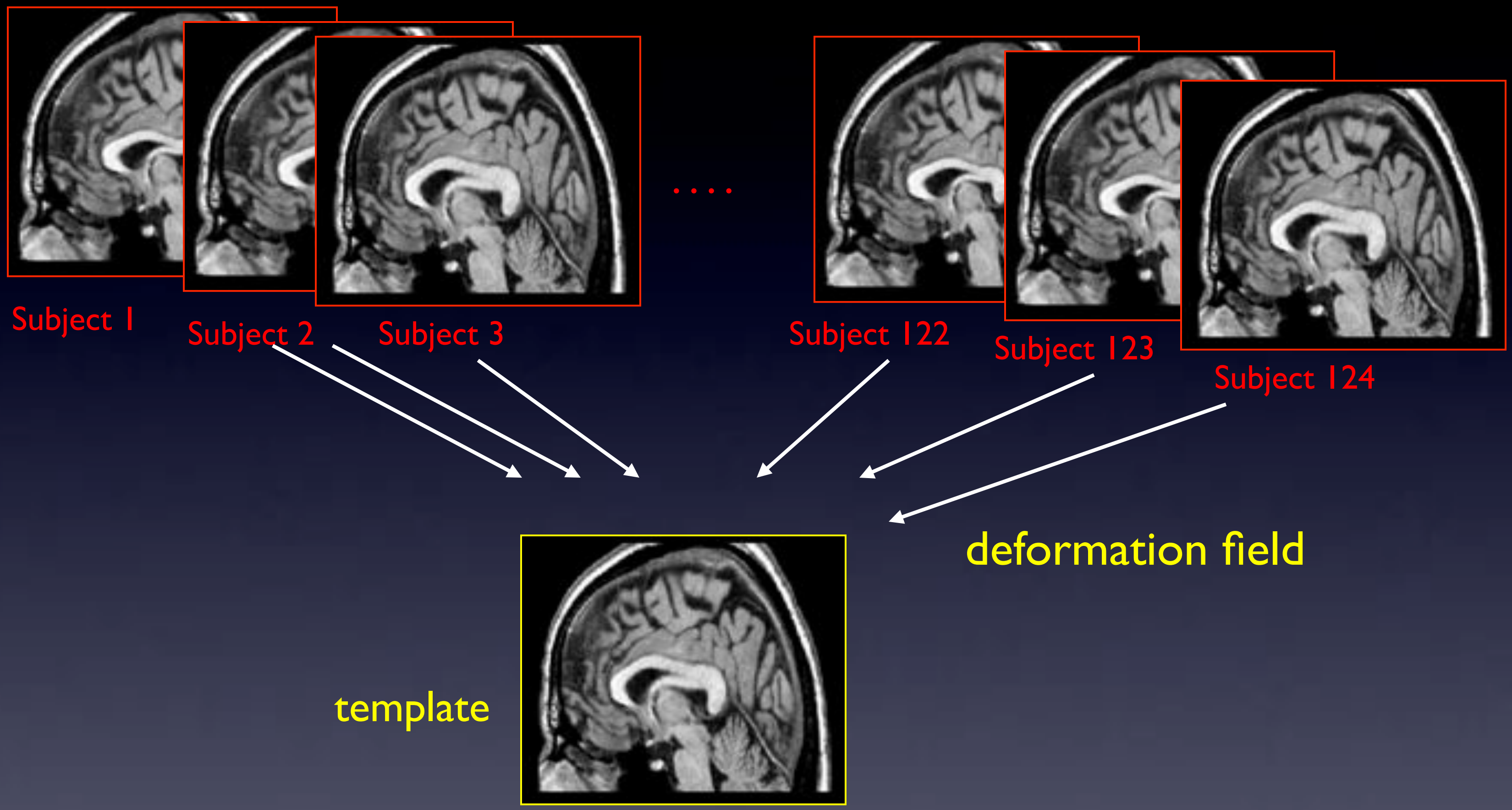
Group 1

Group 2

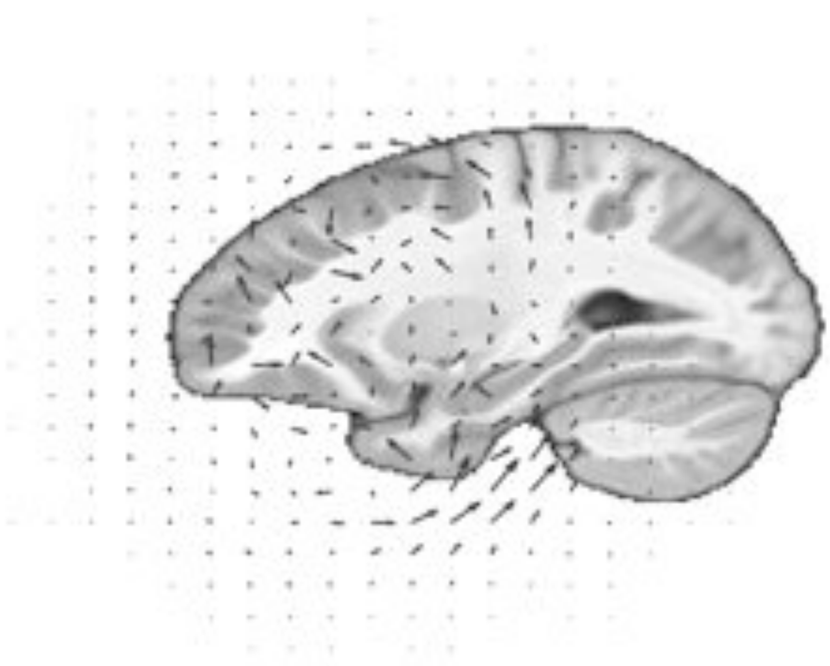
Voxel by voxel comparison causes anatomical mismatching.

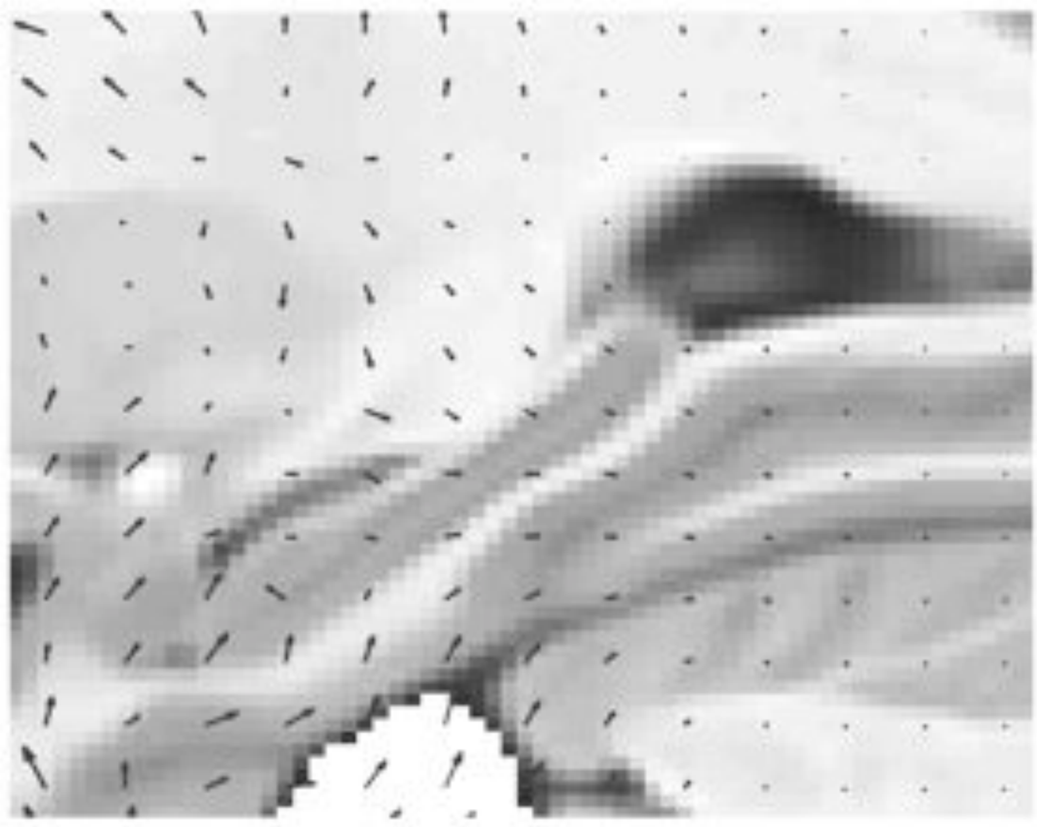
Deformable template framework



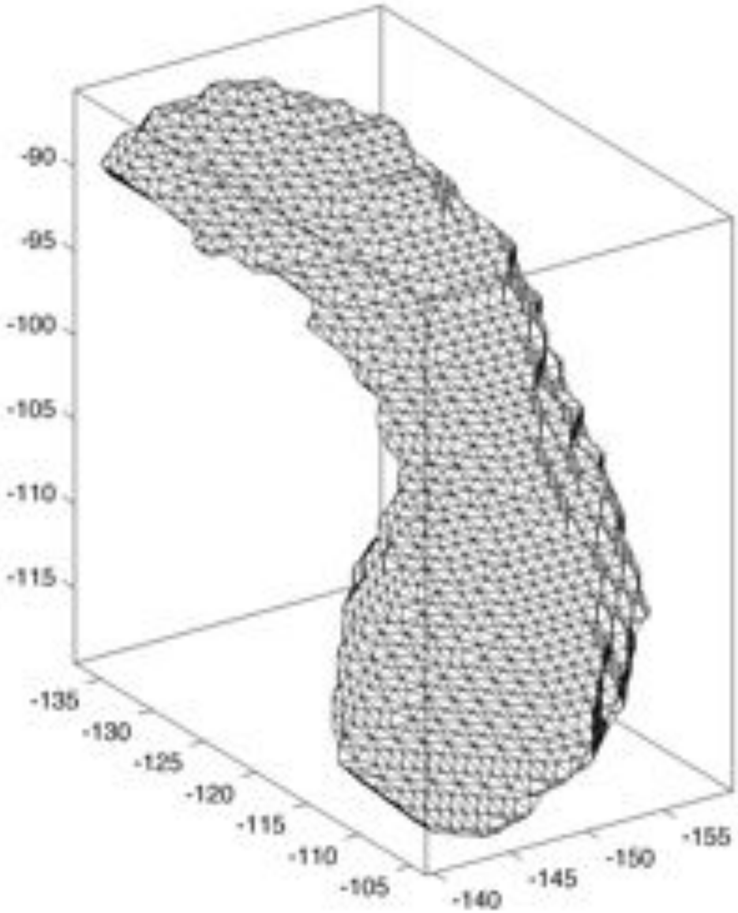
MRIs will be warped into a template and anatomical differences can be compared at a common reference frame.

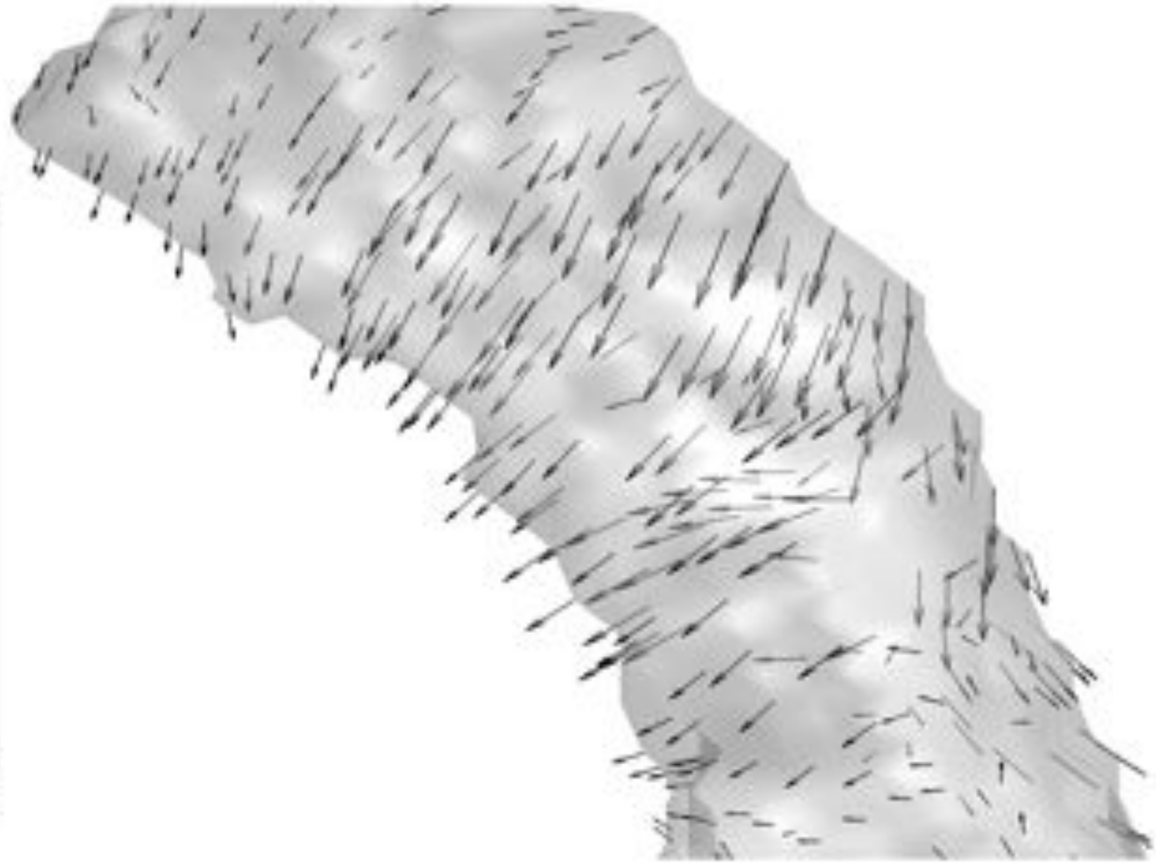
Image registration
Deformation from the template to a subject.
sample size = 124 subjects





Left hippocampus surface template





Deformation field
of warping the template
to a subject

MATLAB demonstration

Basis functions on surface

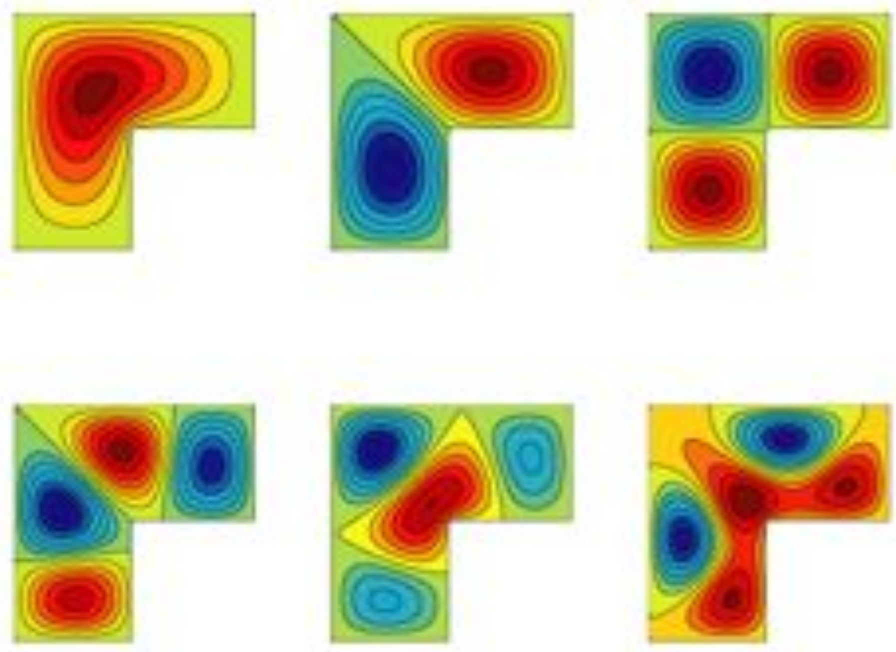
How to compute basis in an arbitrary domain

Steady-state oscillations in wave equation

↓

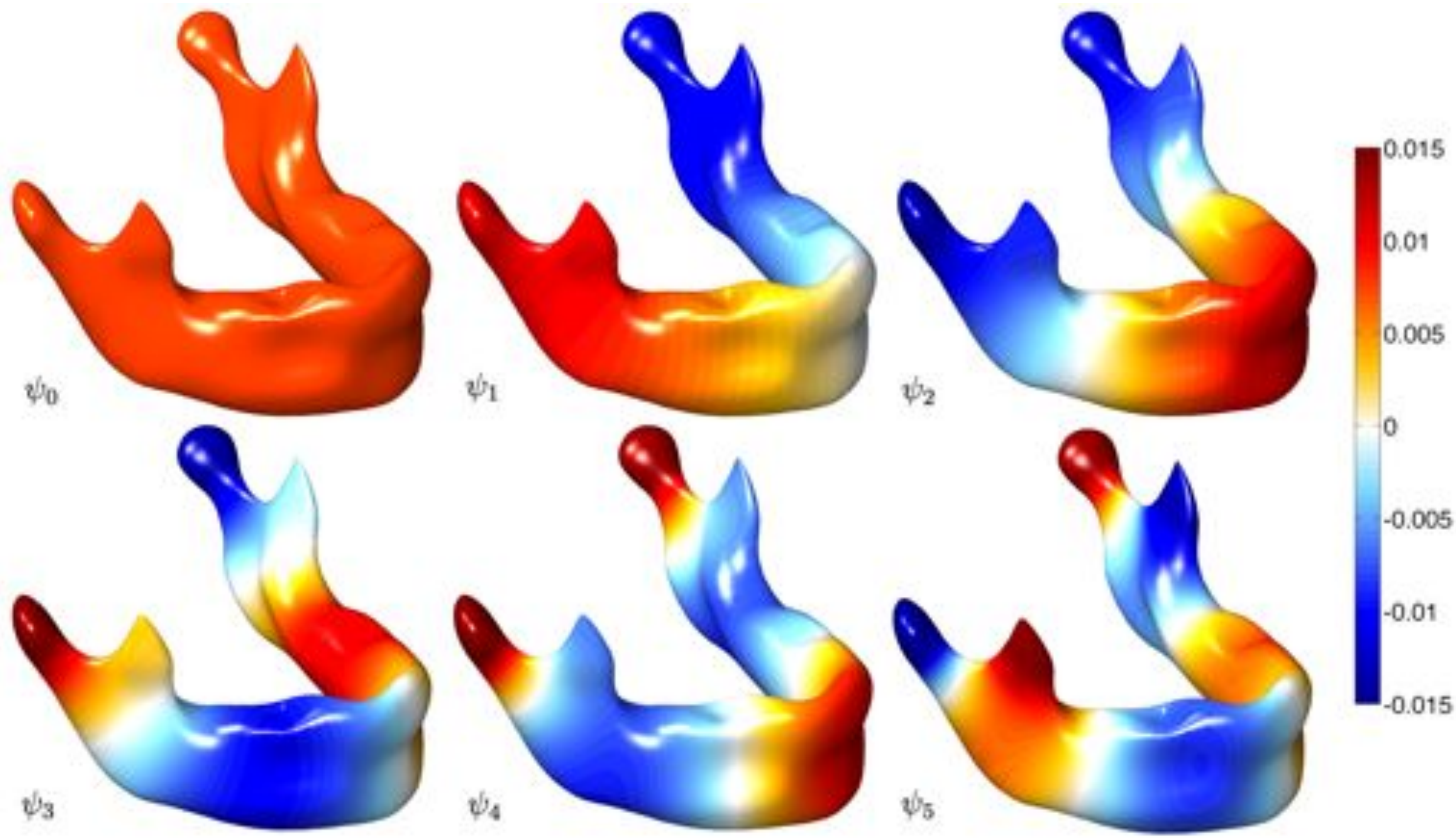
Helmholtz equation $$\Delta_X F = \lambda F$$

↓



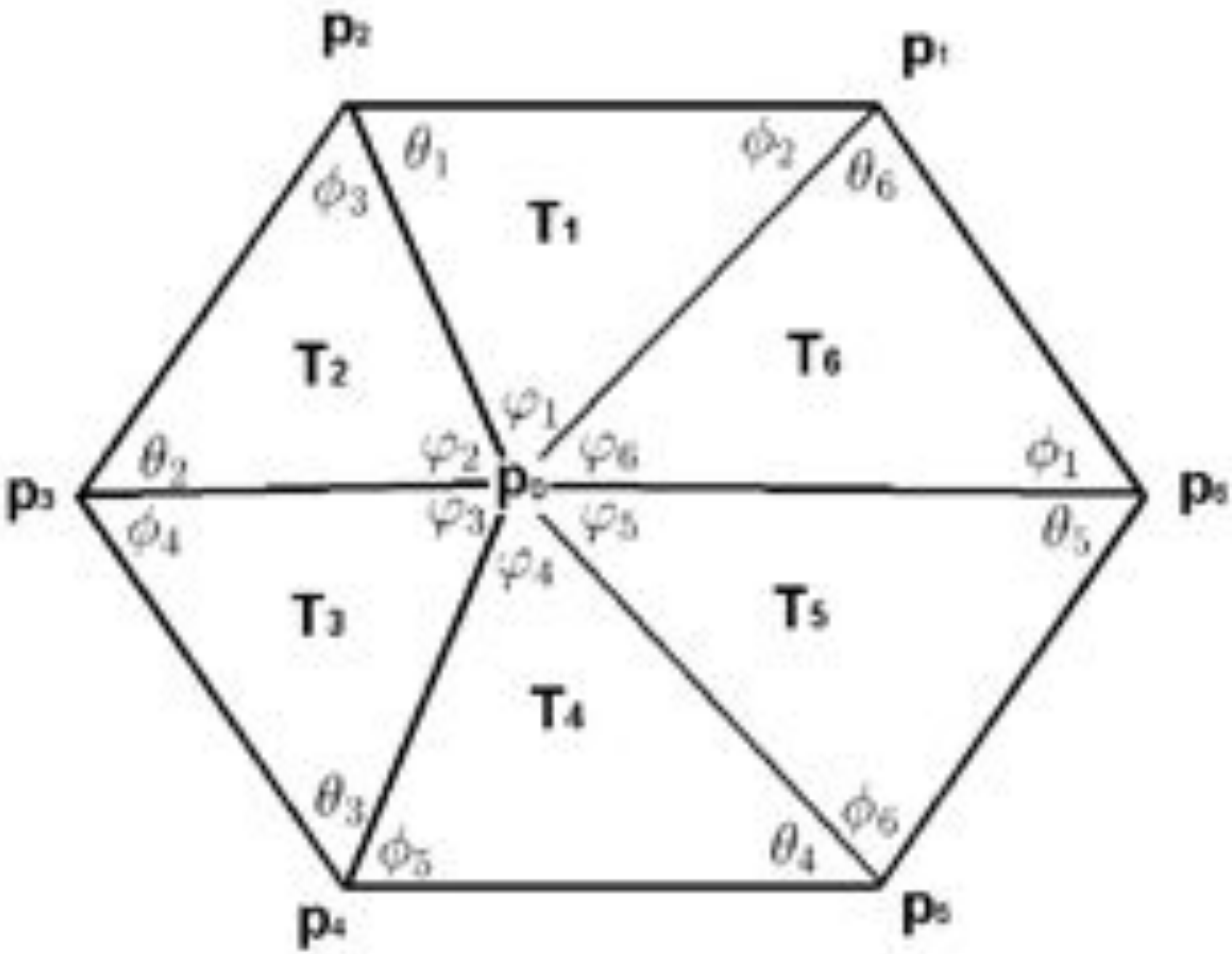
Basis functions in the L-shaped membrane

Orthonormal basis

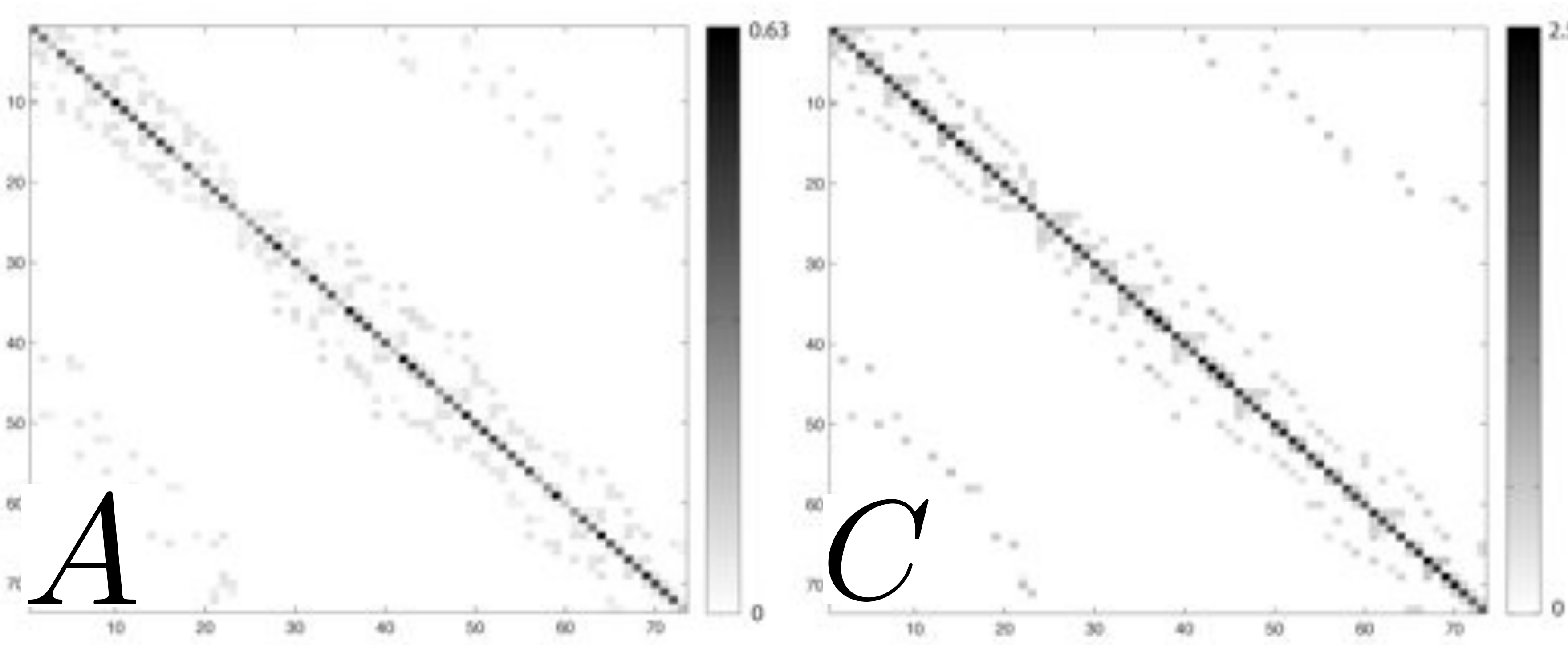


$$\Delta f = \lambda f$$

Finite Element Method

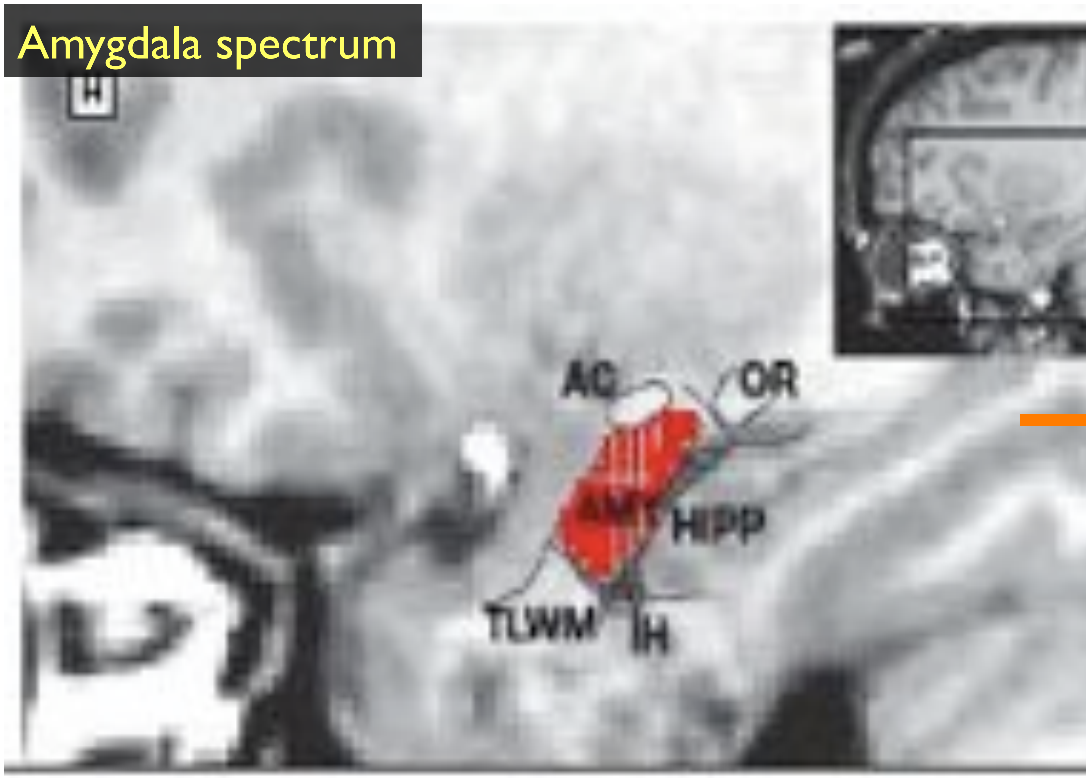


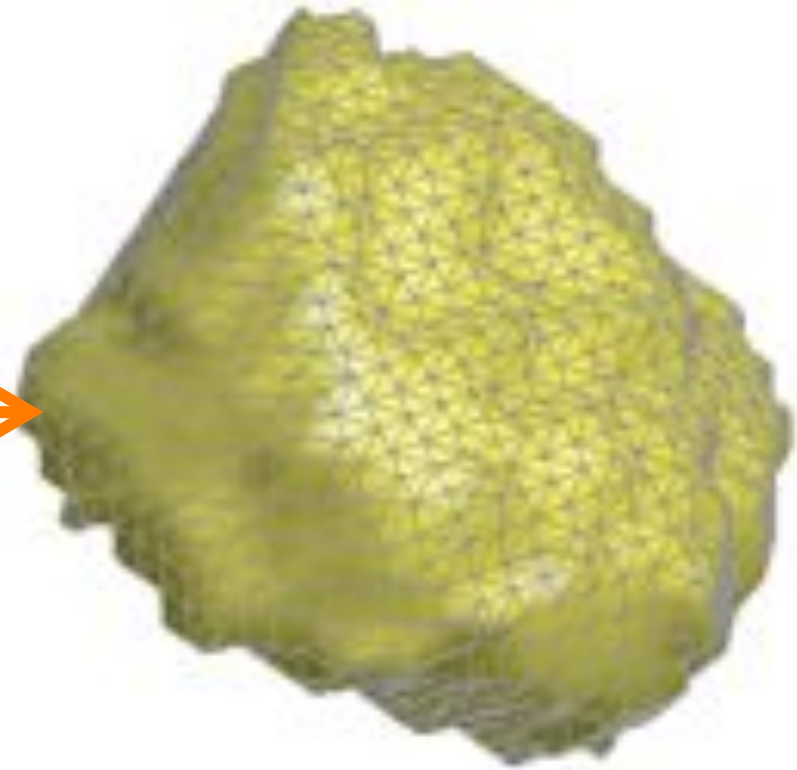
Finite Element Method (FEM)



$$\Delta f = \lambda f \longrightarrow C\psi = \lambda A\psi$$

Amygdala spectrum





Generalized eigenvalue problem

$$\Delta_X F = \lambda F$$

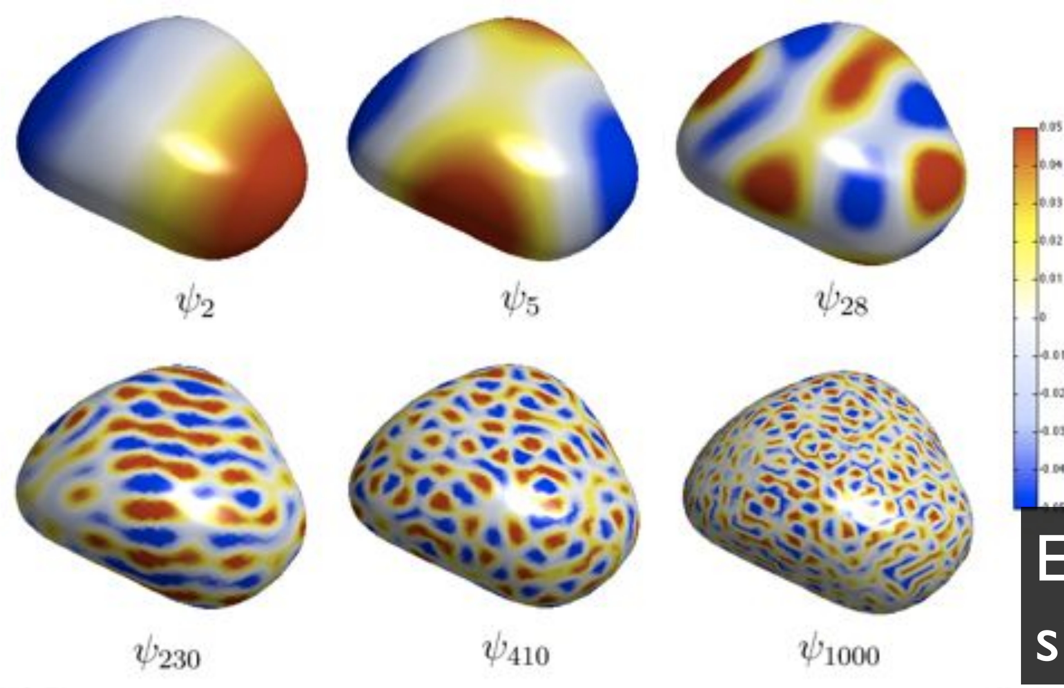
discretization

$$\lambda A \psi = C \psi$$

$$a_{ii} = \frac{1}{12} \sum_{p_j \in N(p_i)} T_{ij}^{+} + T_{ij}^{-}$$

$$c_{ii} = \frac{1}{2} \sum_{p_j \in N(p_i)} (\cot \theta_{ij} + \cot \phi_{ij})$$

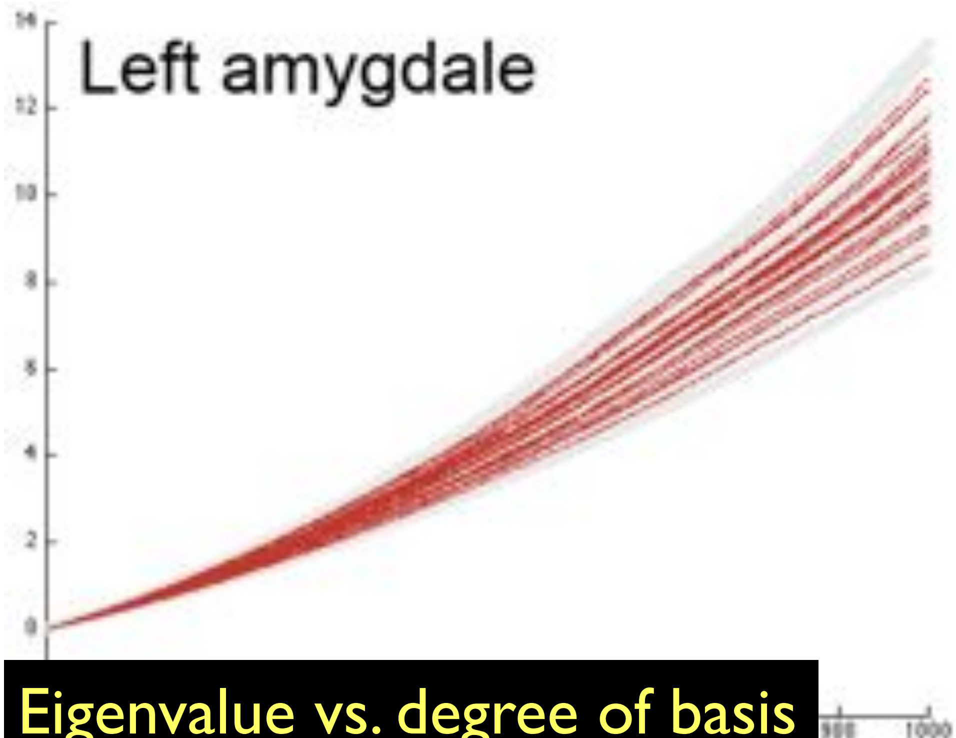
PhD thesis (2001), ISBI (2004) Qiu et al. (2005, IEEE TMI)

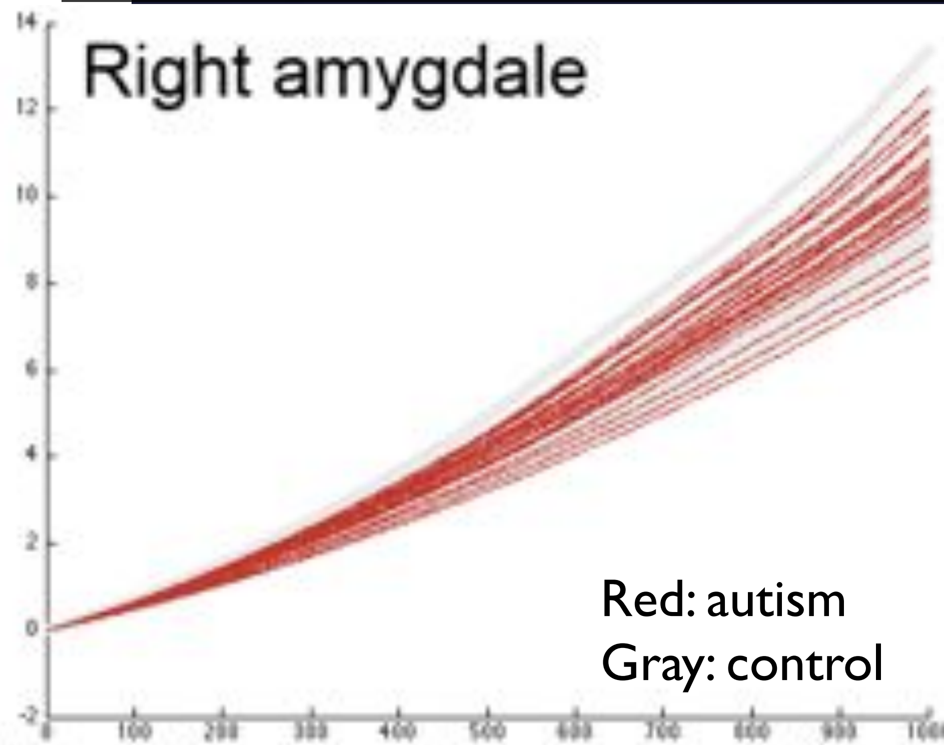


Weyl's formula

$$\lambda_k \to \frac{4\pi k}{\mu(\mathcal{M})}$$

Eigenvalues can't discriminate similarly shaped objects.





Eigenvalue vs. degree of basis

Finite Element Method (FEM)

Let N_T be the number of triangles in the mesh that approximates the underlying manifold $\mathcal{M}$. We seek a piecewise differentiable solution f_i in the i-th triangle T_i such that the solution $f_i(x)$ is continuous across neighboring triangles. A slightly different formulation of FEM for the surface flattening problem is given in (Angenent *et al.*, 1999). The solution f for the whole mesh is then

$$f(x) = \sum_{i=1}^{N_T} f_i(x).$$

Let $p_{i_1}, p_{i_2}, p_{i_3}$ be the vertices of element T_i. In T_i, we estimate f_i linearly as

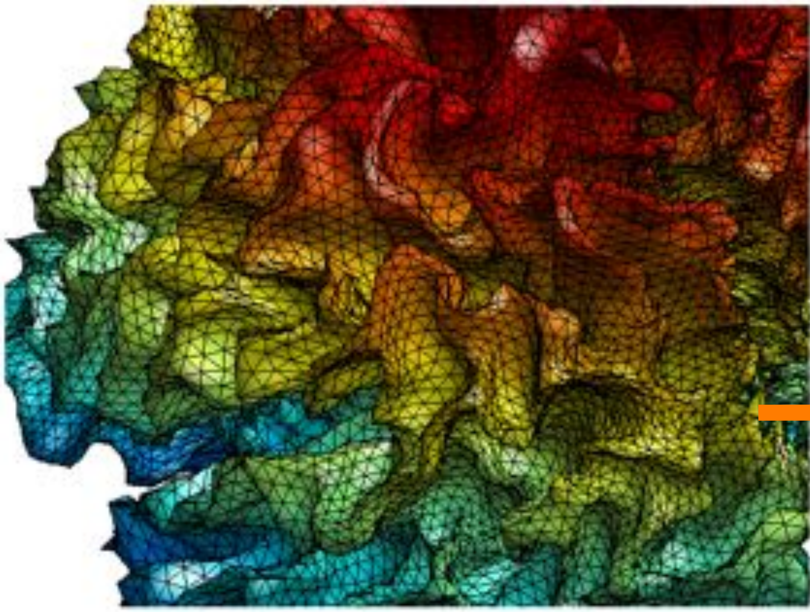
$$f_i(x) = \sum_{k=1}^{3} \xi_{i_k} f(p_{i_k}),$$

where nonnegative ξ_{i_k} are given by the *barycentric coordinates* (Chung, 2001; Sadiku, 1989, 1992; Tang *et al.*, 1999). In the barycentric coordinates, any point $x \in T_i$ is uniquely determined by two conditions:

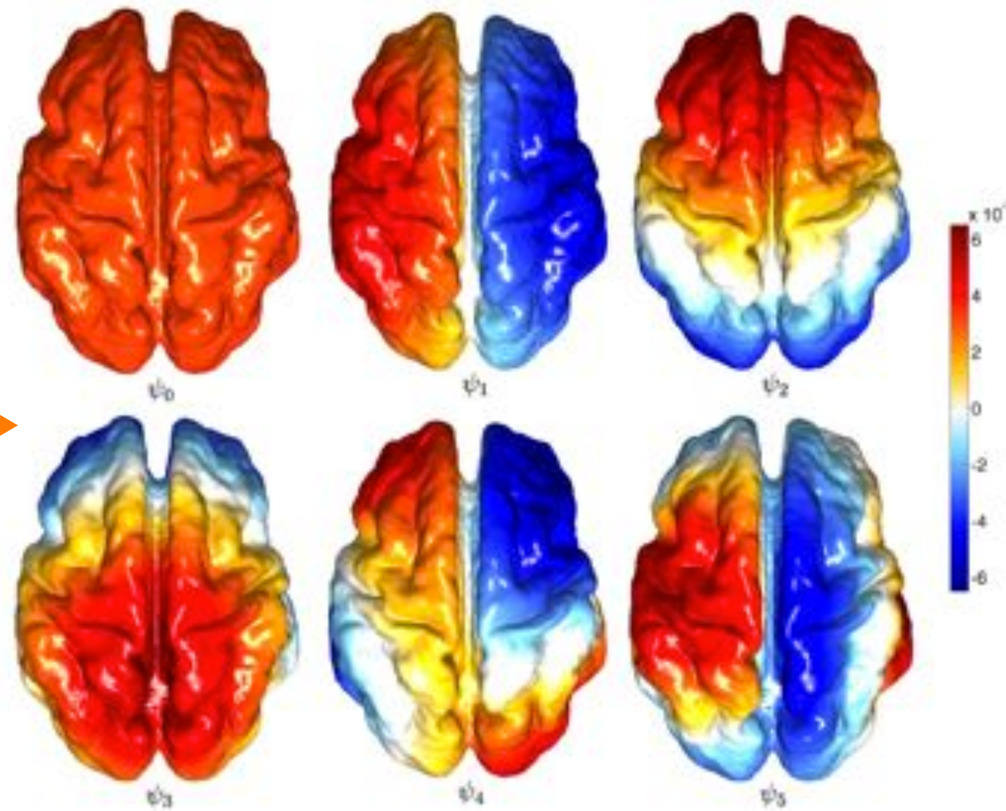
$$x = \sum_{k=1}^{3} \xi_{i_k}(x) p_{i_k}, \quad \sum_{k=1}^{3} \xi_{i_k}(x) = 1.$$

Let's see chapter 4.6

Limitation: Computational bottleneck



$$\lambda A\psi = C\psi$$



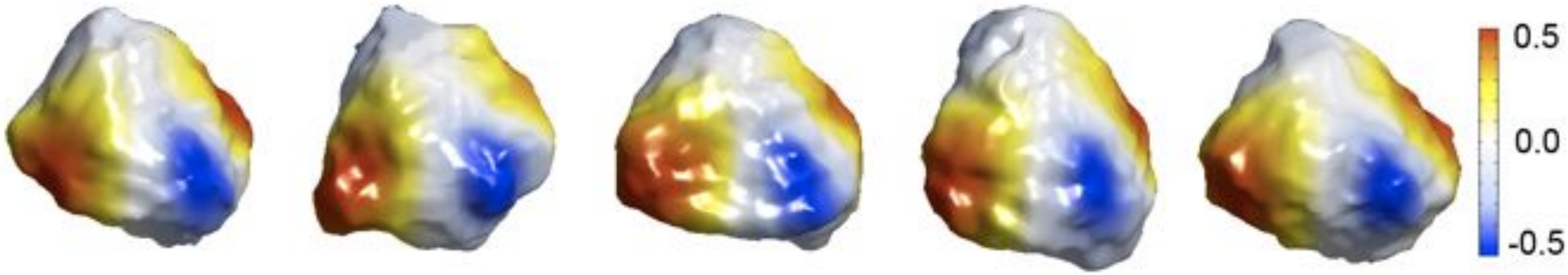
Challenge: Solve the above matrix equation for extremely large matrices.

Implicitly Restarted Arnoldi method

First 10 orthonormal basis on left hippocampus

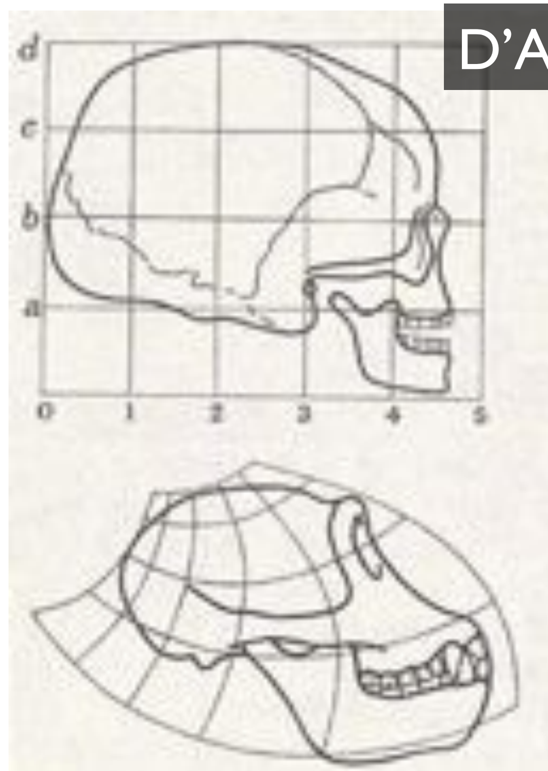
Basis functions will be used to construct a smoothing kernel.

Limitation: No common coordinate system



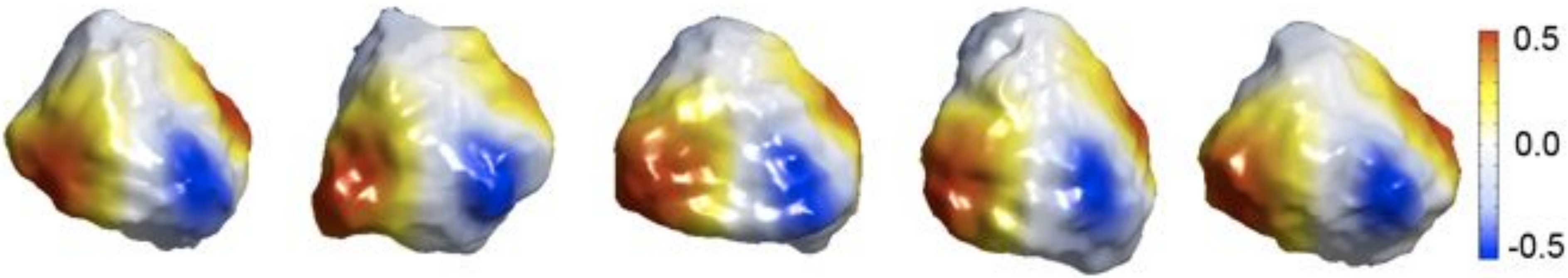
There is no common extrinsic coordinate system for every manifolds.

Need an extrinsic approach for local shape comparison: deformable modeling



D’Arcy Thompson 1860-1948

How to match surfaces intrinsically



Landmarks: Identify min and maximum of eigenfunctions

Develop thin-plate spline with landmarks.

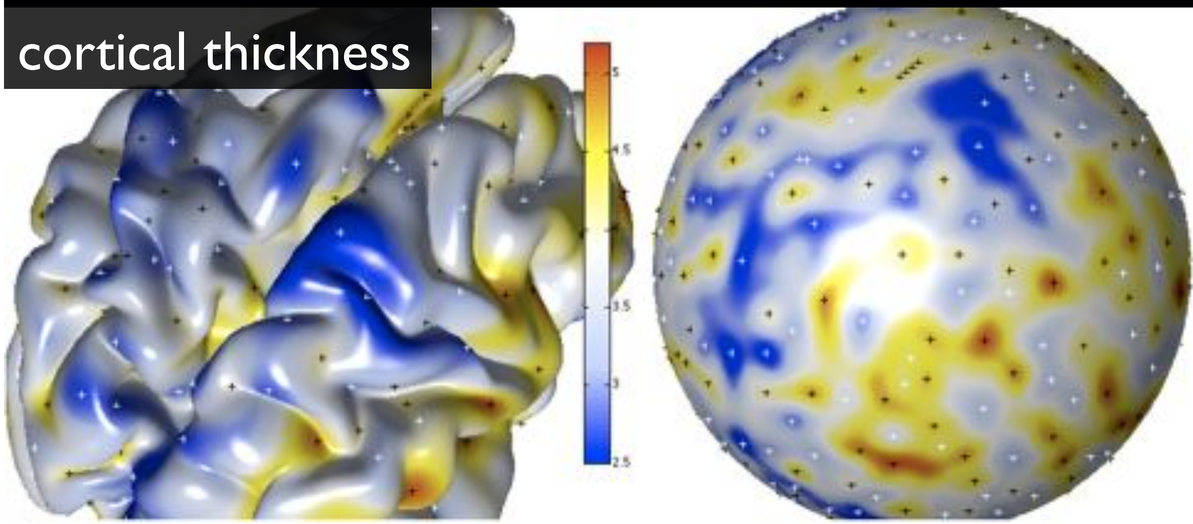
MATLAB demonstration

Data smoothing on surface models

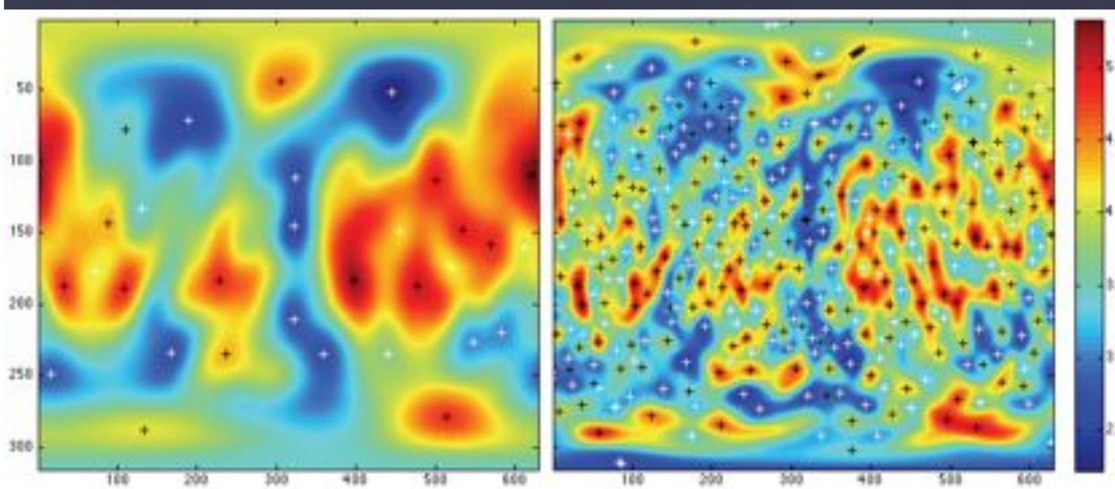
Persistence homology based signal detection

Euler characteristic, Betti numbers, Morse functions,
Worsley's random field theory.

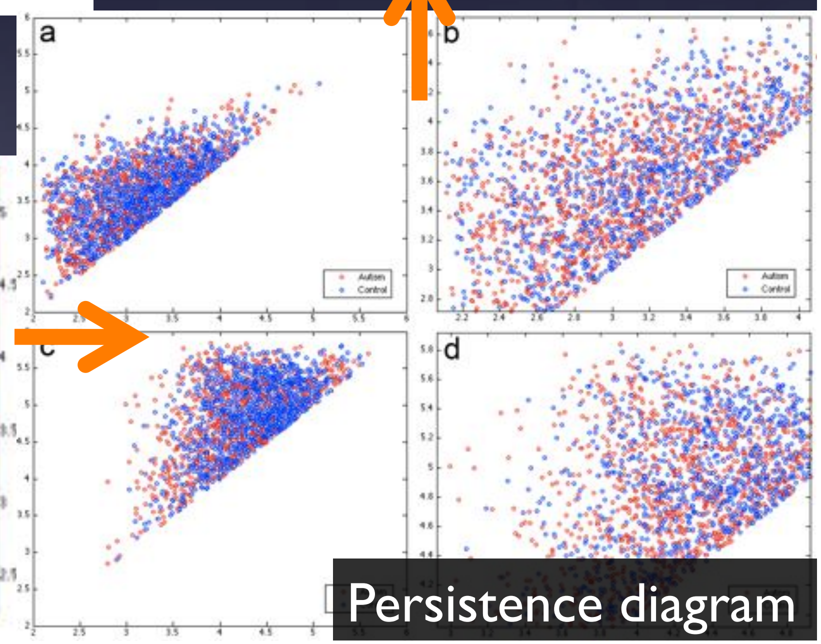
cortical thickness



cortical flattening



Topological classification 96%
Previous method 90%



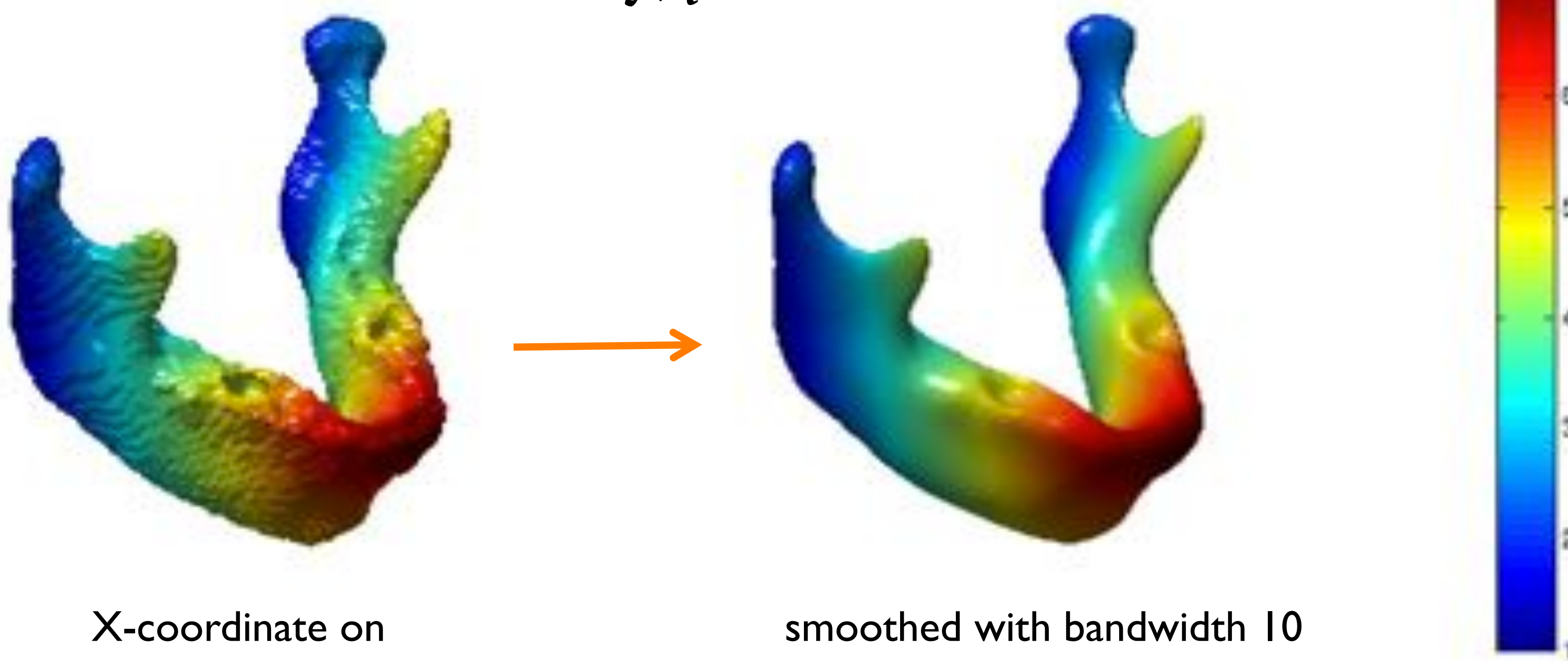
Persistence diagram

Heat kernel smoothing on surface

Heat kernel:

$$K_t(p,q) = \sum_{i=0}^{\infty} e^{-\lambda_i t} \psi_i(p)\psi_i(q)$$

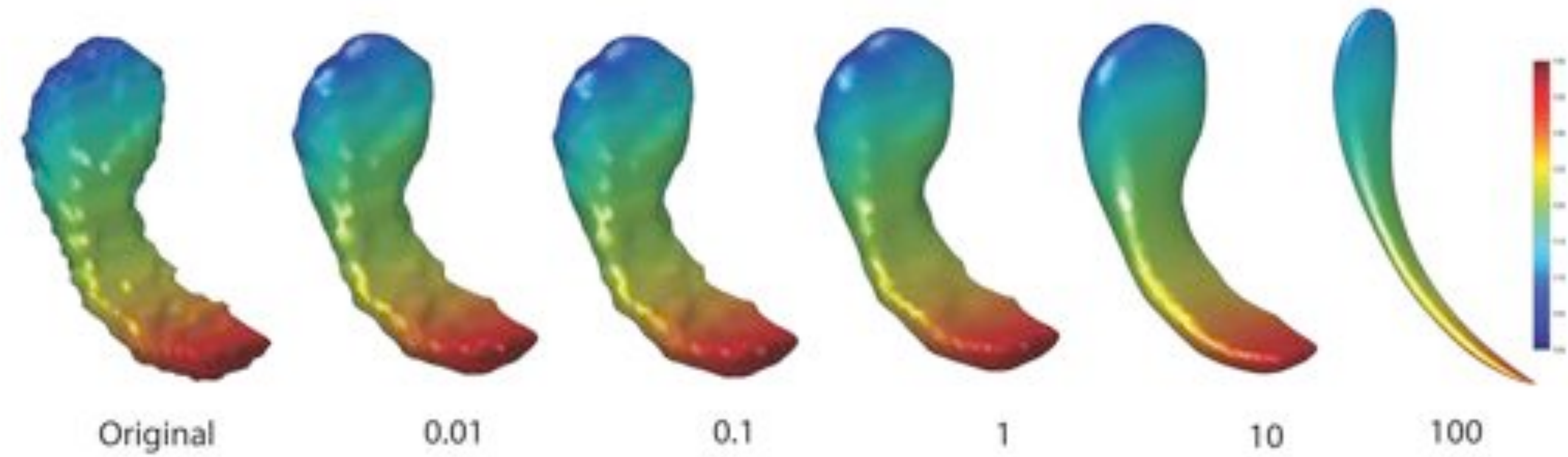
$$K_t * f = \int_{\mathcal{M}} K_t(p,q) f(q)\, dq$$



X-coordinate on mandible surface

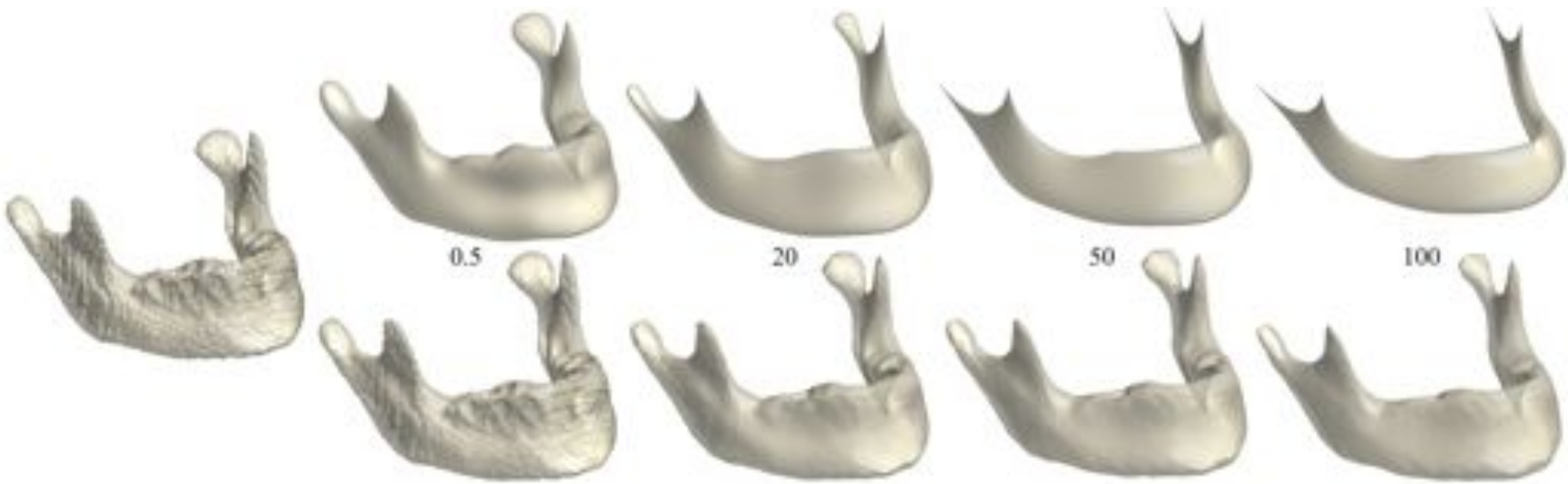
smoothed with bandwidth 10 and 1269 eigenfunctions

Heat kernel smoothing of hippocampus



Heat kernel smoothing on mandible shape

Heat kernel smoothing (Seo et al., 2010 MICCAI)

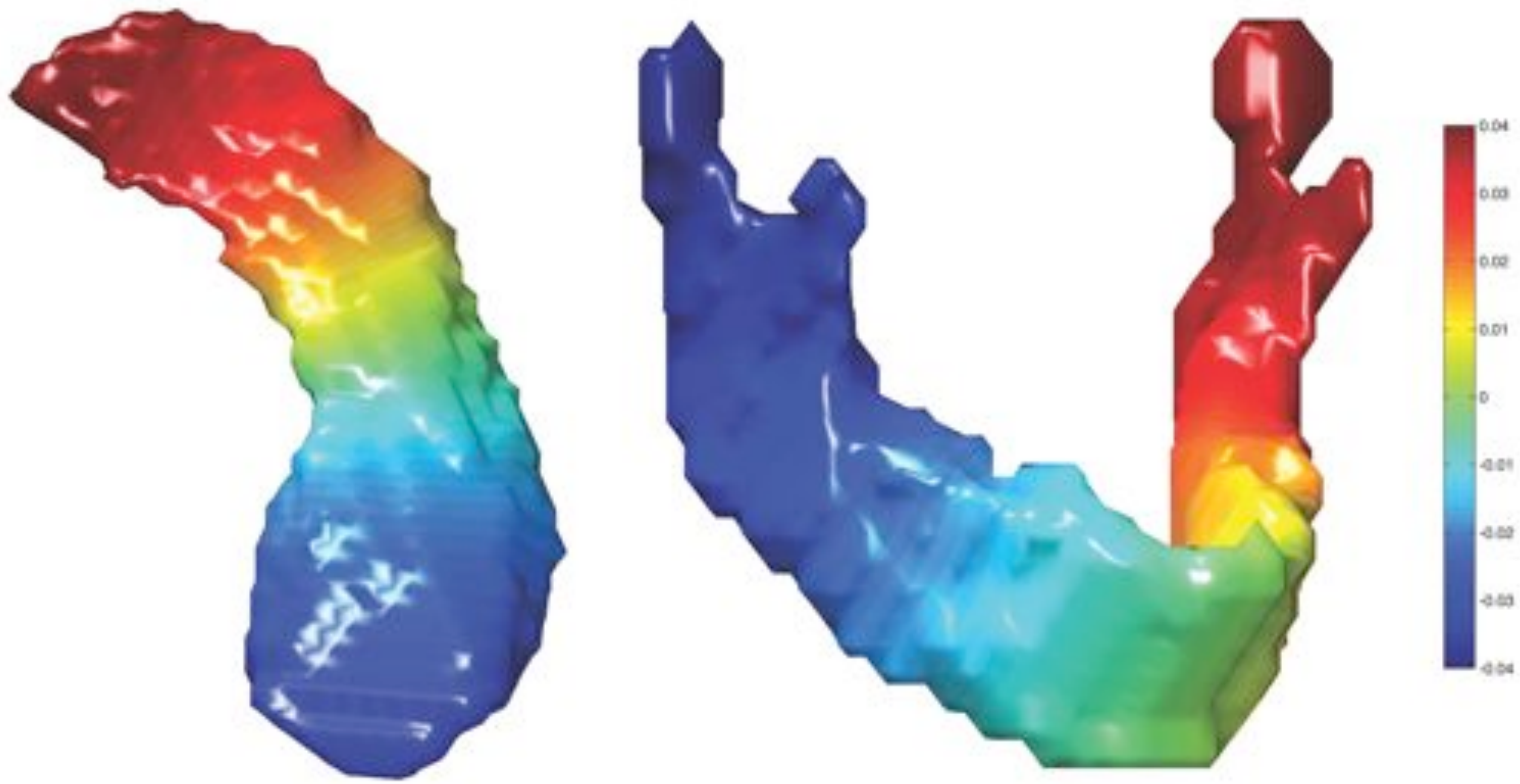


Iternated kernel smoothing (Chung et al., 2005 NeuroImage)

MATLAB demonstration

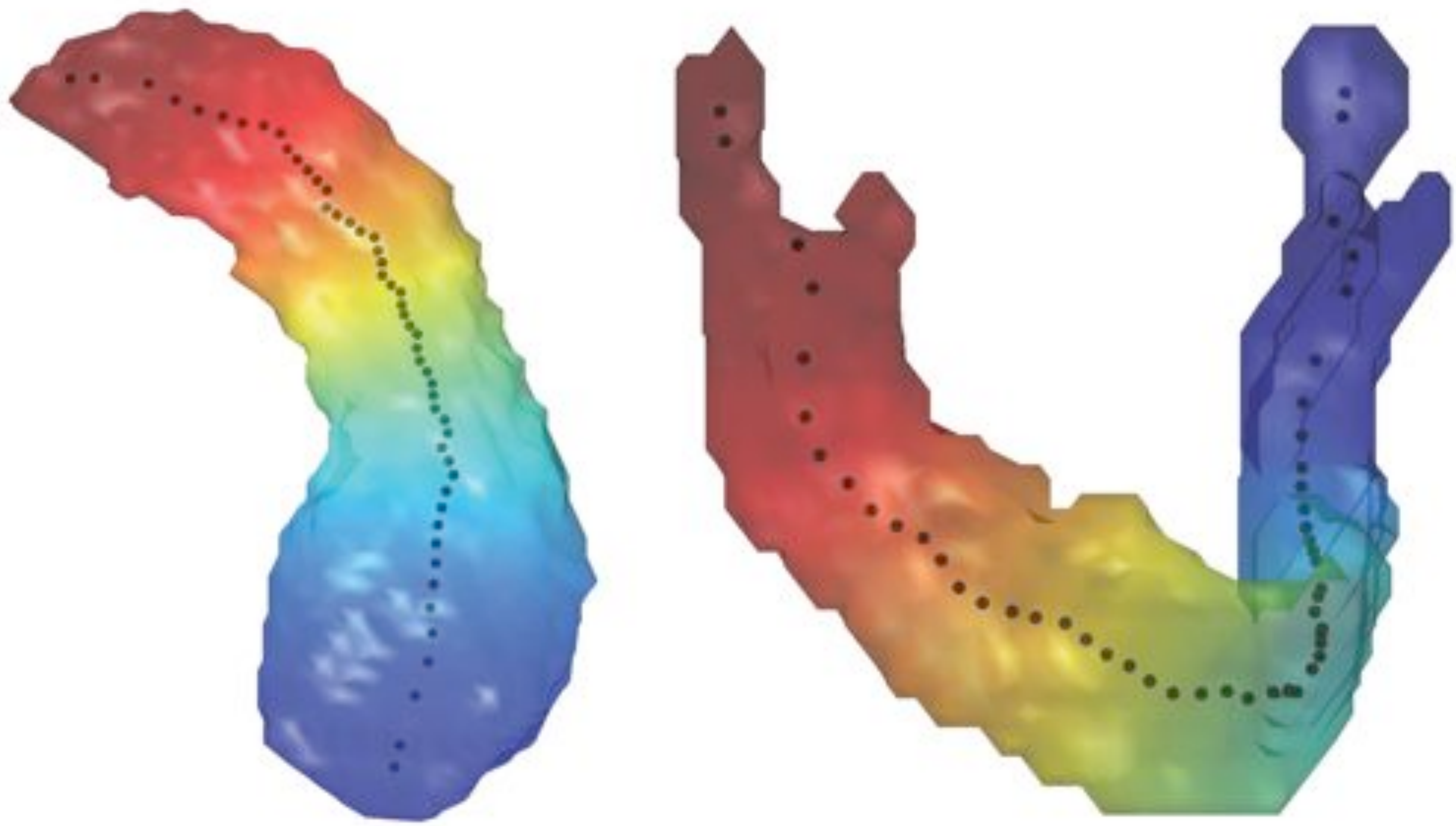
Statistical Analysis

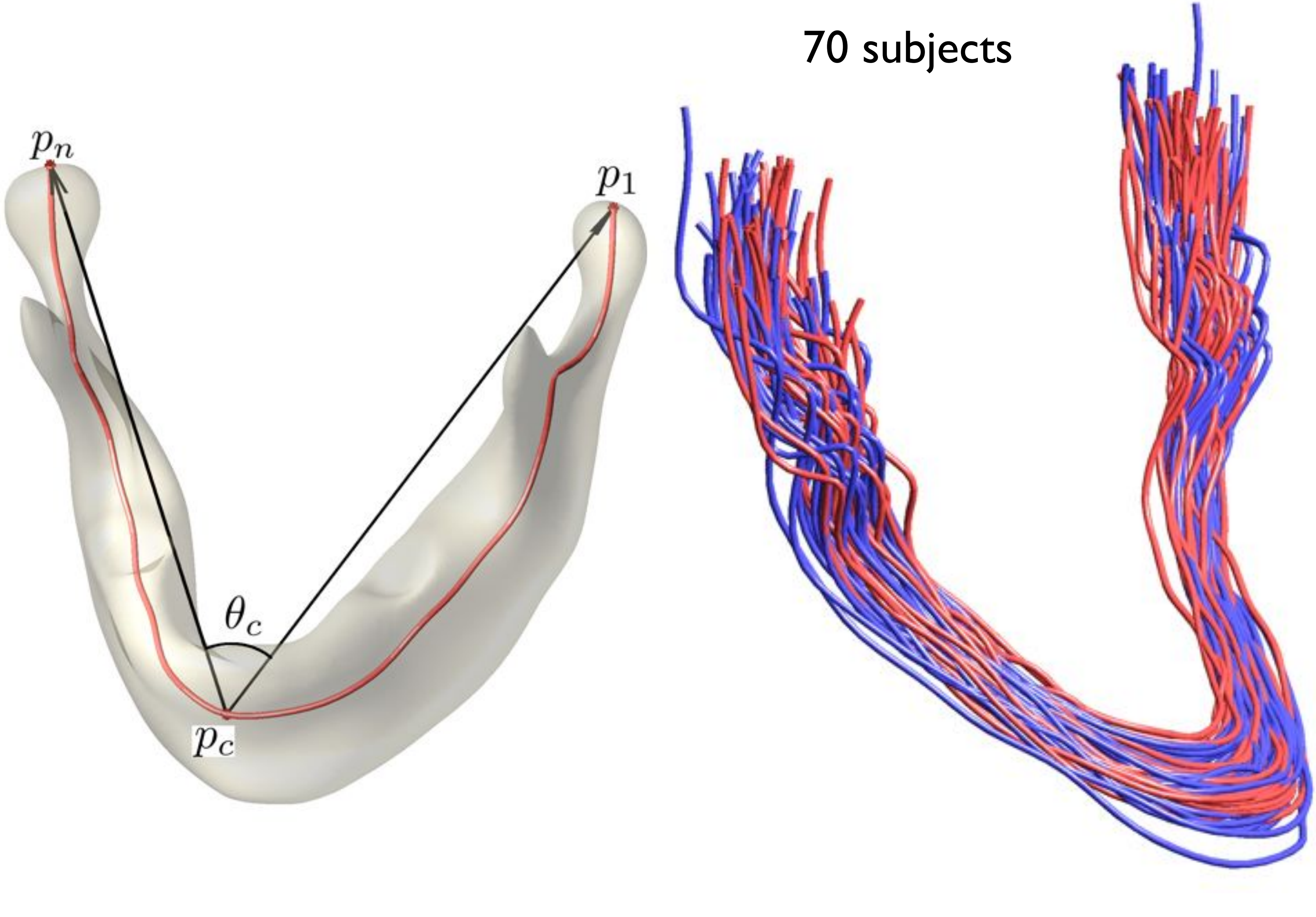
2nd eigenfunction of elongated objects



Hot spot conjecture:
Min and max always occur at the extreme end points

Center of isocontour circles

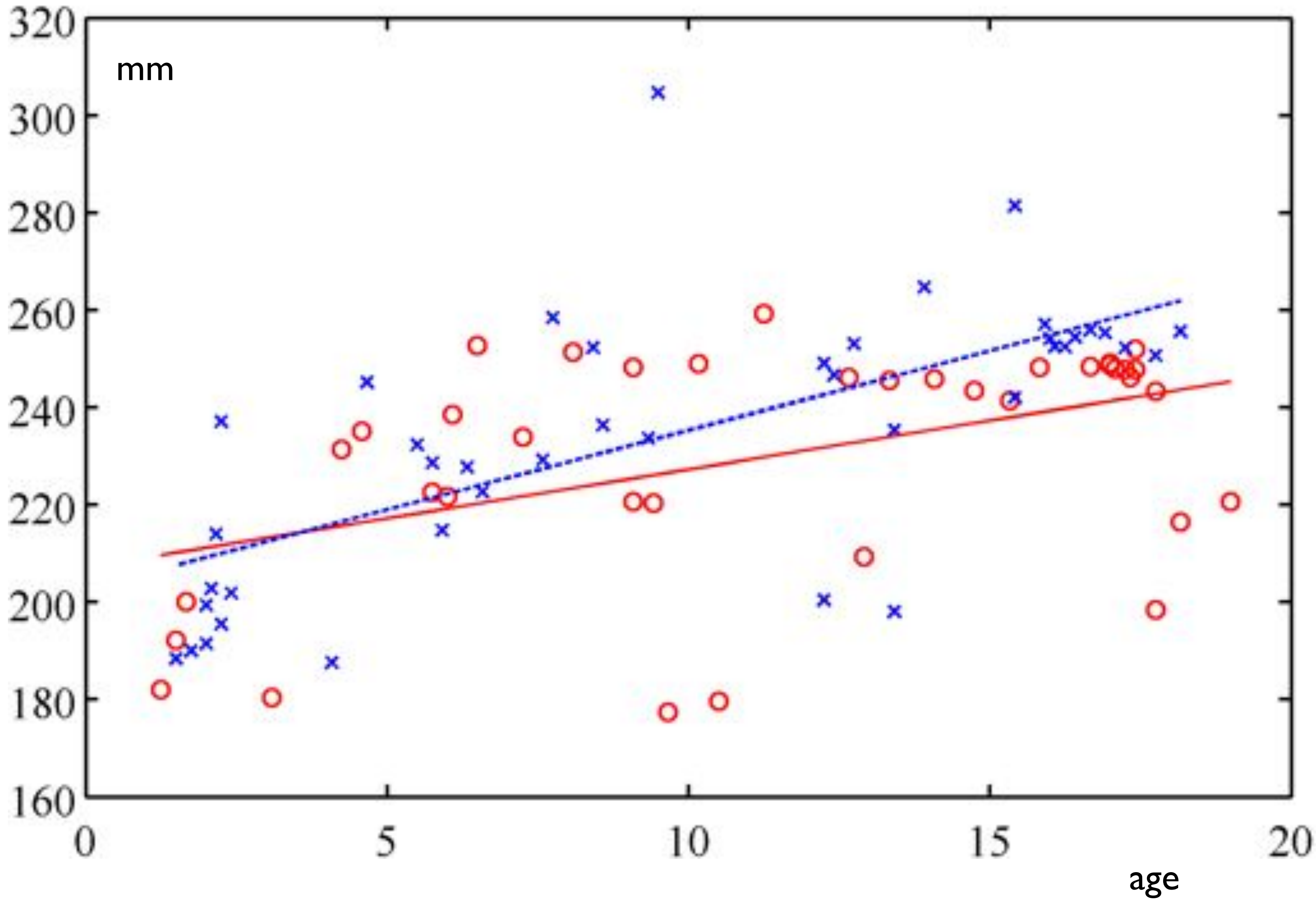




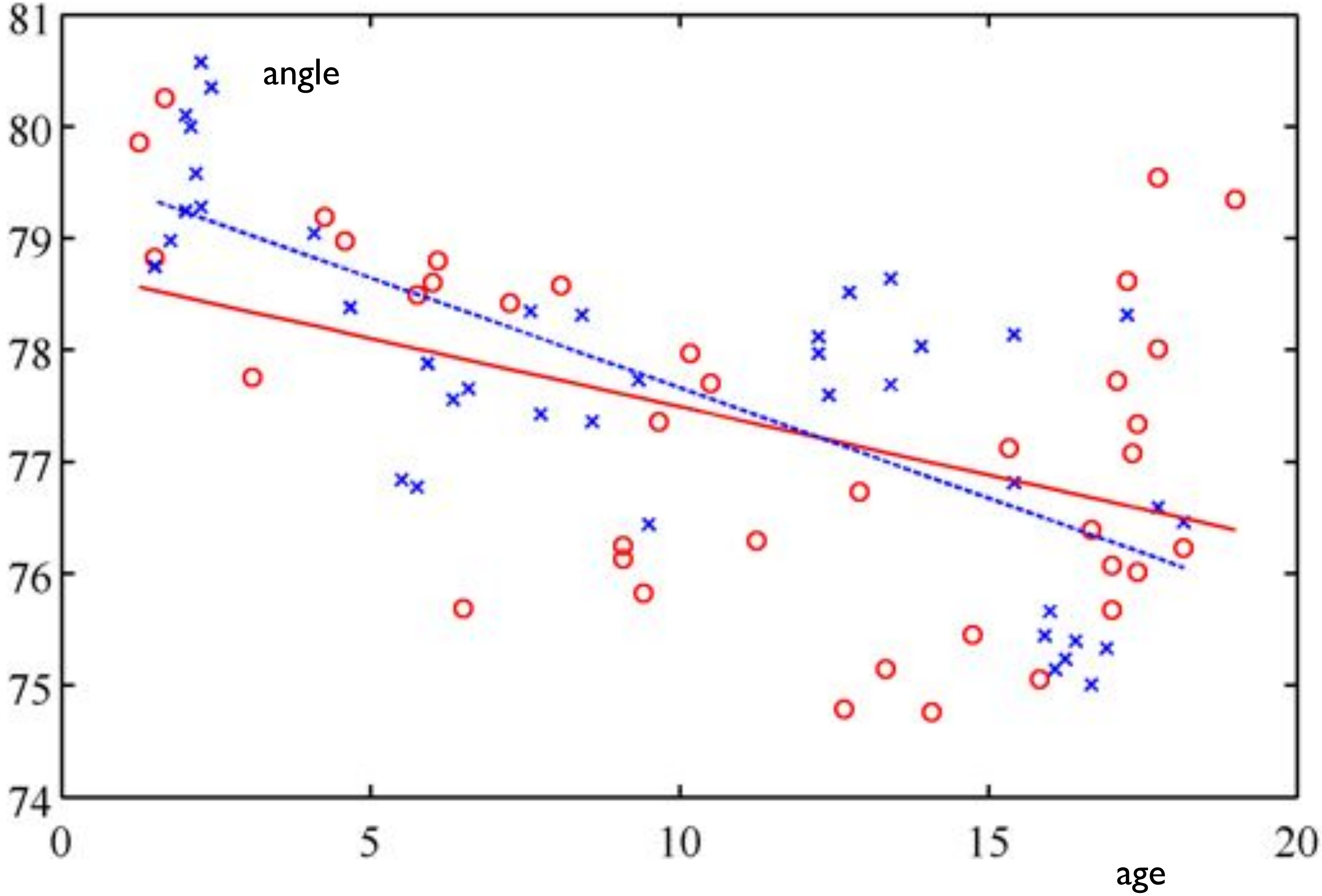
70 subjects
p_n
p_1
θ_c
p_c

MATLAB demonstration

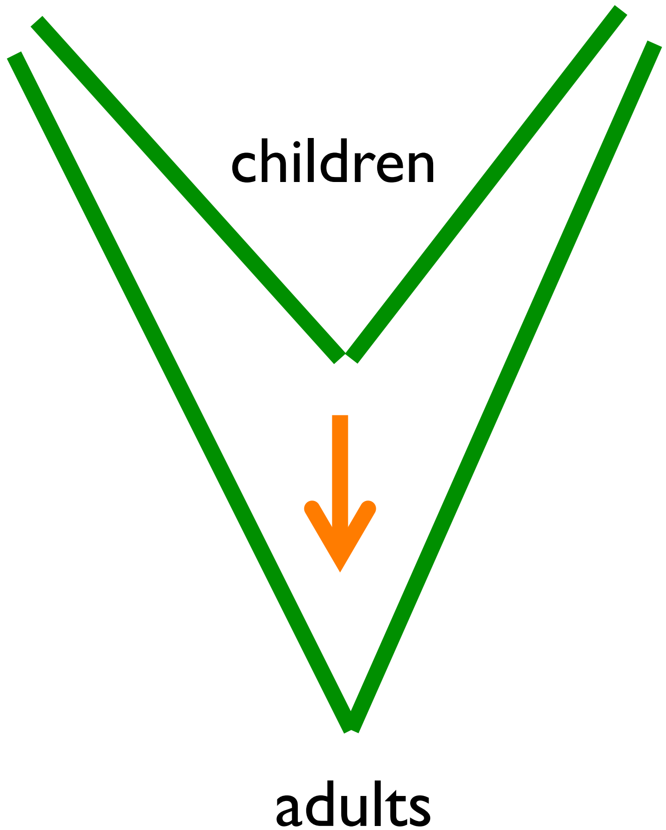
Elongation of mandible (length increase)



Elongation of mandible (angle decrease)



Growth projectory



We are becoming more 주걱 턱 as we gets older.

Need more data to differentiate gender specific growth difference.

Left hippocampus surface template

Total number of subjects **124**

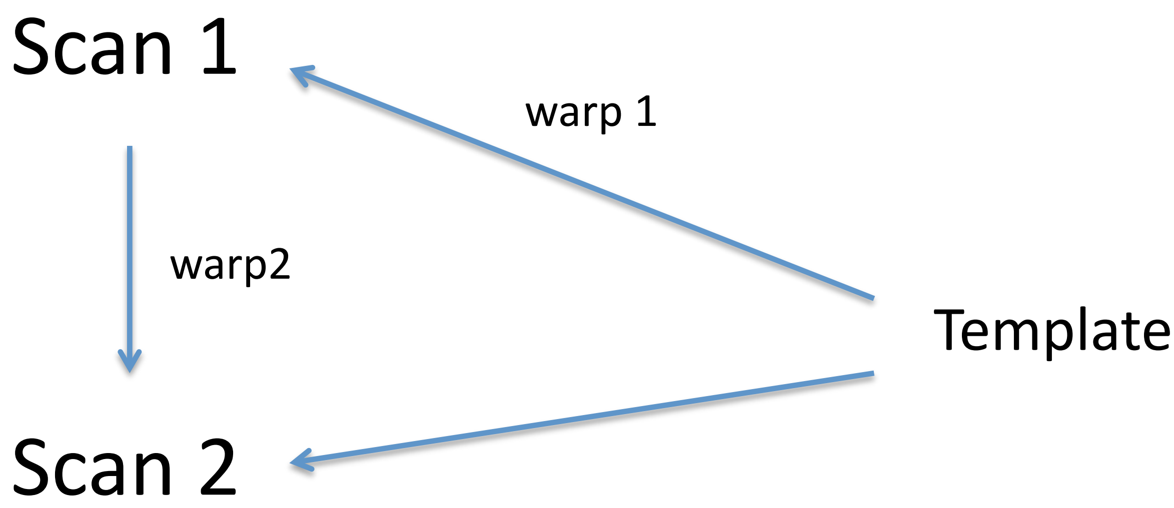
High income family **86** = 24 males + 62 females
Average age = 140 +/- 45 months = **12+/- 4 years old**

Low income family **38** = 13 males + 25 females
Average Age= 137 +/- 45 months = **12 +/- 4 years old**

Balanced study design except there is no info on handness.

Each subject has multiple MRI scans (1-2 scans).

Longitudinal Image processing pipeline

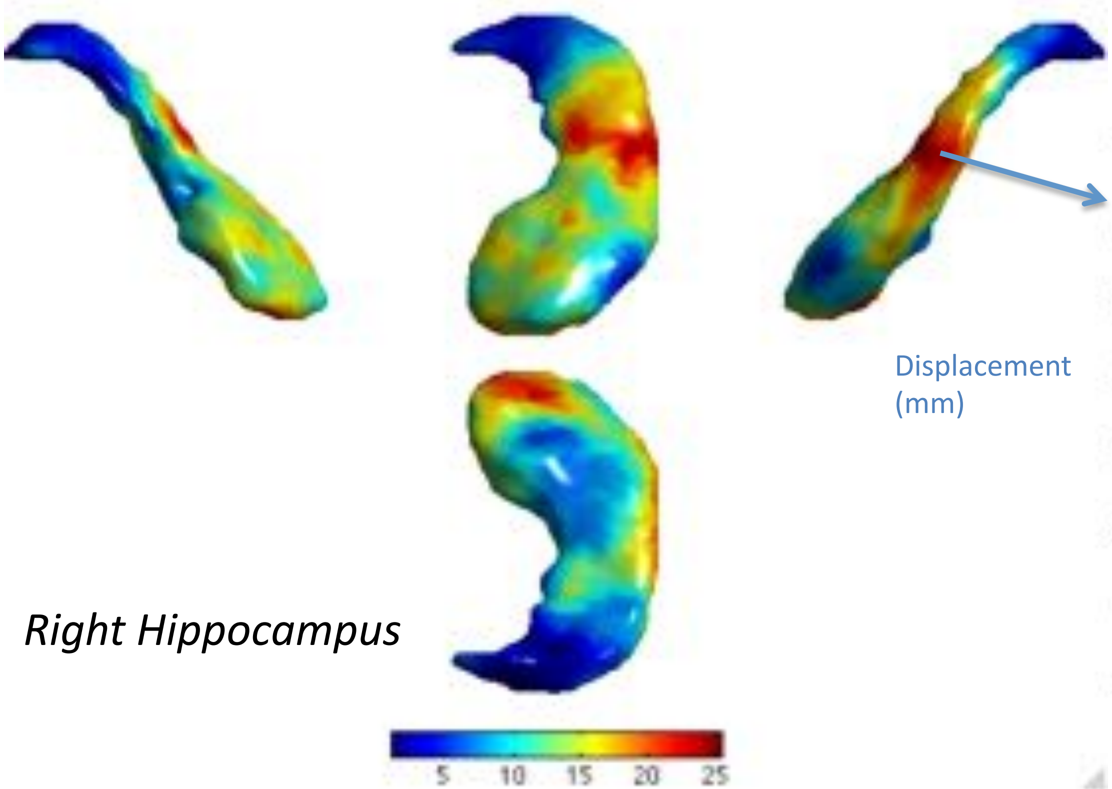


Deformation form the template to Scan2 is given by warp1 + warp2.

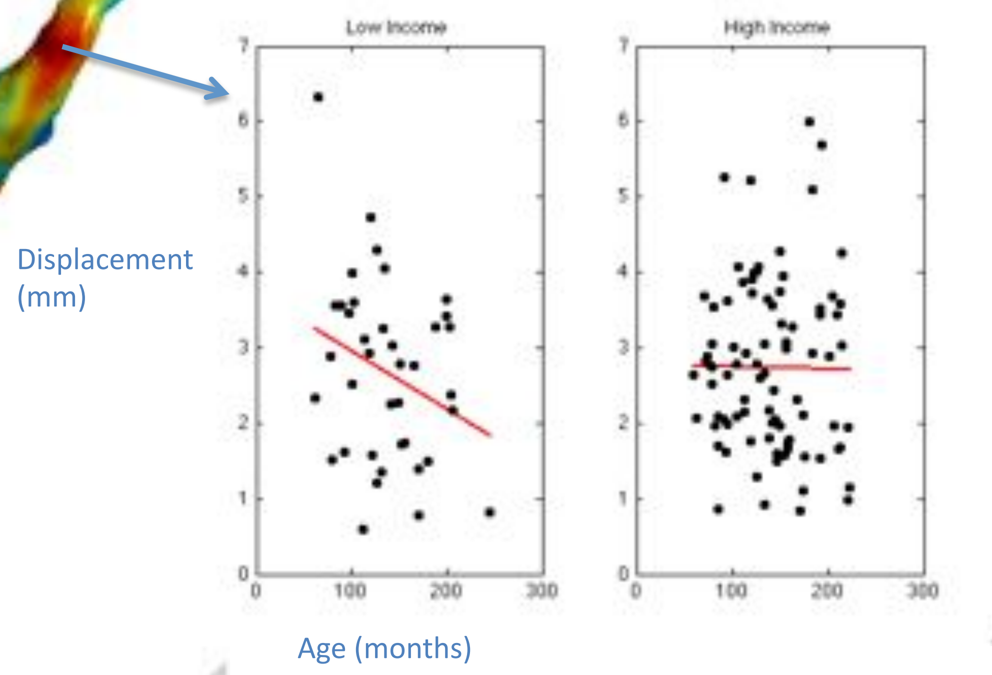
Fixed effect model accounting treating multiple scans within a subject as indepdenent

displacement = age + gender + group + <u>age*group</u>

max F = 25.4
corrected pvalue < 0.001
it inflates statistical significance



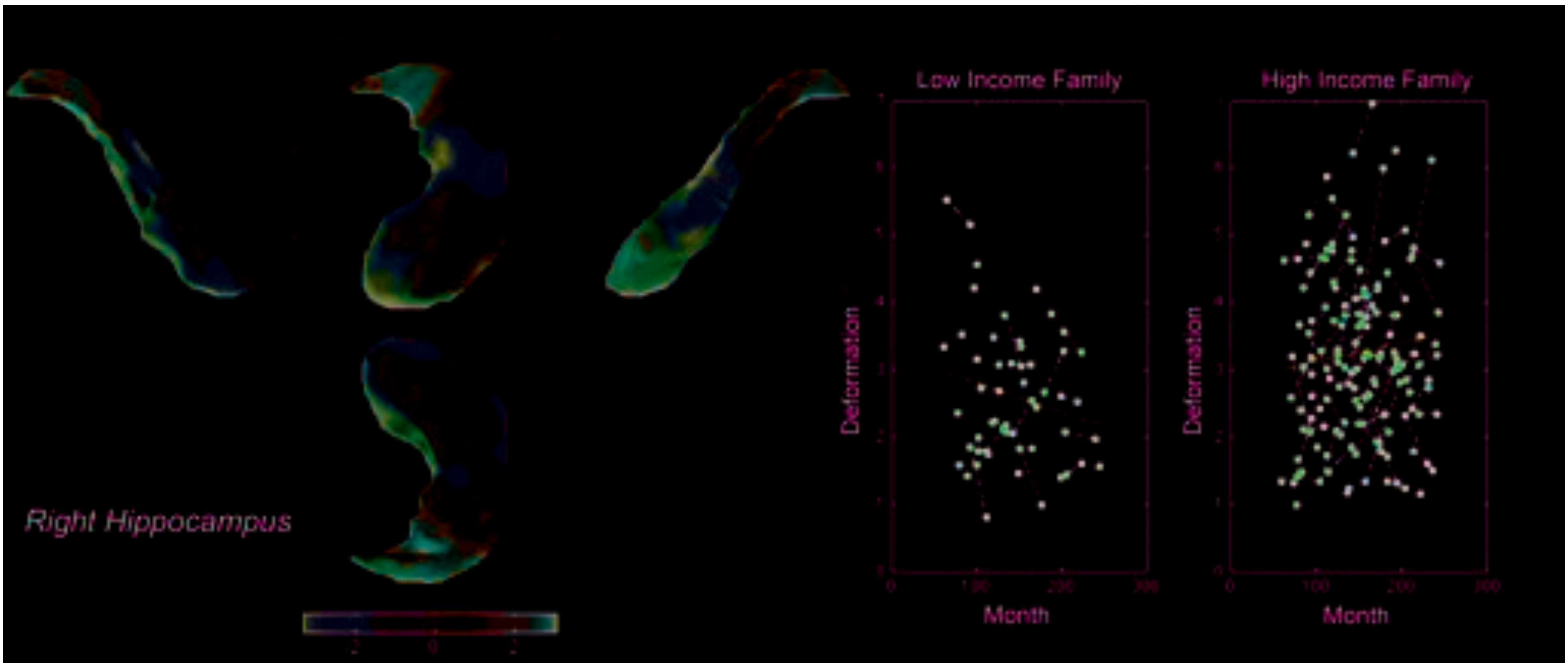
Right Hippocampus



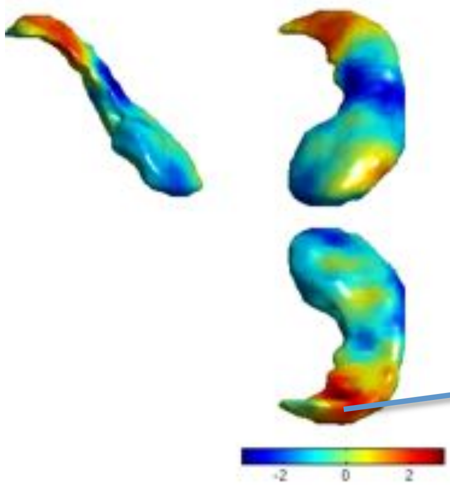
Linear mixed effect model accounting for inter-correlation of multiple scans within a subject

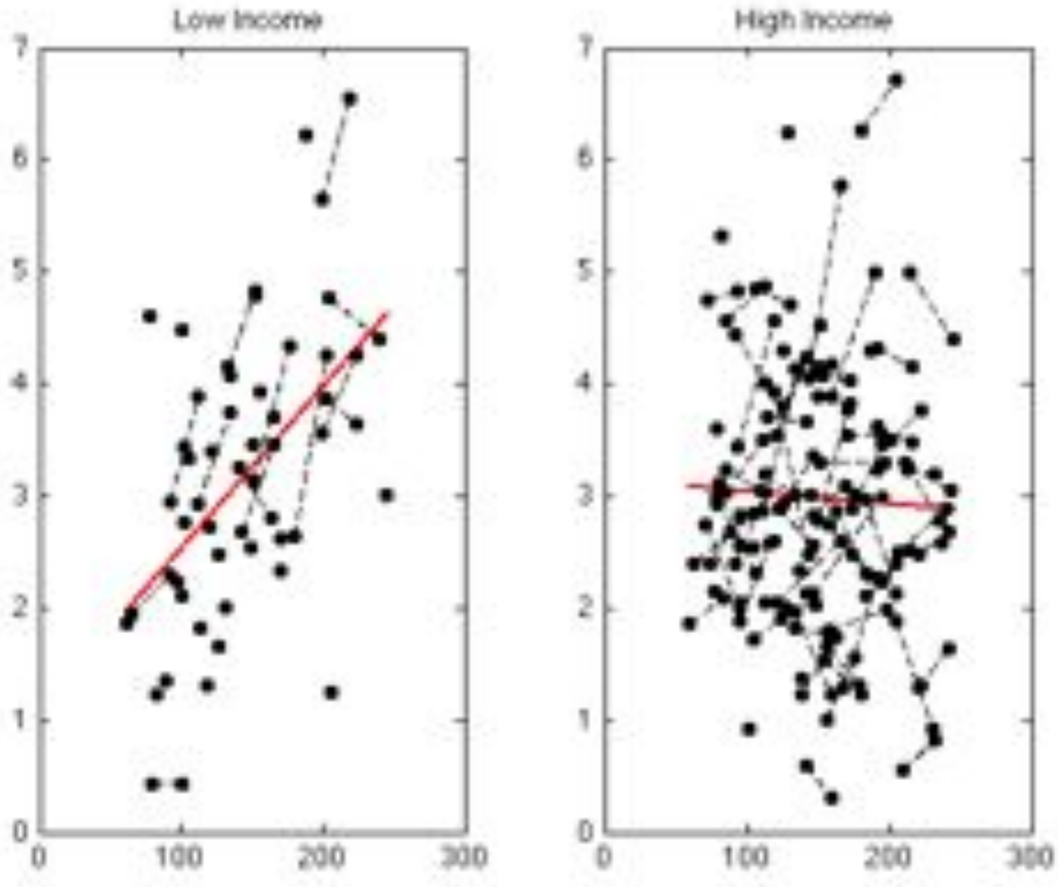
displacement = age + gender + group + age*group

min t = -3.3398,
corrected pvalue = 0.025



Right hippocampus



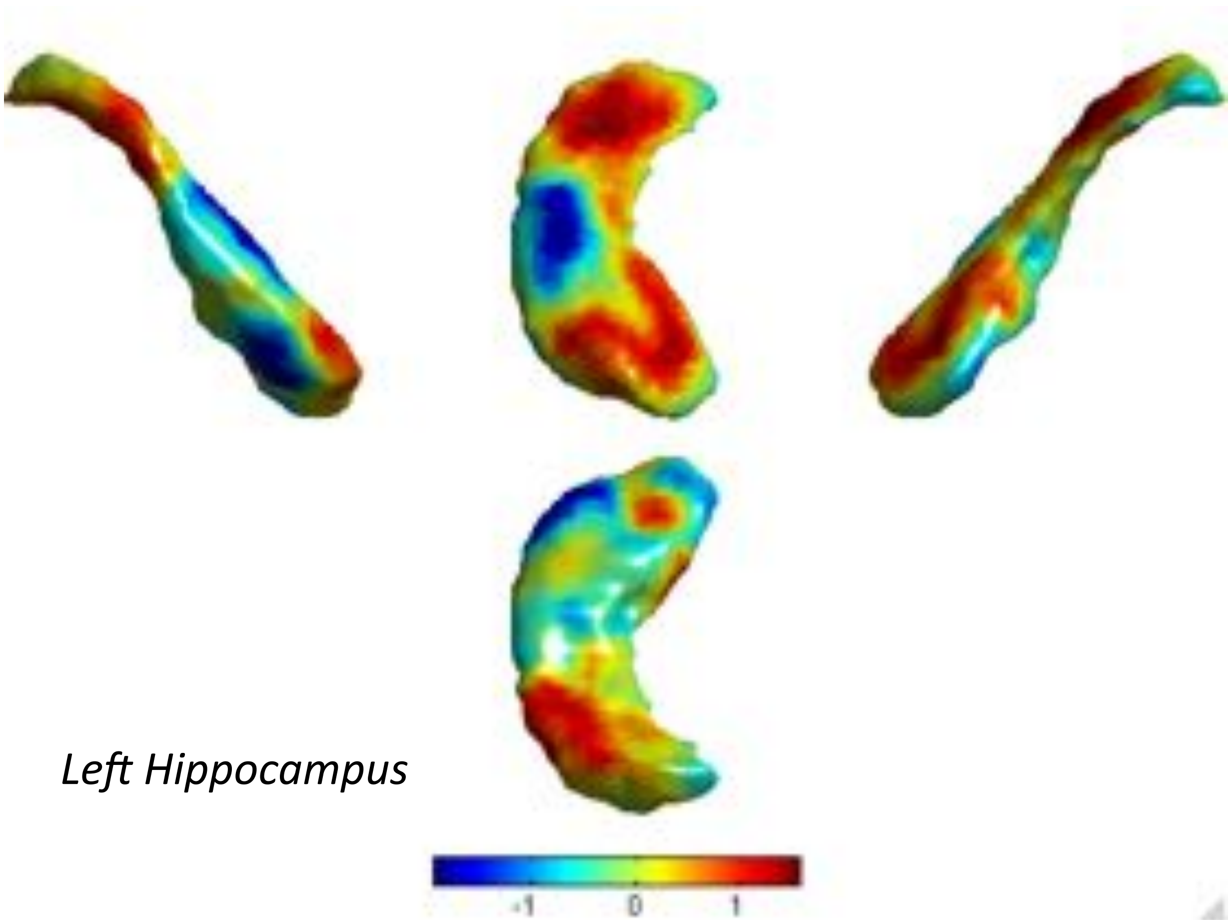


max t = 3.0531
corrected pvalue = 0.05

Linear mixed effect model on left hippocampus

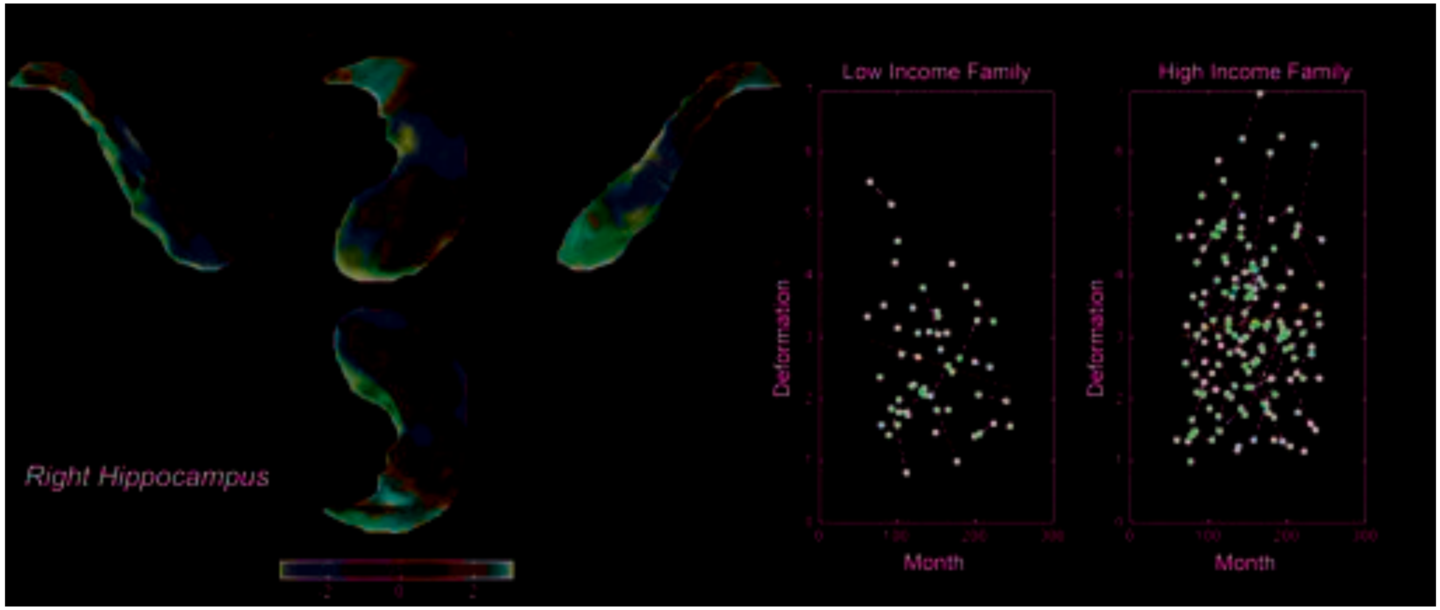
displacement = age + gender + group + age*group

min t = -1.9589,
corrected pvalue > 0.4



Left Hippocampus

Summary



Family income level is highly correlated with hippocampus growth.

Adverse environment, stress
→ hippocampus → memory loss

Lecture 9
Brain Network Modeling

Read
lee.2011.SPIE – sparse network modeling
Chung.2011.SPIE – epsilon neighbor
lee.2011.ISBI -- rips filtration
kim.2011.ISBI -- epsilon neighbor

MATLAB demonstration