Computational Methods in NeuroImage Analysis

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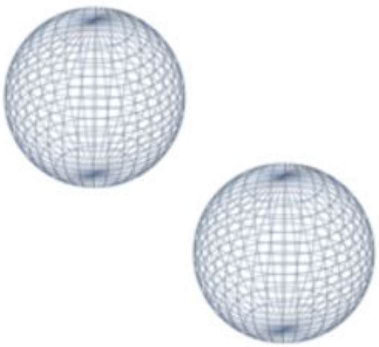
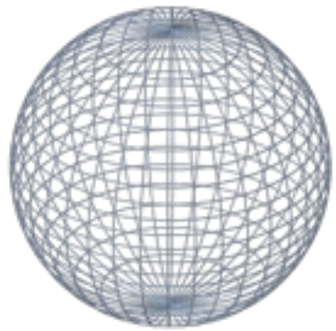
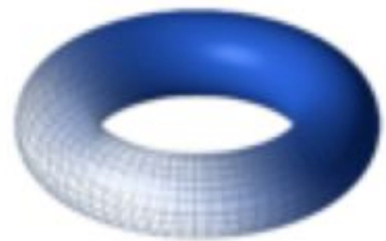
Lecture 6
Topological computation
Brain Network Modeling

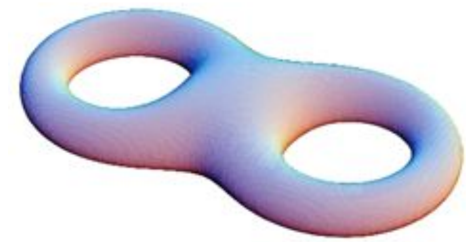
October 8, 2010

Euler characteristic: most widely used topological invariant





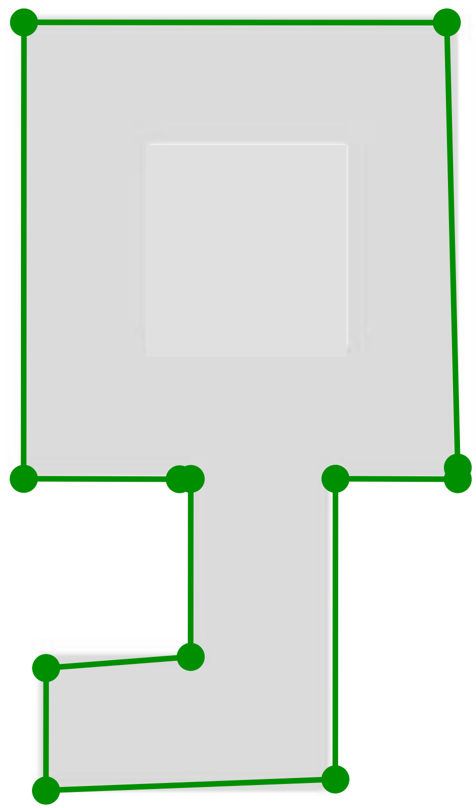


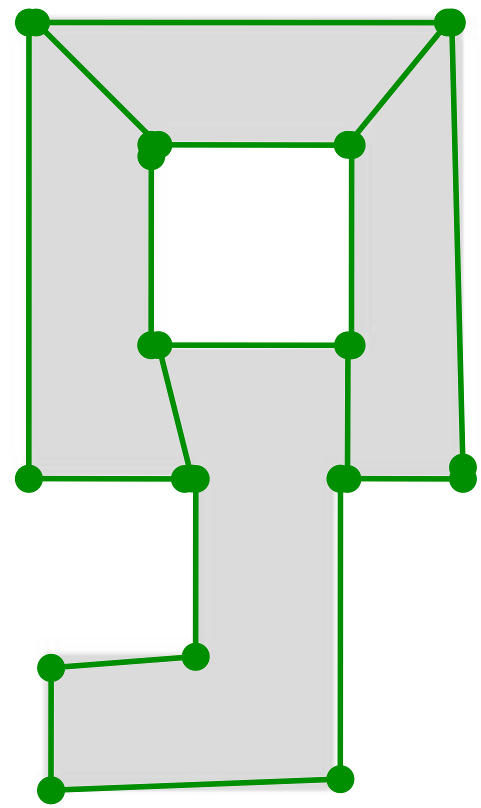


4 2 0 -2

For an object with n-handles, EC= $-2n$

Computing Euler characteristic



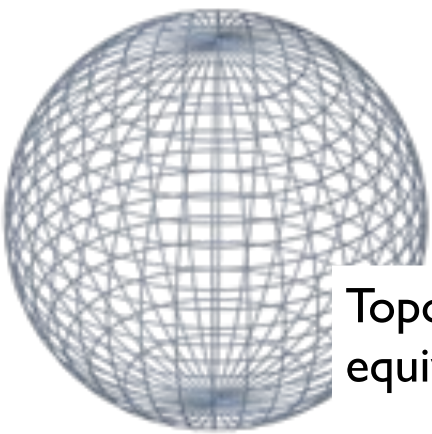


Enclose the object with a graph

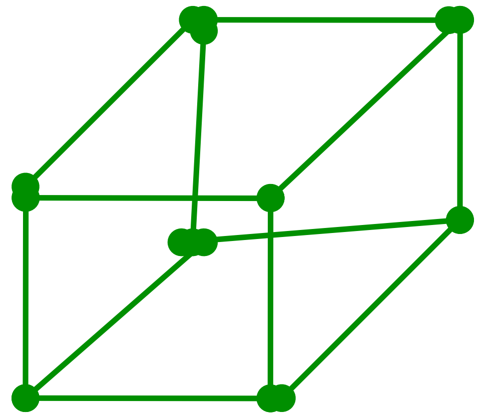
$EC = N - E + F$
$= 10 - 10 + 1$
$= 1$

$EC = N - E + F$
$= 14 - 18 + 4$
$= 0$

Computing Euler characteristic



Topologically equivalent

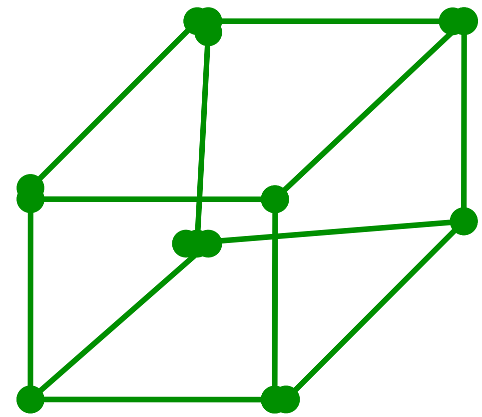


Sphere

EC = N – E + F

=8 – 12 + 6

= 2



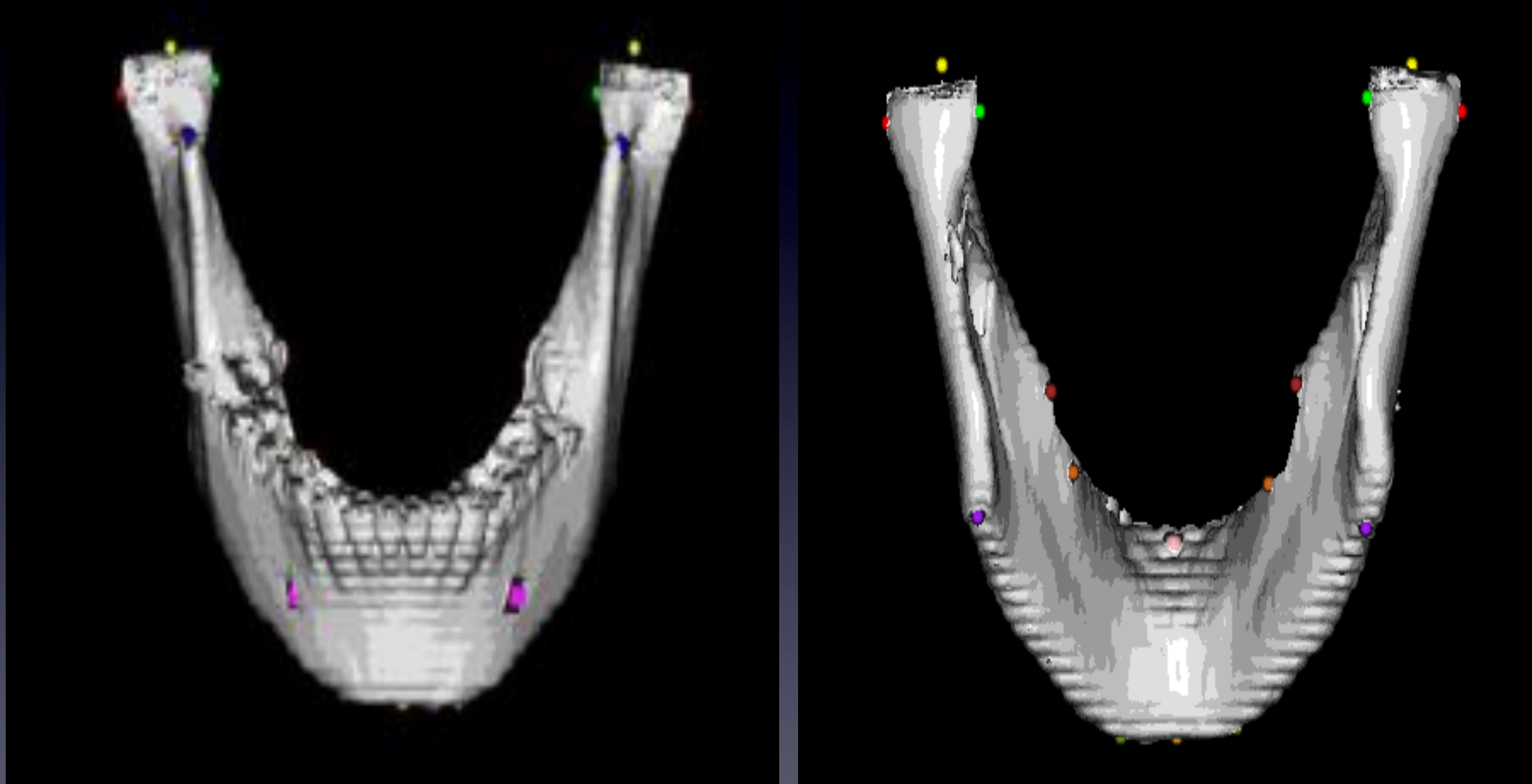
Solid ball

EC = N – E + F - V

=8 – 12 + 6 - 1

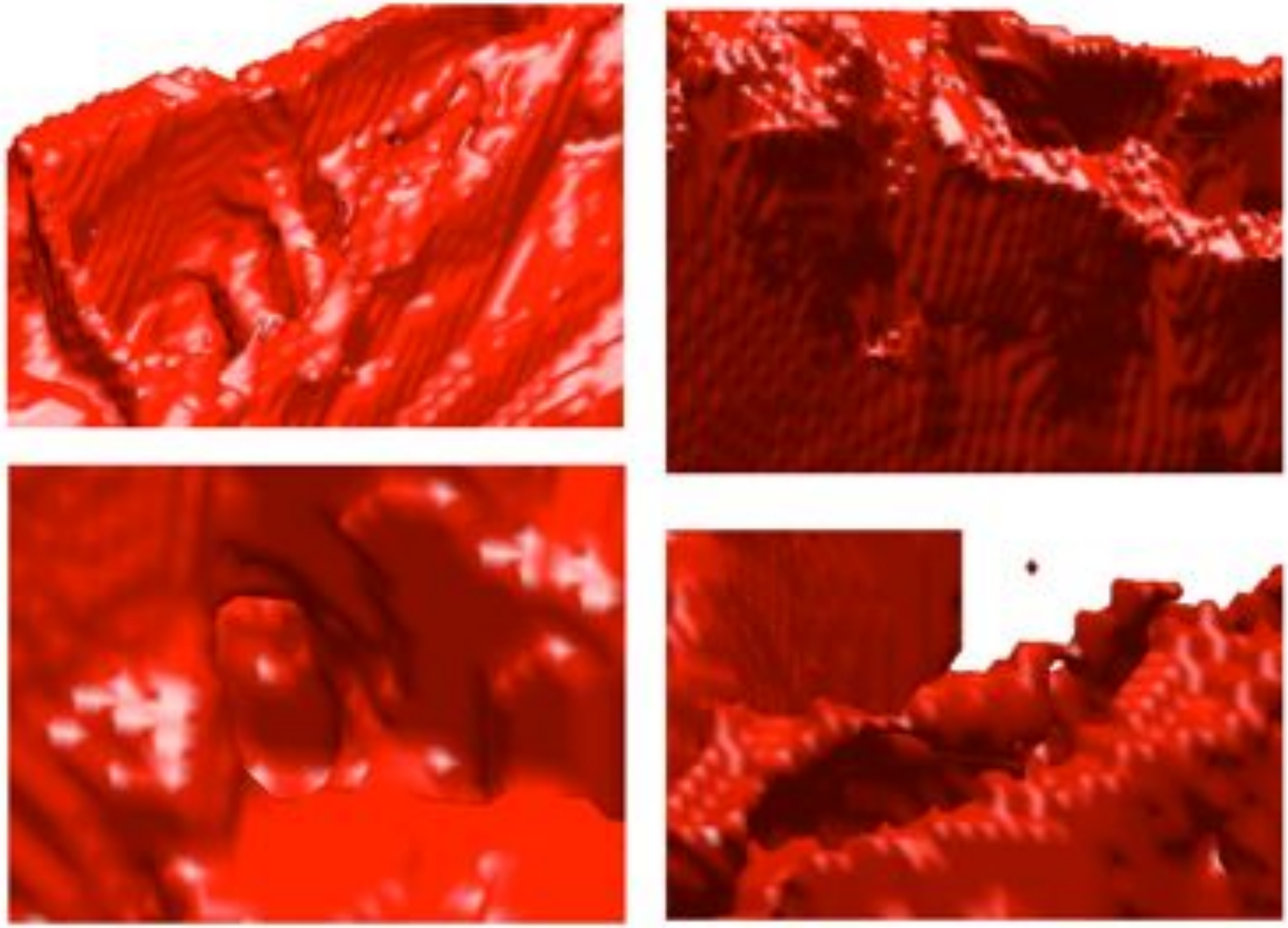
= 1

Mandible binary segmentation from CT



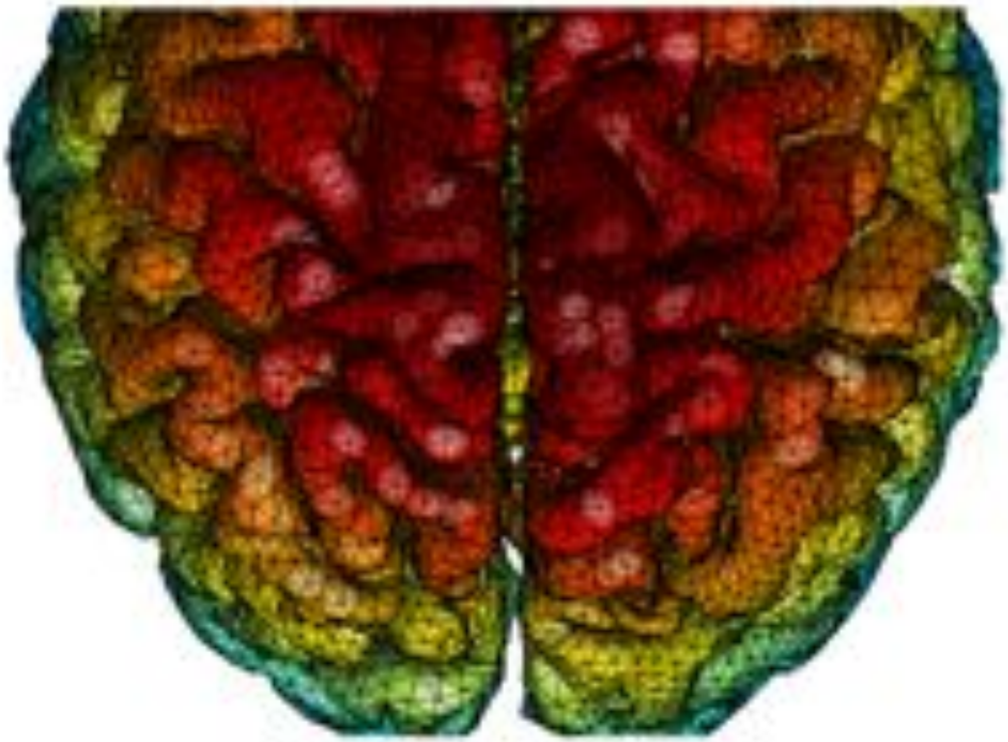
Colored dots are manually identified landmarks

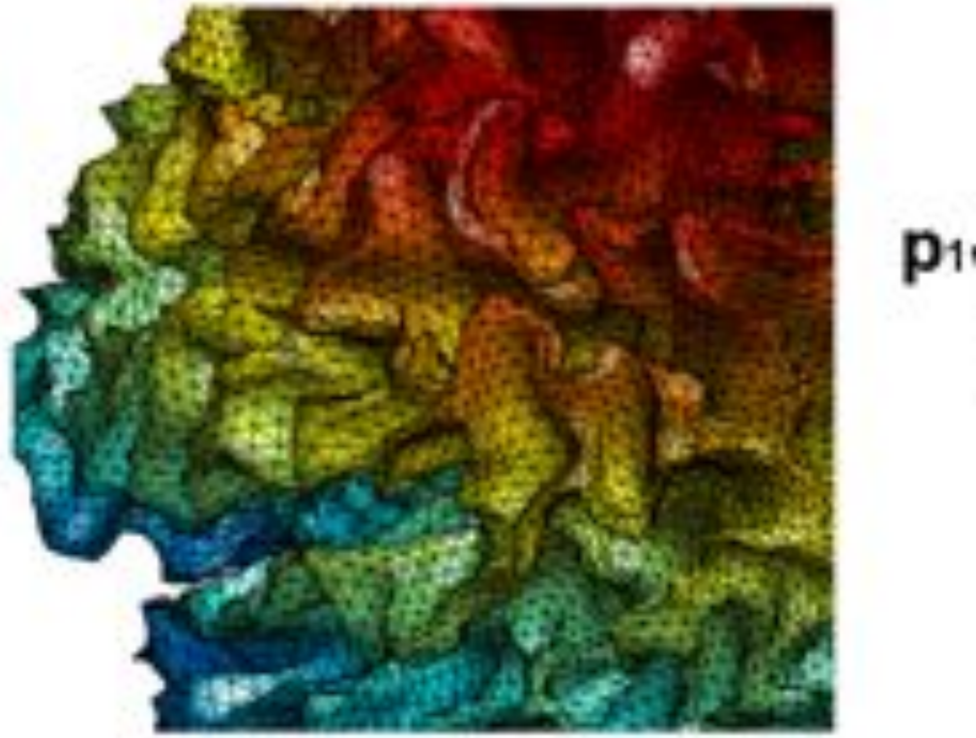
By checking the Euler characteristic of the original binary volume of a mandible, holes in the binary volume have been patched up. This process is necessary to make the mandible binary volume to be topologically equivalent to a solid sphere for subsequent modeling and analysis.



Holes and handles in binary segmentation

Additional morphological closing operation was done to patch up the space that was occupied by teeth. Without this morphological operation, the final statistical result will be highly biased in teeth regions.





Euler characteristic for cortical surface

N - E +F = 2 for cortical surface .
For each triangle, there are three edges. Since two adjacent triangles share the same edge, the total number of edges is E = 3F/2. Hence, we have F=2N - 4.

MATLAB demonstration

Why do we need topological approaches?

Chung et al. 2009. Information processing in Medical Imaging (IPMI)
Read chung.2009.IPMI.pdf

Usual scientific model: $f = \mu + \epsilon$

Correlated test statistic: $T(x)$

Type-I error computation: $P\Big(\sup_{x \in \mathbb{M}} T(x) > h\Big)$

Euler characteristic based random field theory

Worsley et al., Human Brain Mapping, 1996

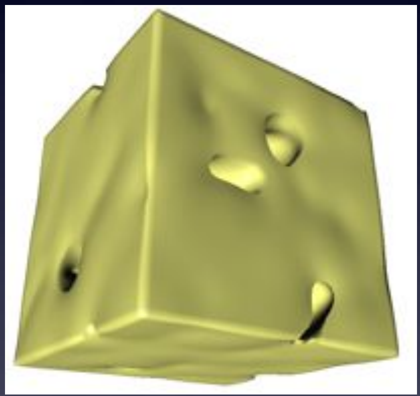
Uses Morse Theory to link analytical & geometric problem to topology

Excursion Probability

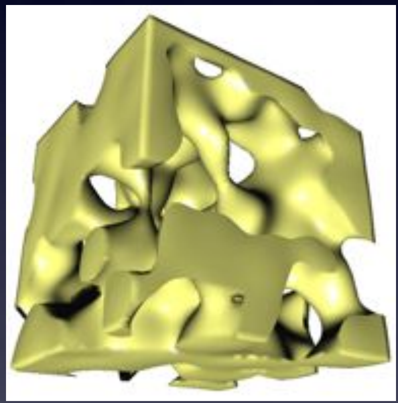
$Z(x)$: Stationary isotropic random field in $x \in \Omega \subset \mathbb{R}^N$

$A_z = \{x : Z(x) > z\}$ excursion set

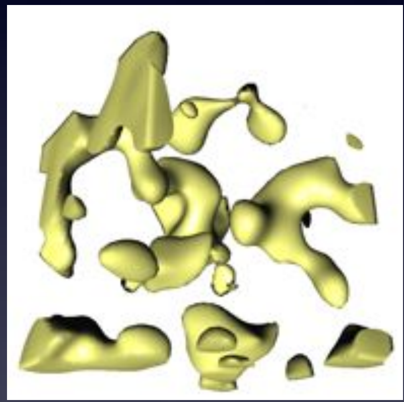
$\chi(A_z)$: Euler characteristic



z = -10



z = 0



z = 10

$$P\left(\max_{x \in \Omega} Z(x) > z\right) \approx \mathbb{E}\left(\chi(A_z)\right)$$

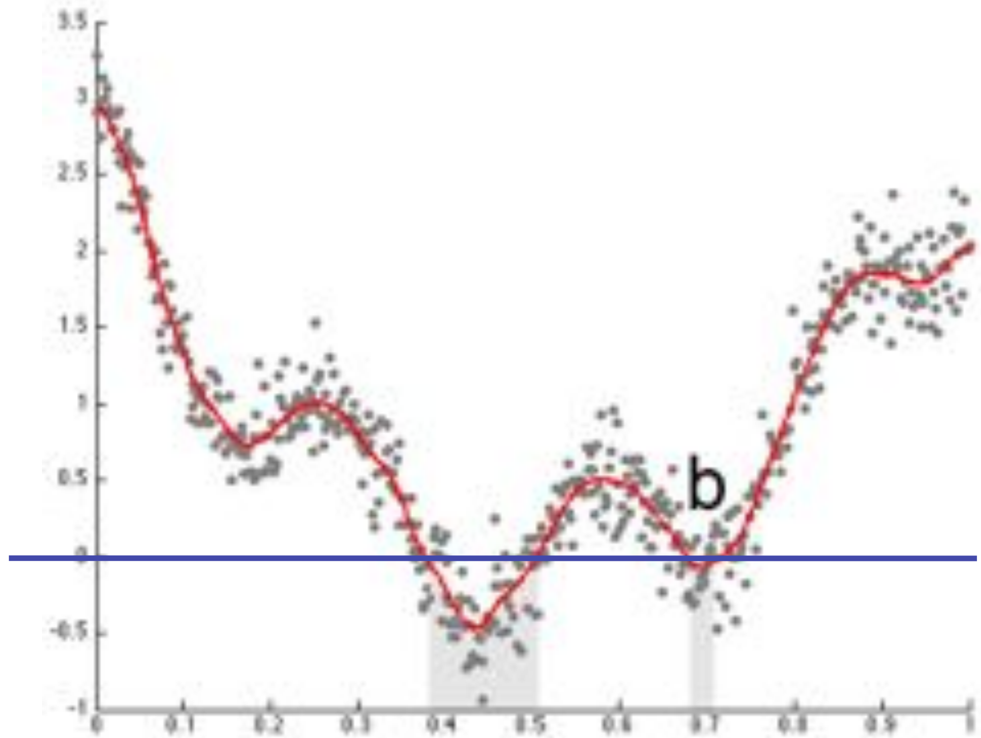
(Adler, 1984)

Morse Theory

Assume underlying signal μ to be a Morse function (all critical values are unique).

Define a sublevel set $R(y) = \mu^{-1}(-\infty, y]$

$y = 0$



Morse Theory

Number of connected components $\#R(y)$

Local min: $g_1, \cdots, g_n$

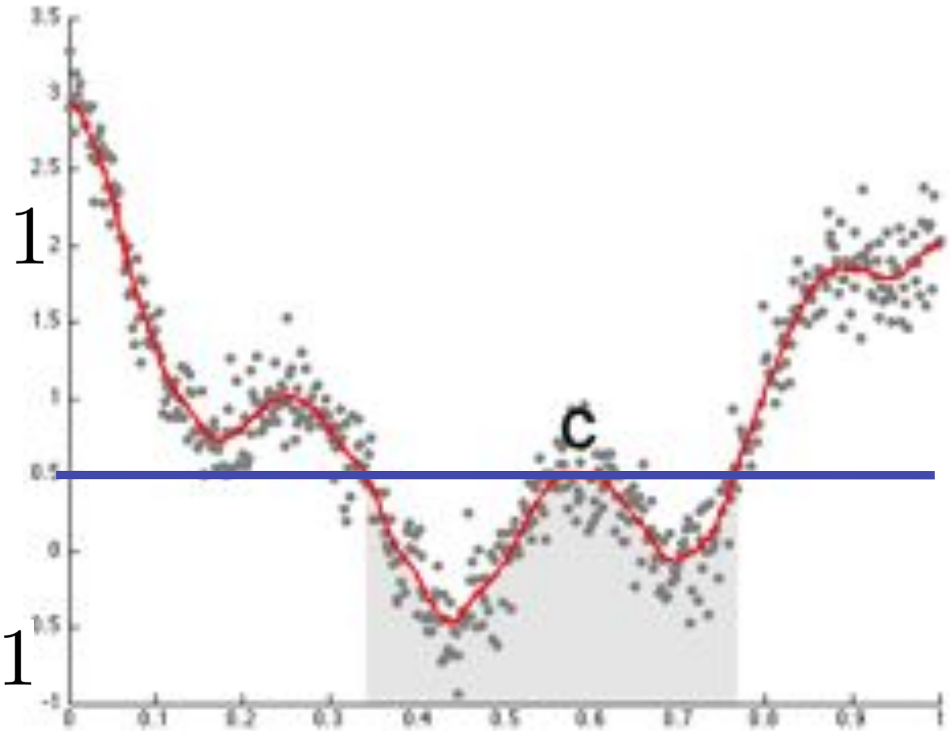
Birth:

$$\#R(g_i) = \#R(g_i - \epsilon) + 1$$

Local max: $h_1, \cdots, h_n$

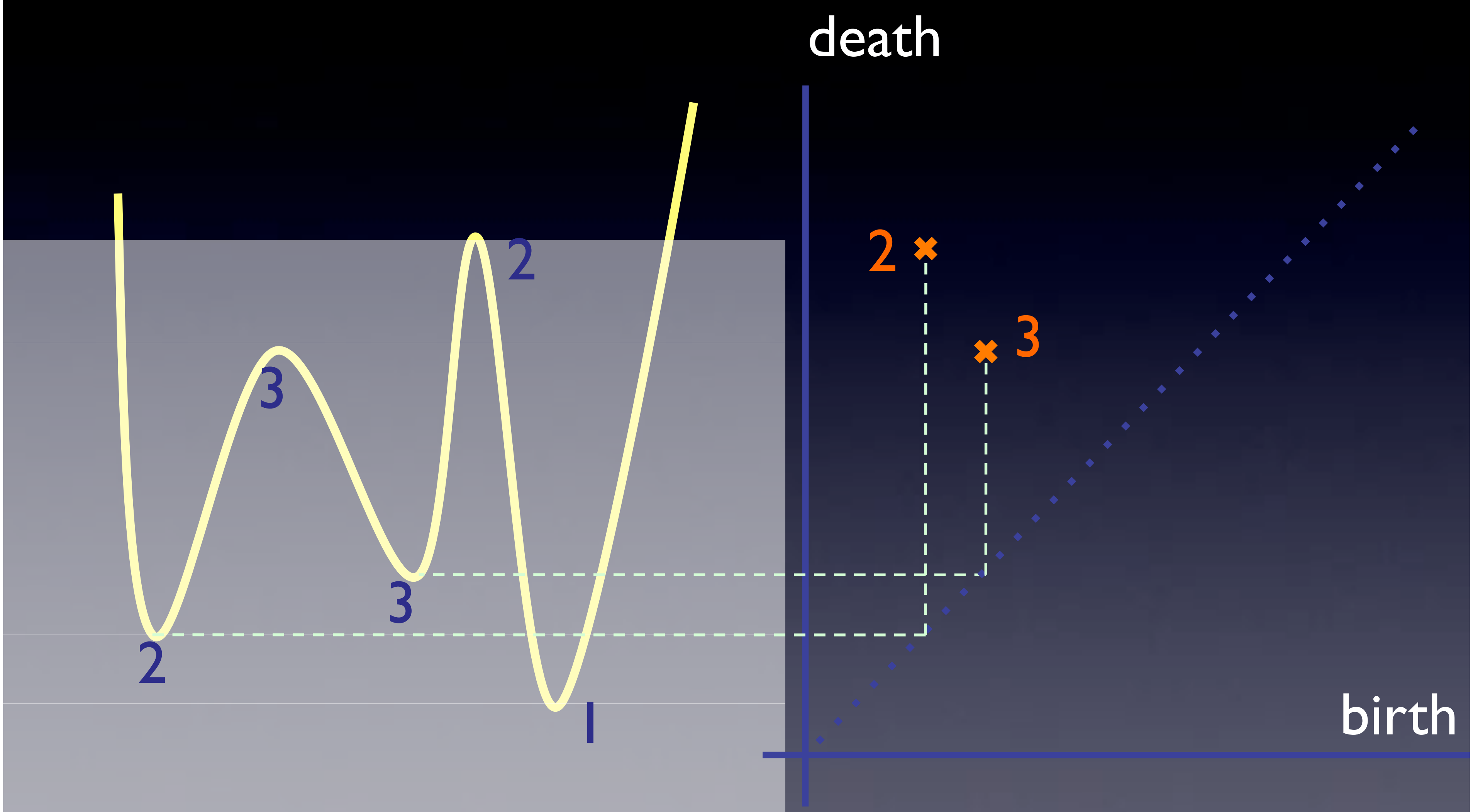
Death:

$$\#R(h_i) = \#R(h_i - \epsilon) - 1$$



Topological characteristic of sublevel set is completely characterized by tabulating the occurrence of critical values.

Persistence diagram

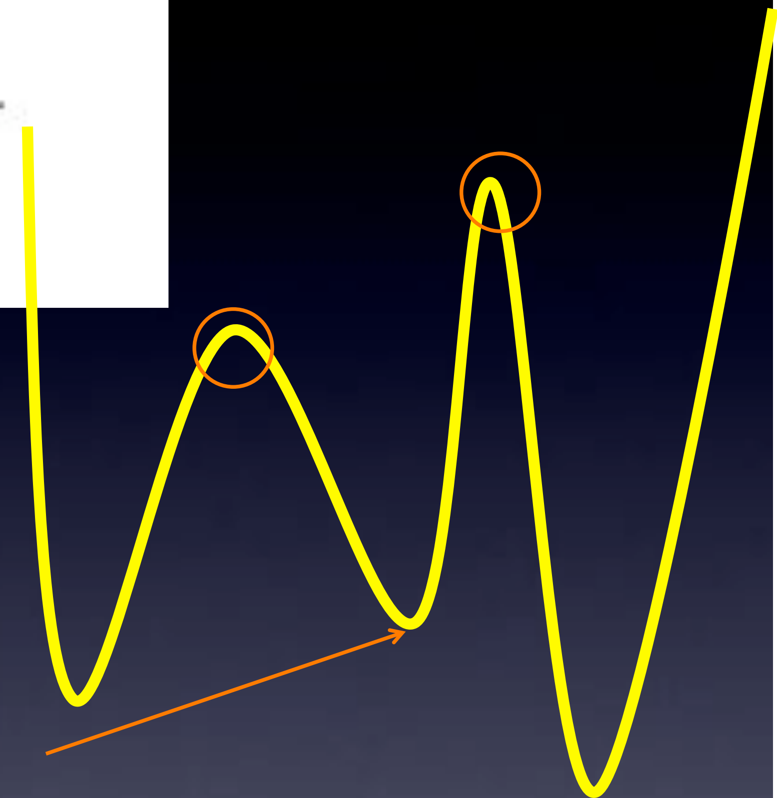


Pair the time of death with the time of the closest earlier birth

MICCAI 2009

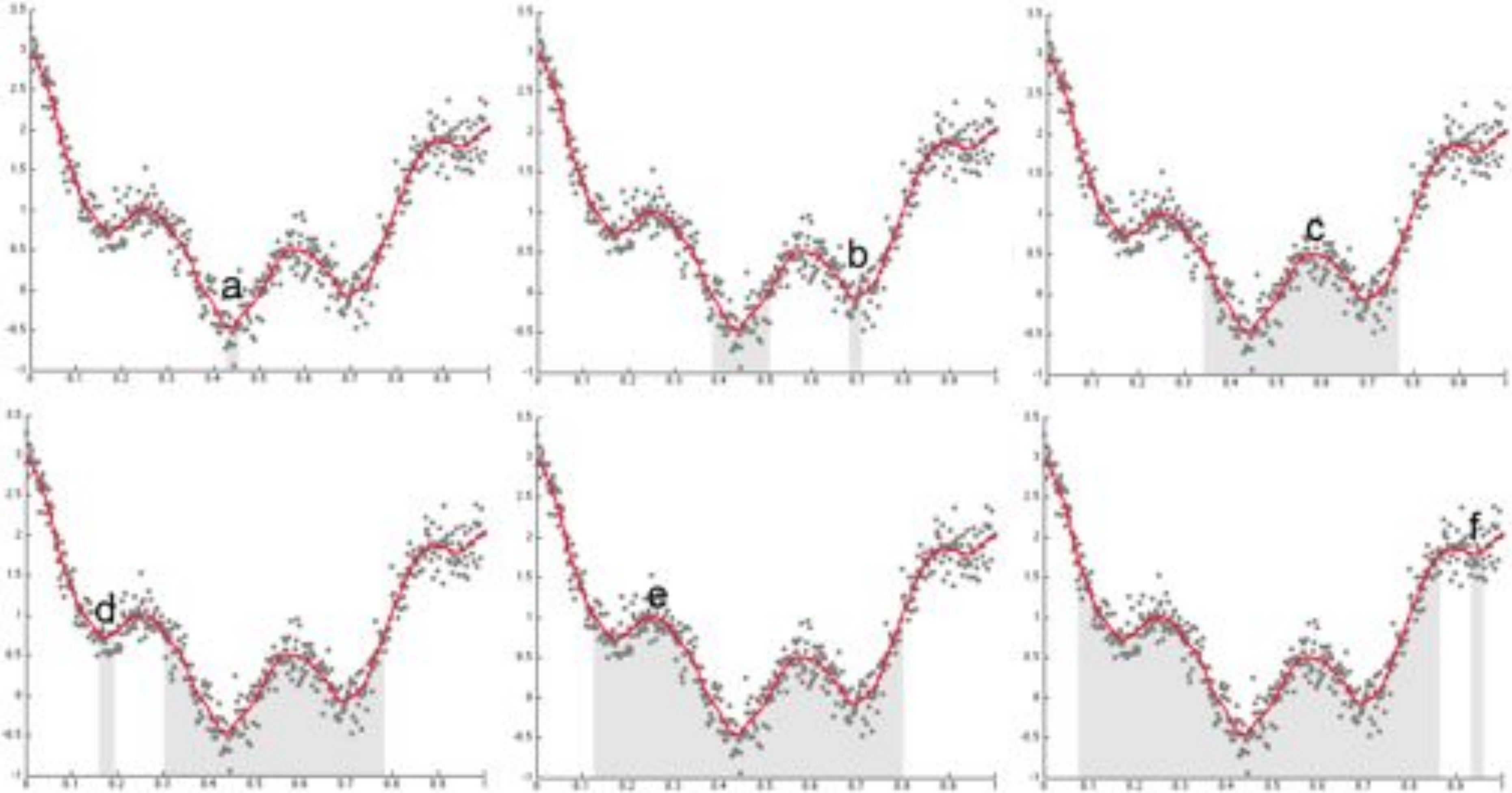
Algorithm 1 Iterative Pairing and Deletion

1. $H \leftarrow \{h_1, \cdots, h_n\}$.
2. $i \leftarrow m$.
3. $h_i^* = \arg\min_{h_j \in H} \{h_j | h_j > g_{(i)}, h_j \sim g_{(i)}\}$.
4. *If* $h_i^* \neq \emptyset$, *pair* $(g_{(i)}, h_i^*)$
5. $H \leftarrow H - h_i^*$.
6. *If* $i > 1$, $i \leftarrow i - 1$ *and go to Step 3.*



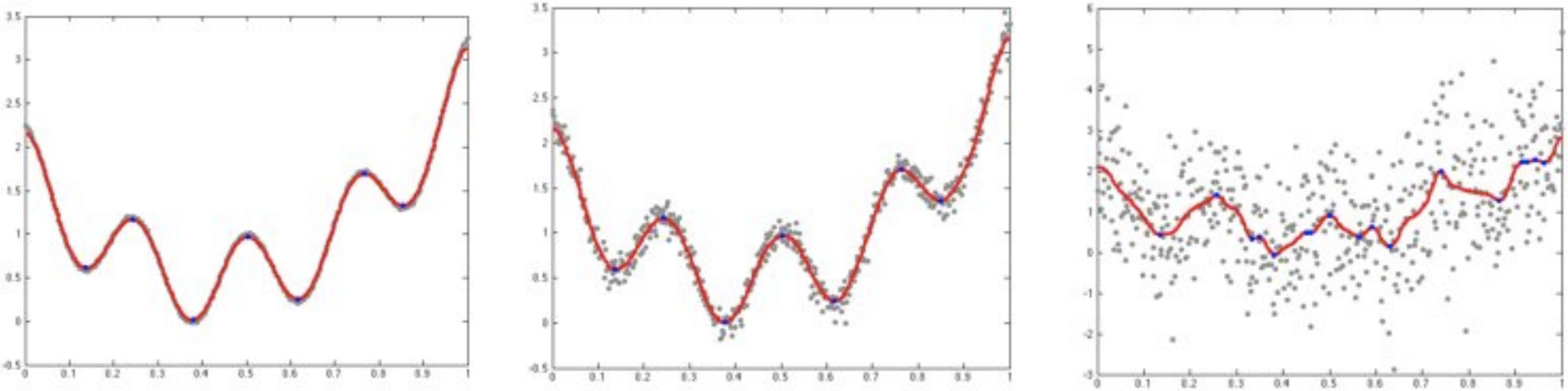
Essence of the algorithm:

1. start with the largest minimum
3. look above and check if there is a smallest local max.
4. Pair min. and max., and delete them
6. Go to the second largest minimum

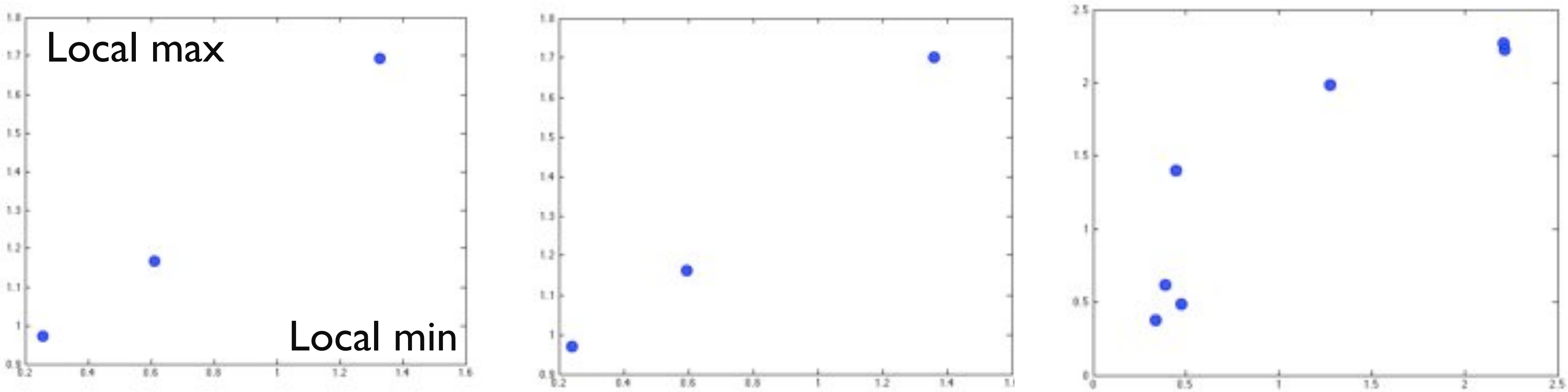


Pairing rule: when we pass through a maximum and merge two components, we pair the maximum with the higher of the two minimums of the two components → (c, b), (e, d)

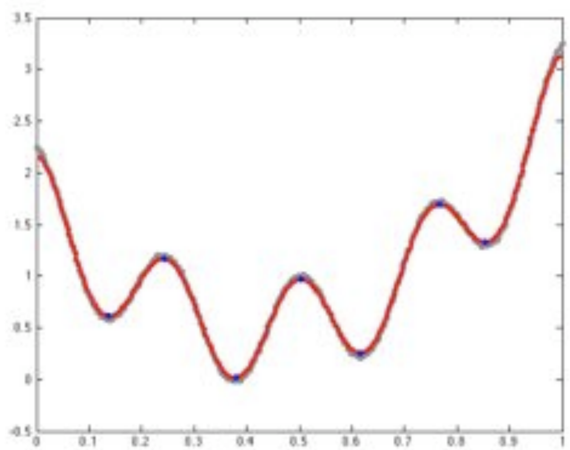
Simulation examples:



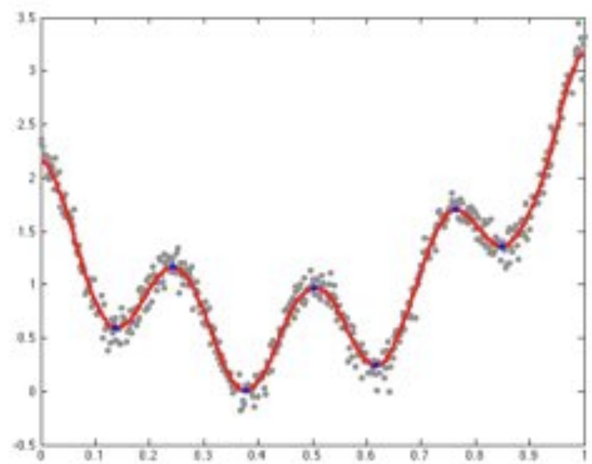
$\epsilon \sim N(0, 0.01^2)$ $\epsilon \sim N(0, 0.1^2)$ $\epsilon \sim N(0, 1^2)$



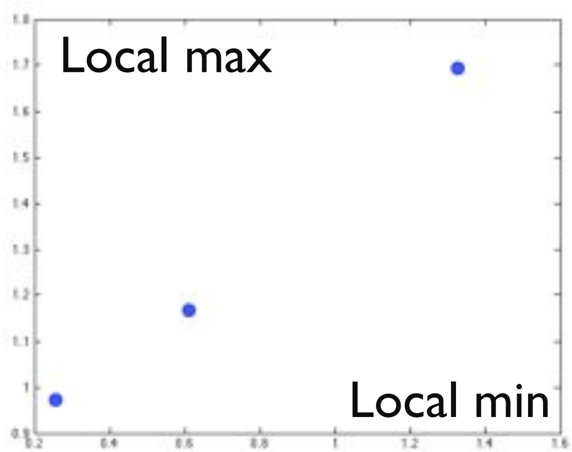
Stability of persistence diagram?

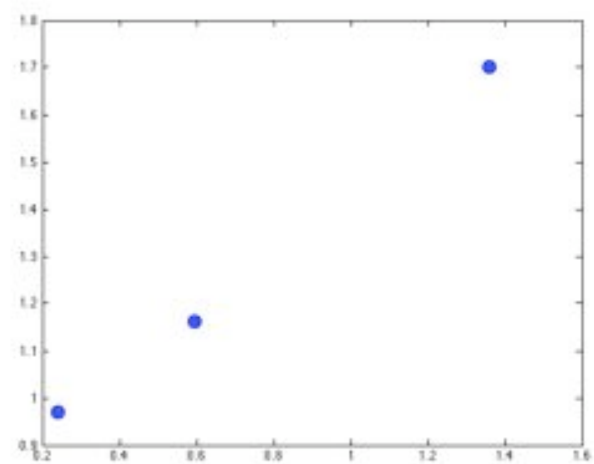


$\epsilon \sim N(0, 0.01^2)$



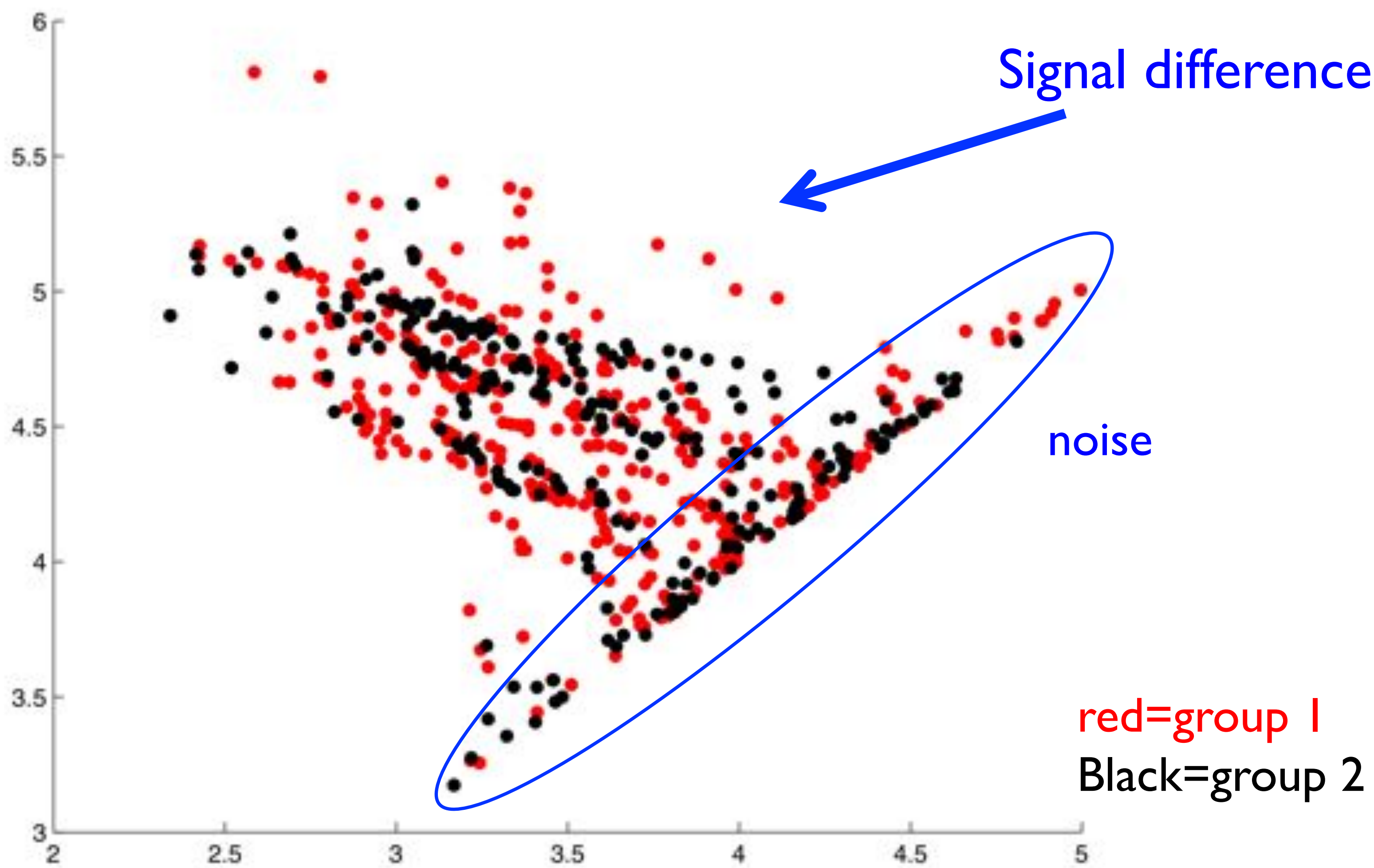
$\epsilon \sim N(0, 0.1^2)$





Stable under small perturbation

Statistical analysis on persistence diagram

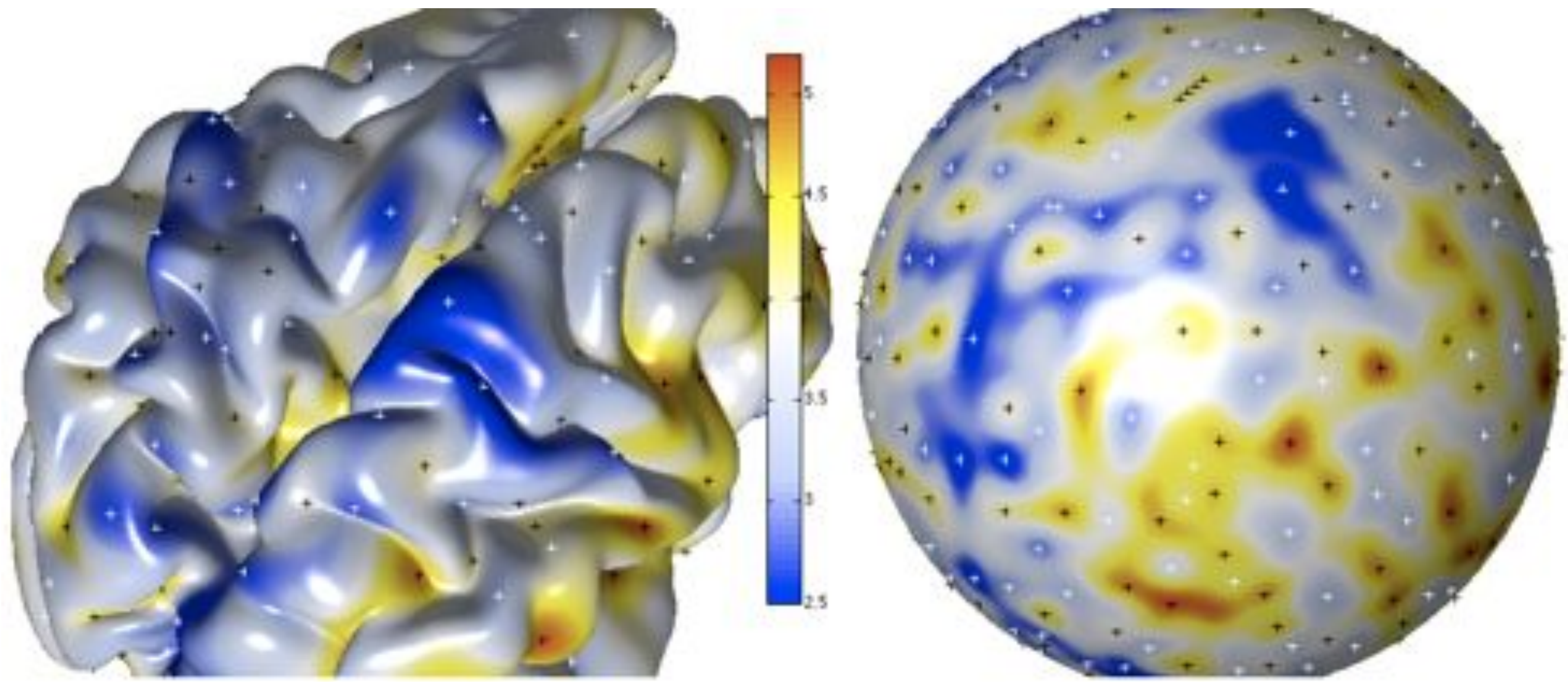


How to do a population study?

MATLAB demonstration

Read chung.2010.IPMI.pdf

Cortical thickness data on cortical manifolds



White cross = min, black cross = max

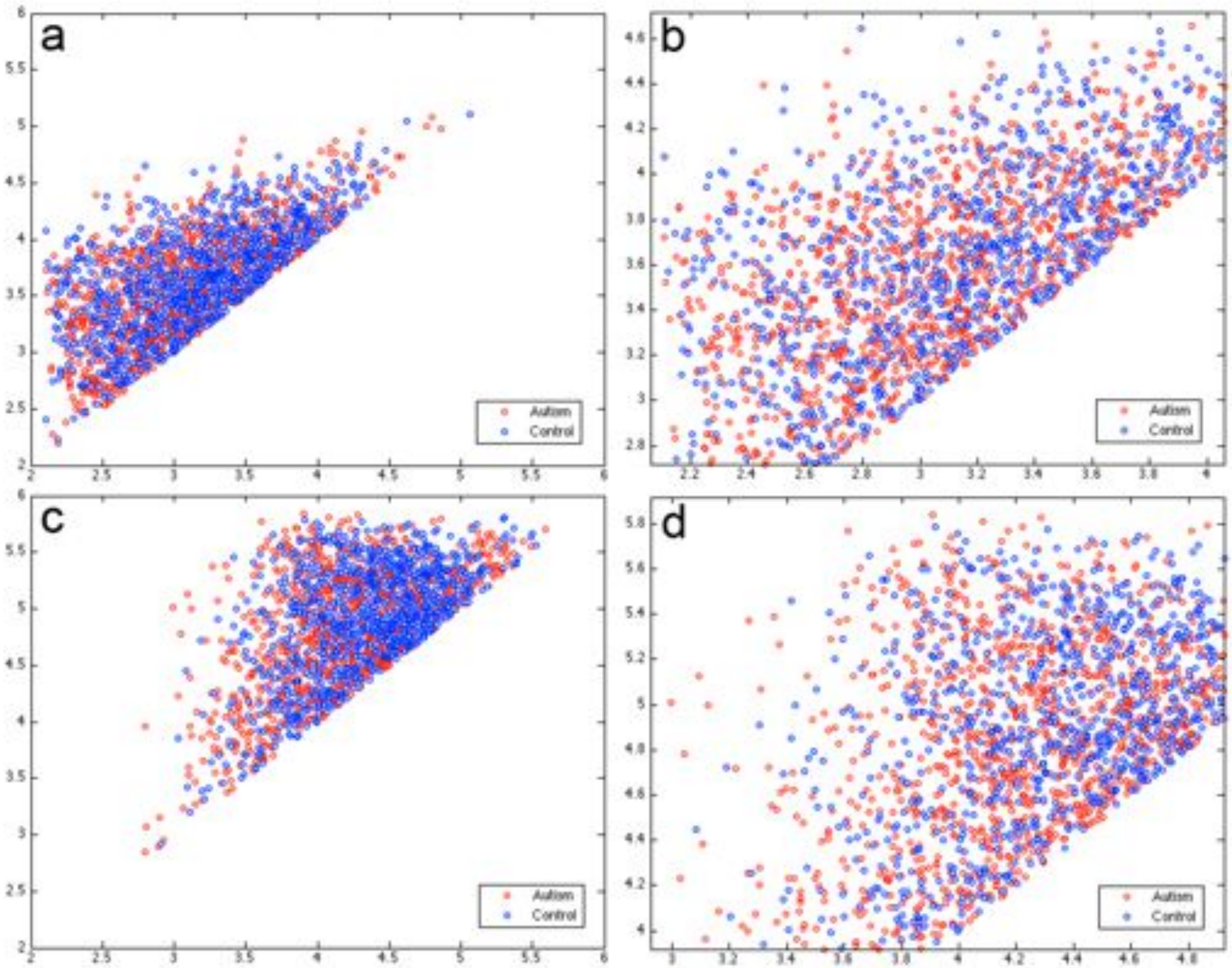
Saddle points are not shown

Persistence Diagrams

blue= control (n=11), red= autism (n=16)

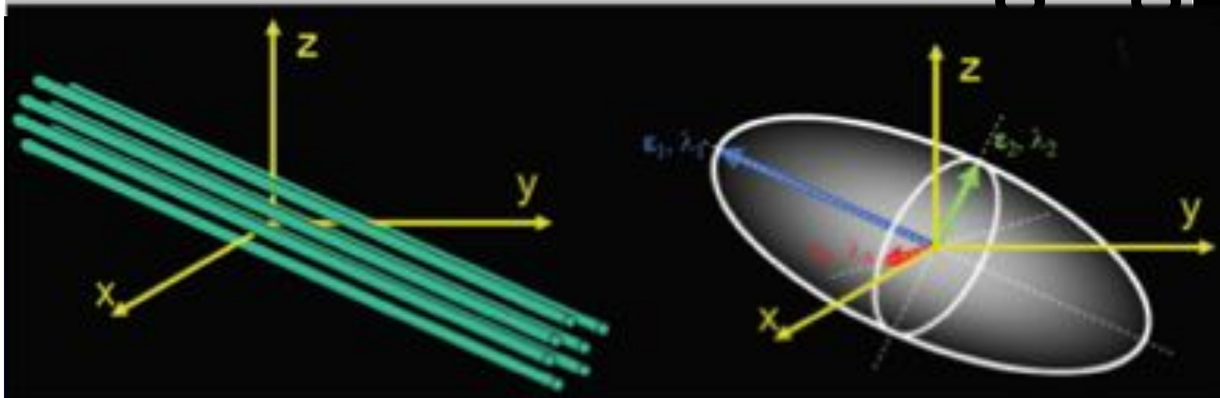
degree 0
pairing of saddle points to minimums

degree 1
pairing of saddle points to maximums



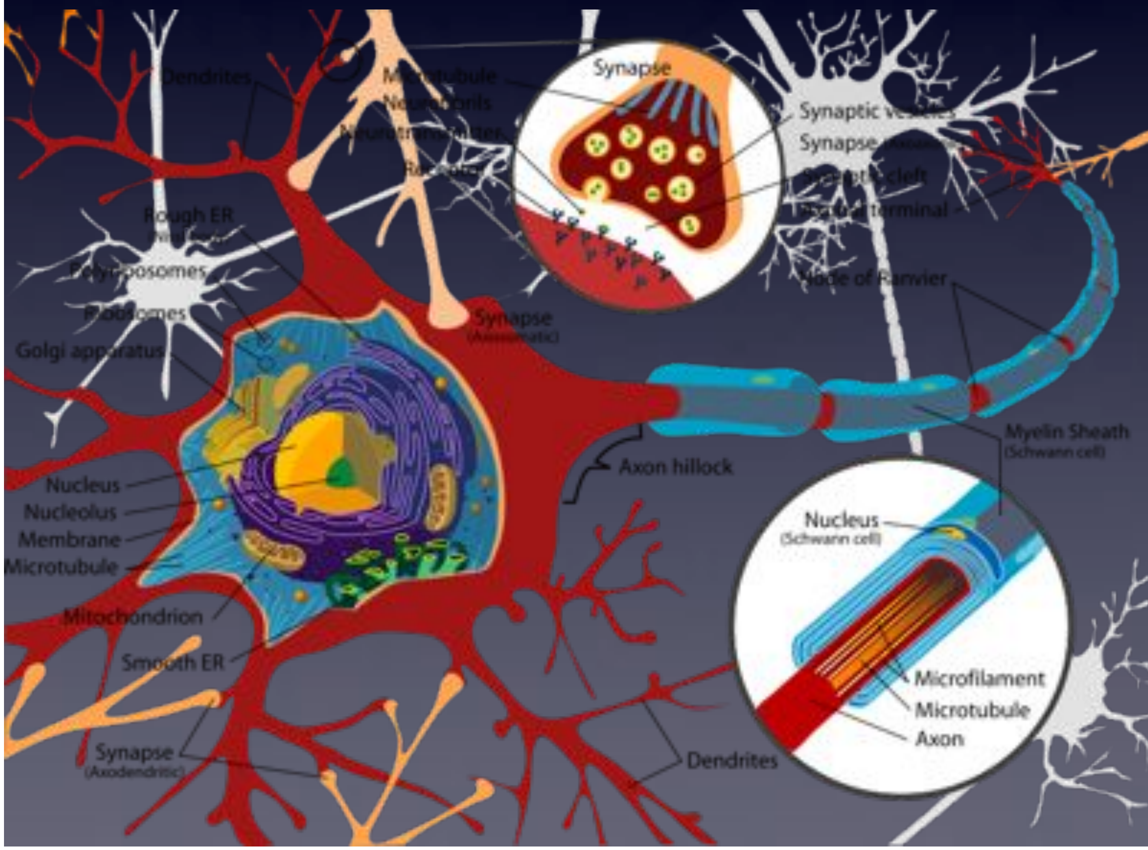
Diffusion Tensor Imaging

Mori and van Zijl NMR Biomed 2002

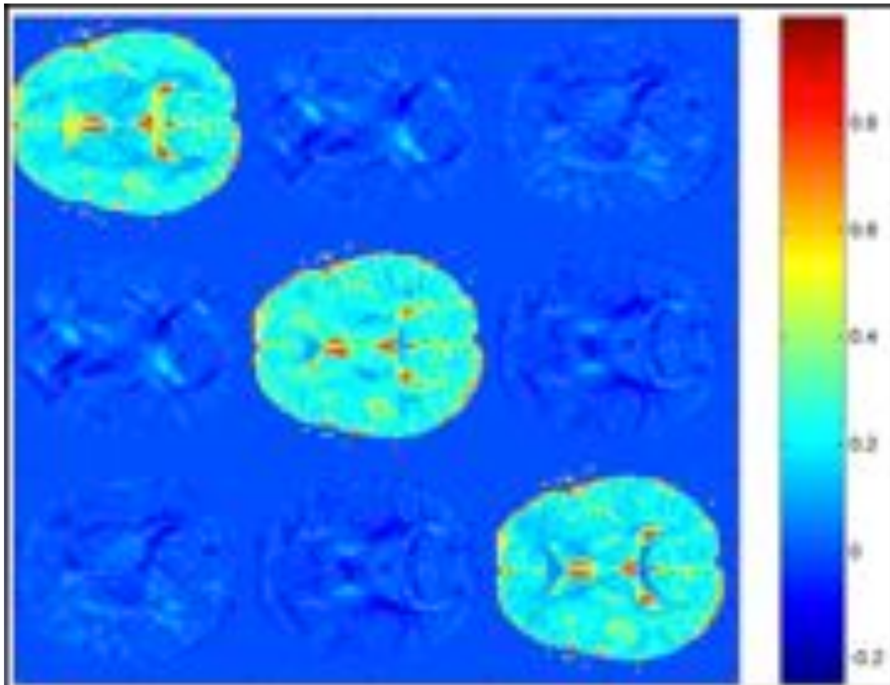


The movement of anisotropic water diffusion can be measured using DTI

↓

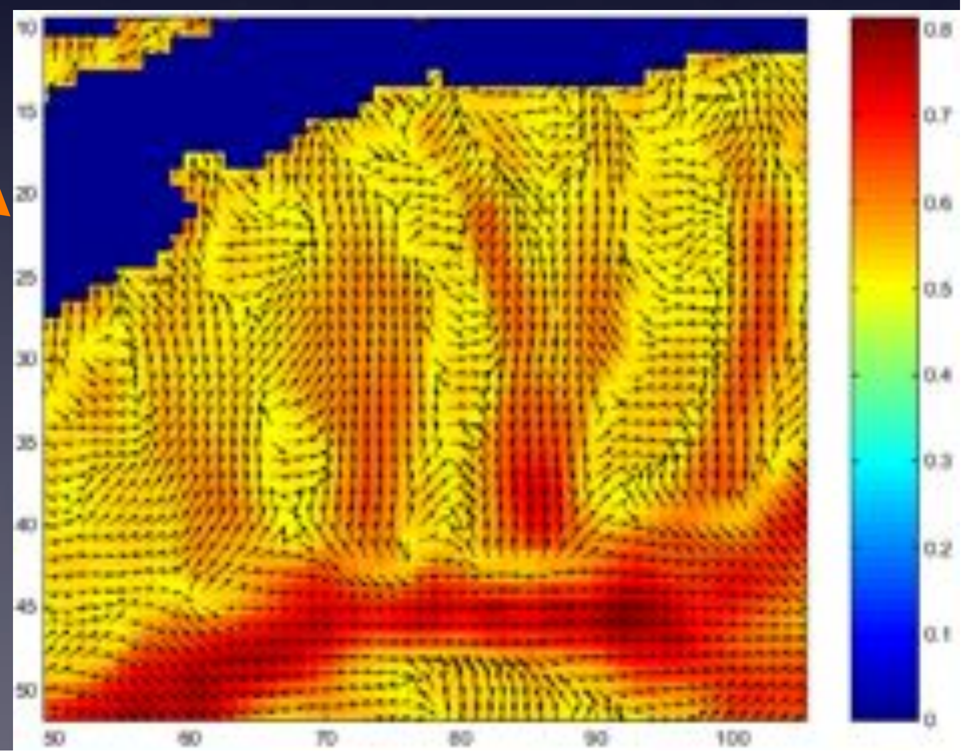


The direction of neuronal filaments in the axon dictates the movement of water diffusion.



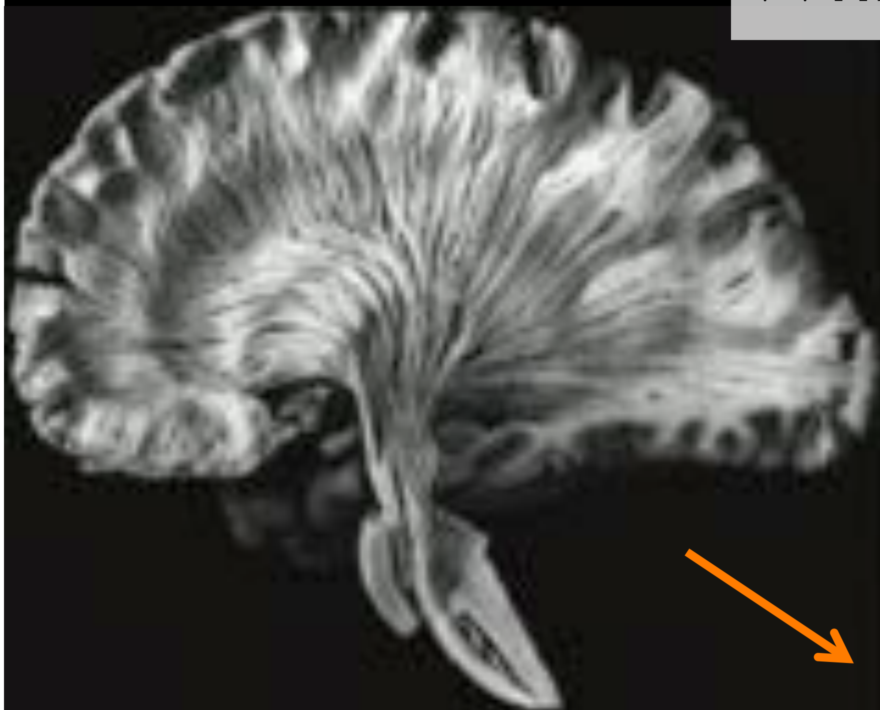
Direction of diffusion is encoded in 3x3 matrix *D* (diffusion tensor)

Principal eigenvectors of D

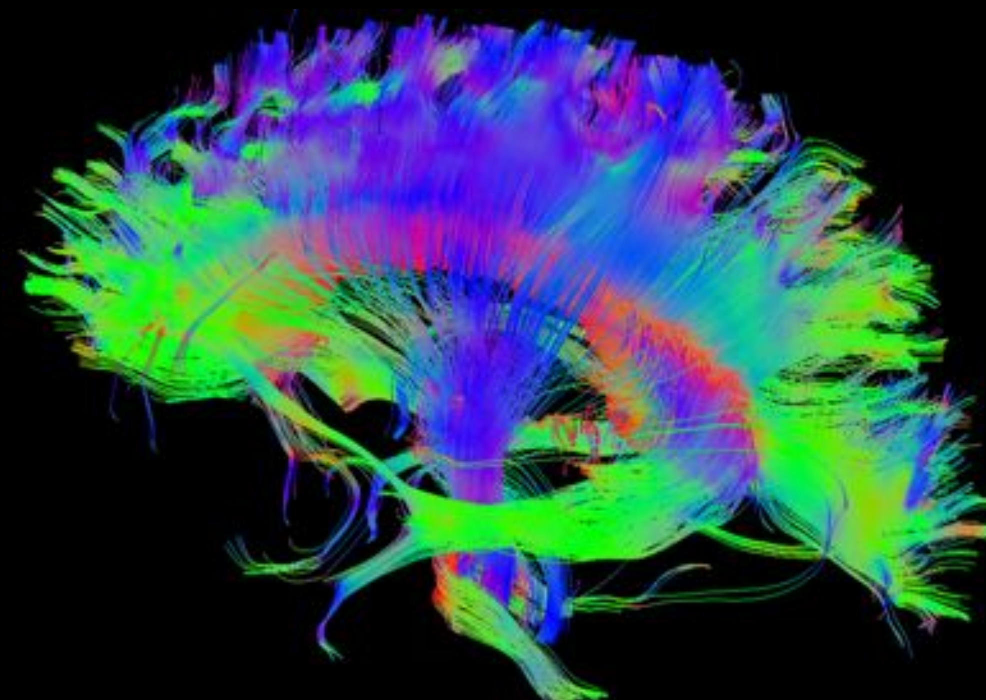


Tractography done using the second order Runge-Kutta algorithm with TEND (Lazar et al., HBM 2003)

White Matter Fiber Tractography

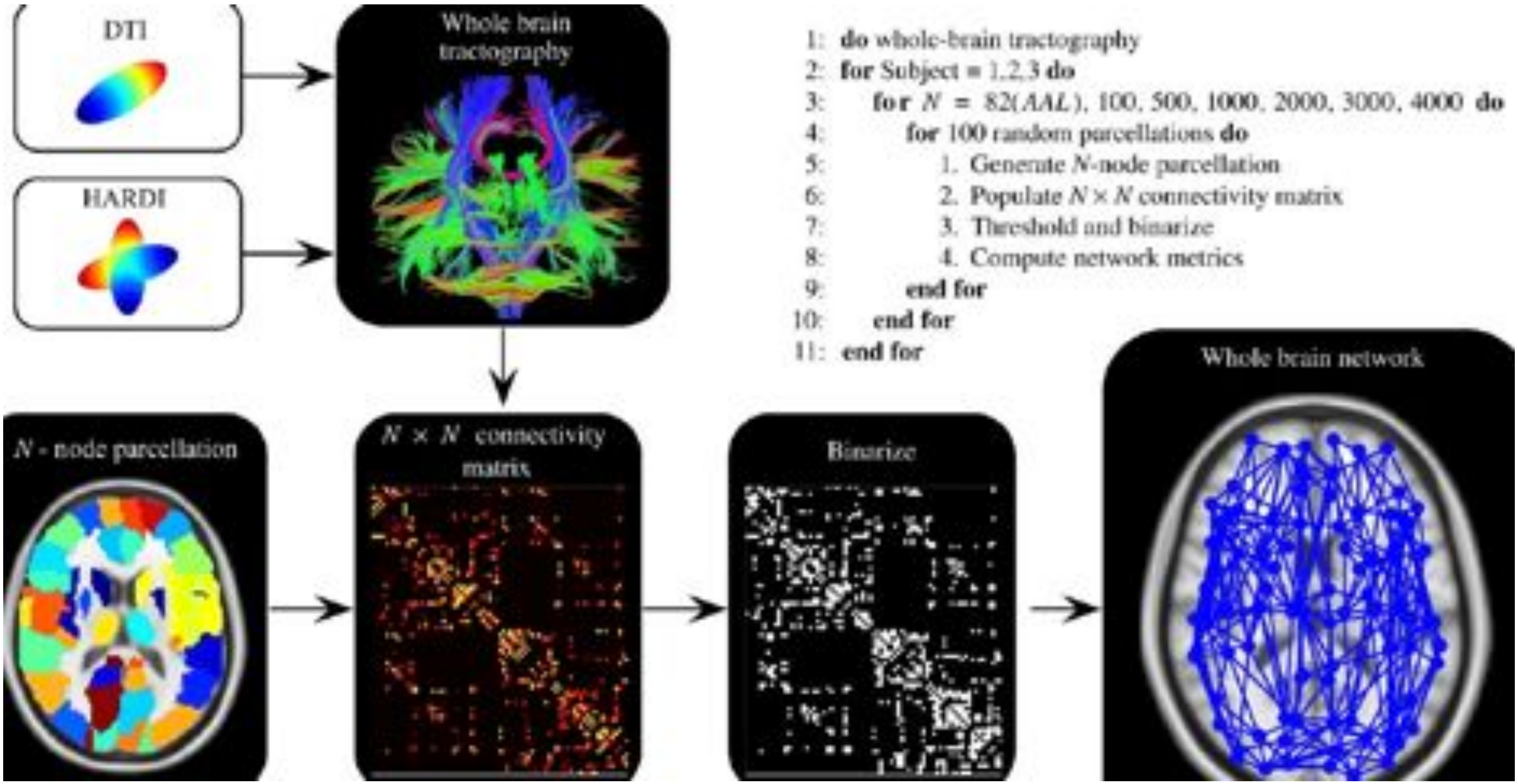


Postmortem



Reconstructed
0.5 million tracts

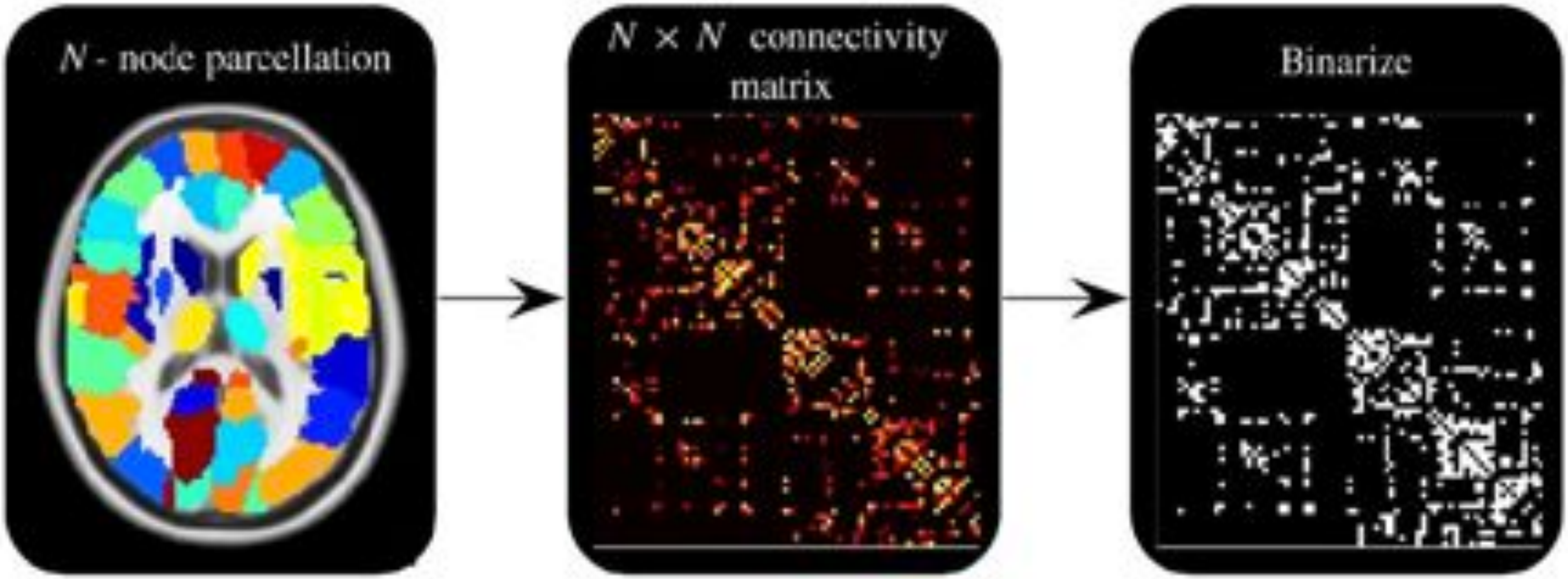
Standard graph construction pipeline



Zalesky et al. 2010

Structural connectivity in DTI:

DTI connectivity graphs are constructed using the epsilon neighbor method



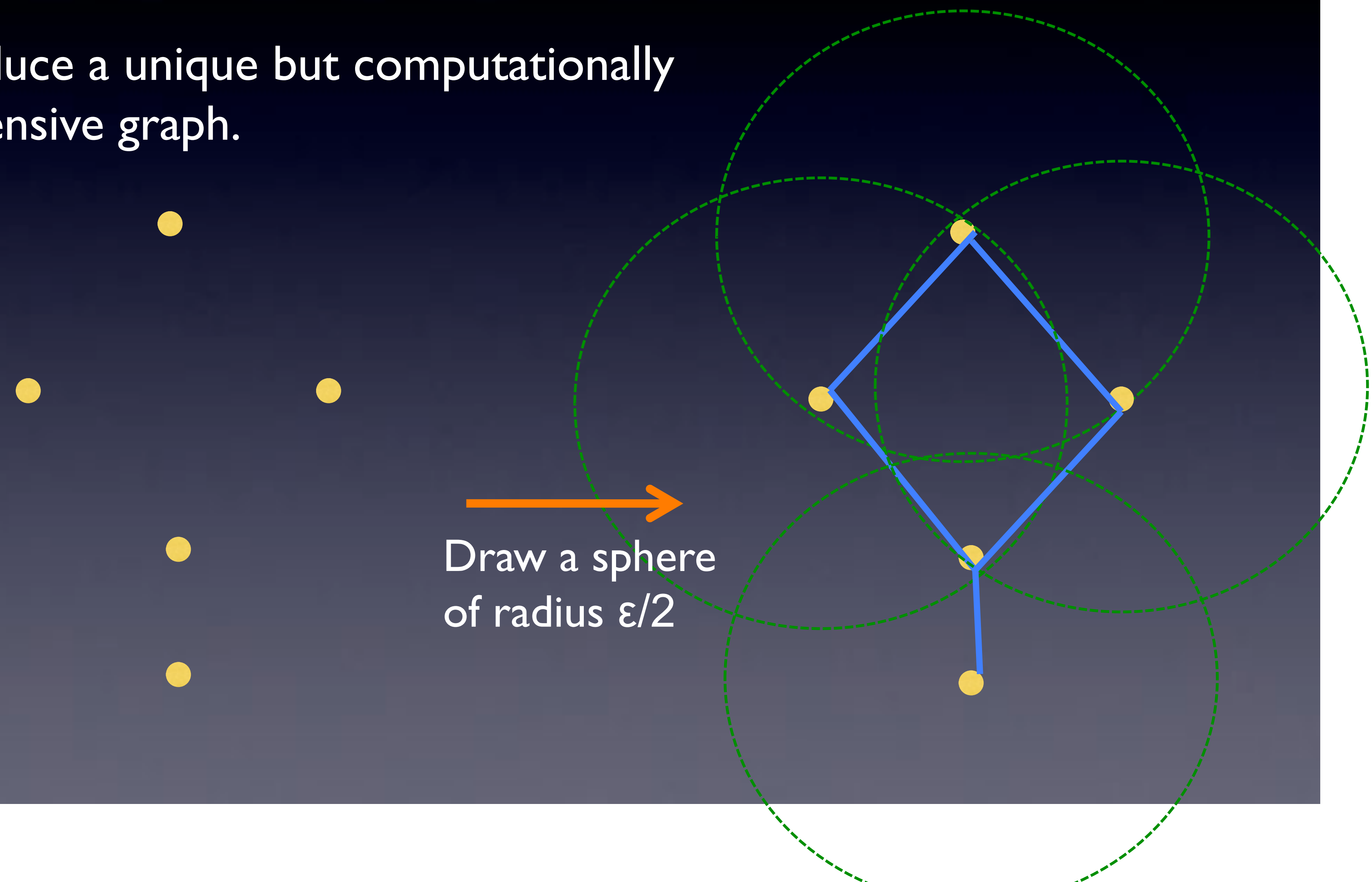
No parcellation

No thresholding

Rips complex of cloud point data

Used for obtaining the topological data structure of cloud point data.

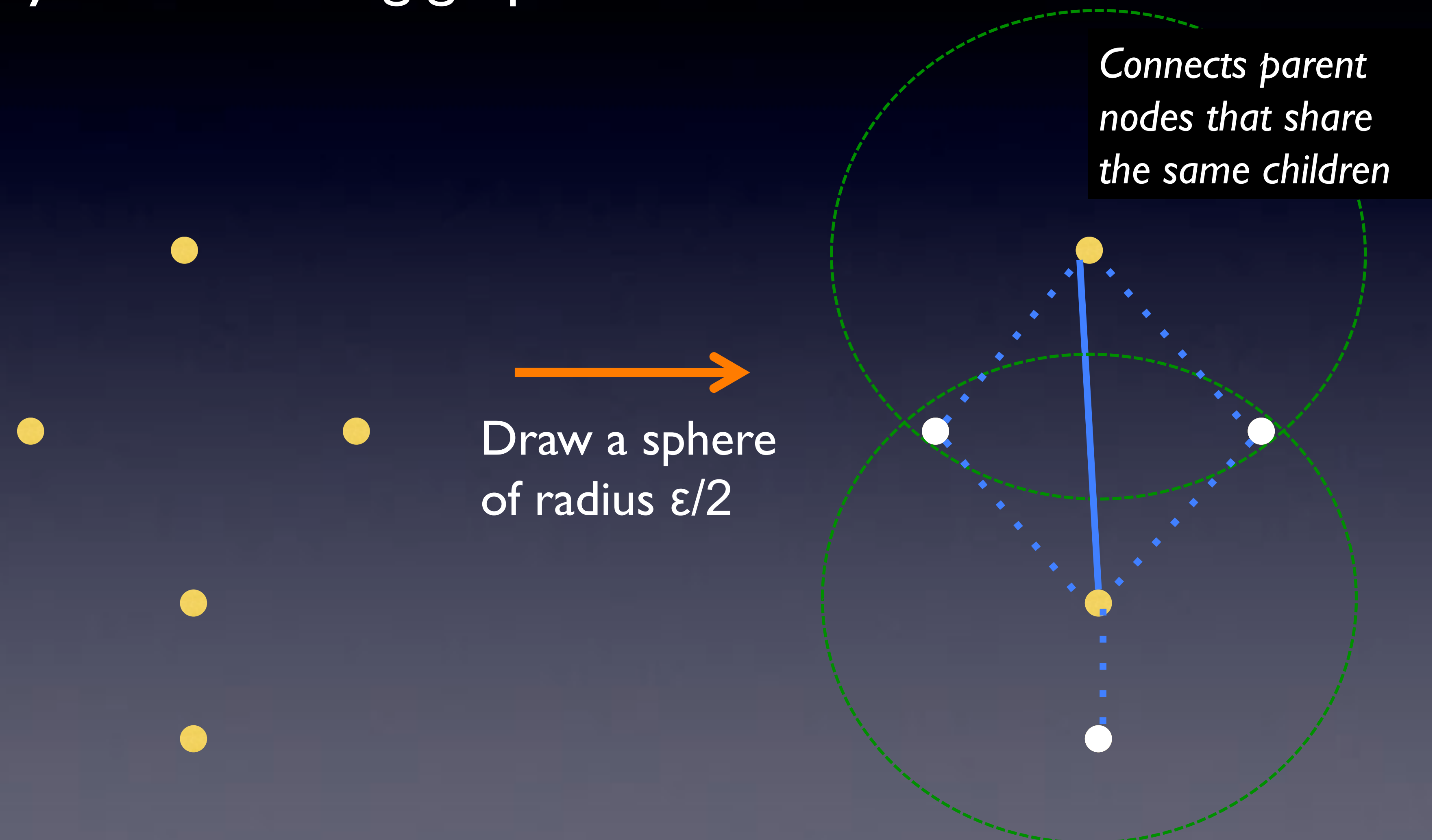
Produce a unique but computationally expensive graph.



ε-neighbor graph

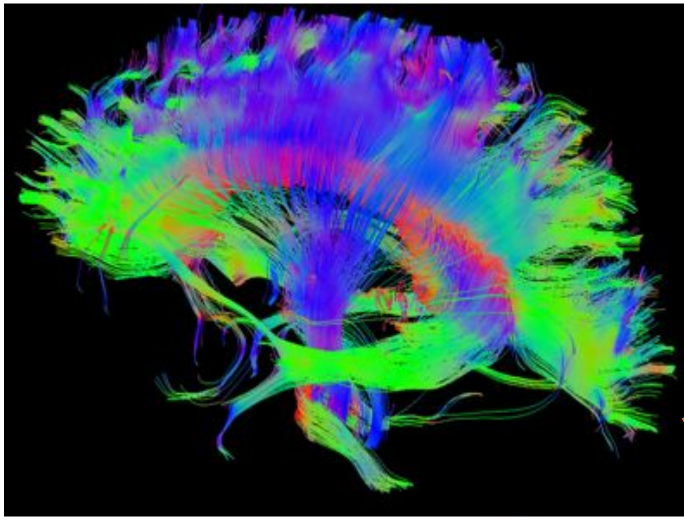
Produce a nonunique but simpler graph structure

A way of smoothing graph



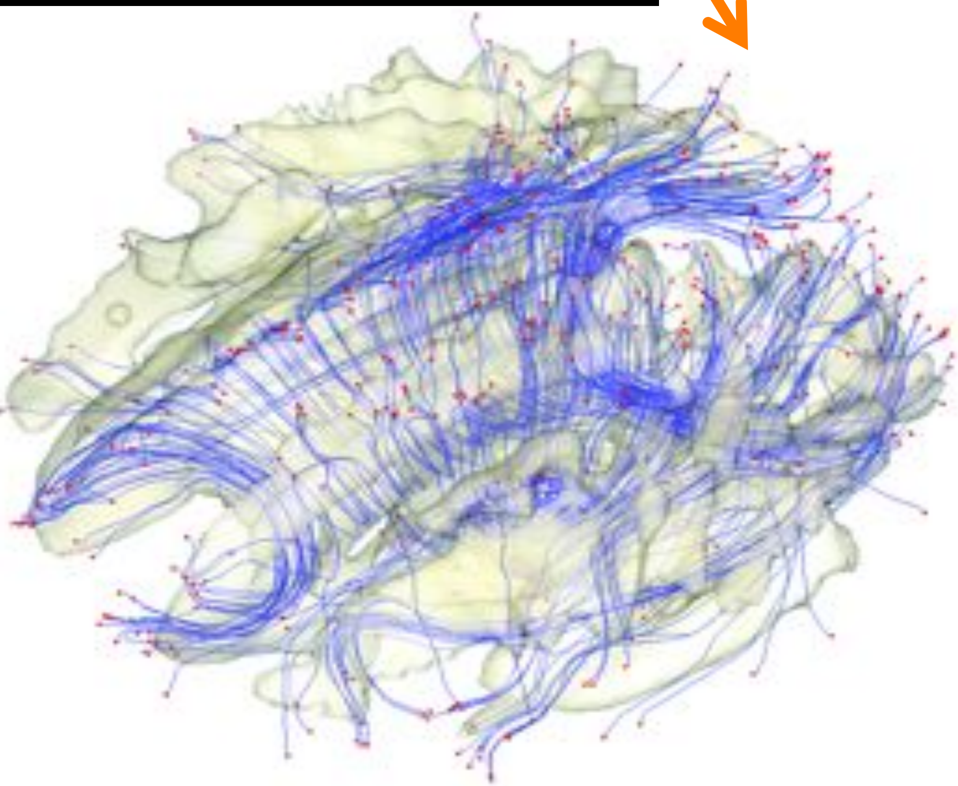
MATLAB demonstration

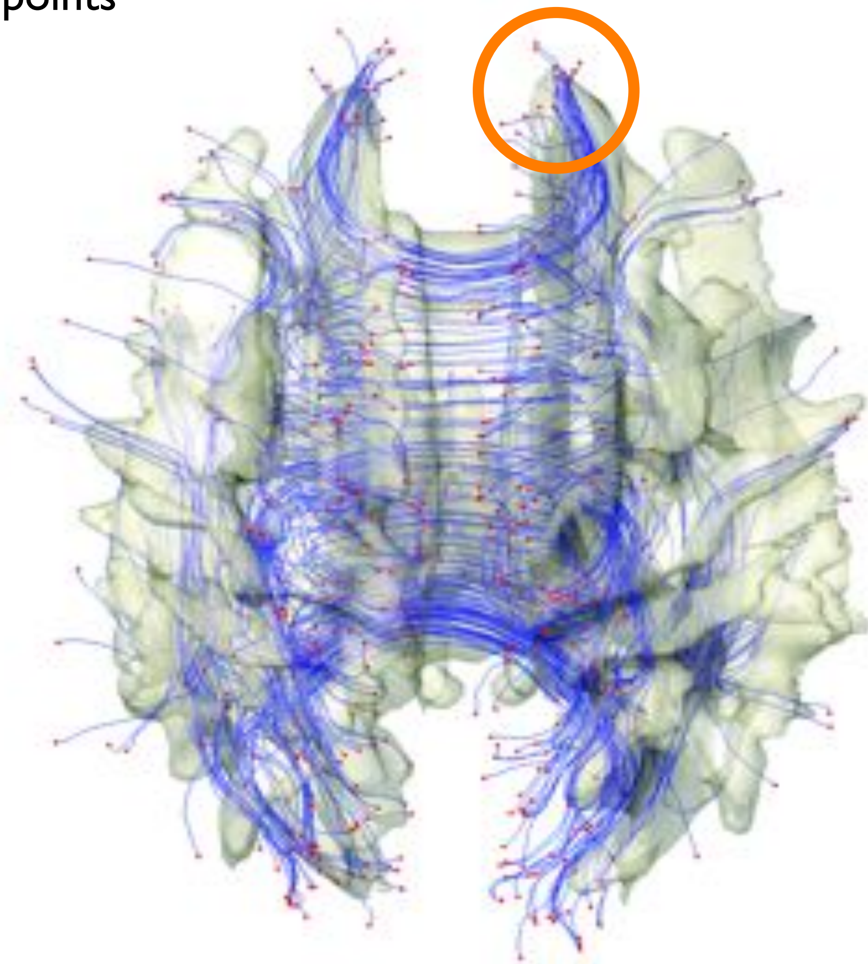
Graph construction



Identify end points

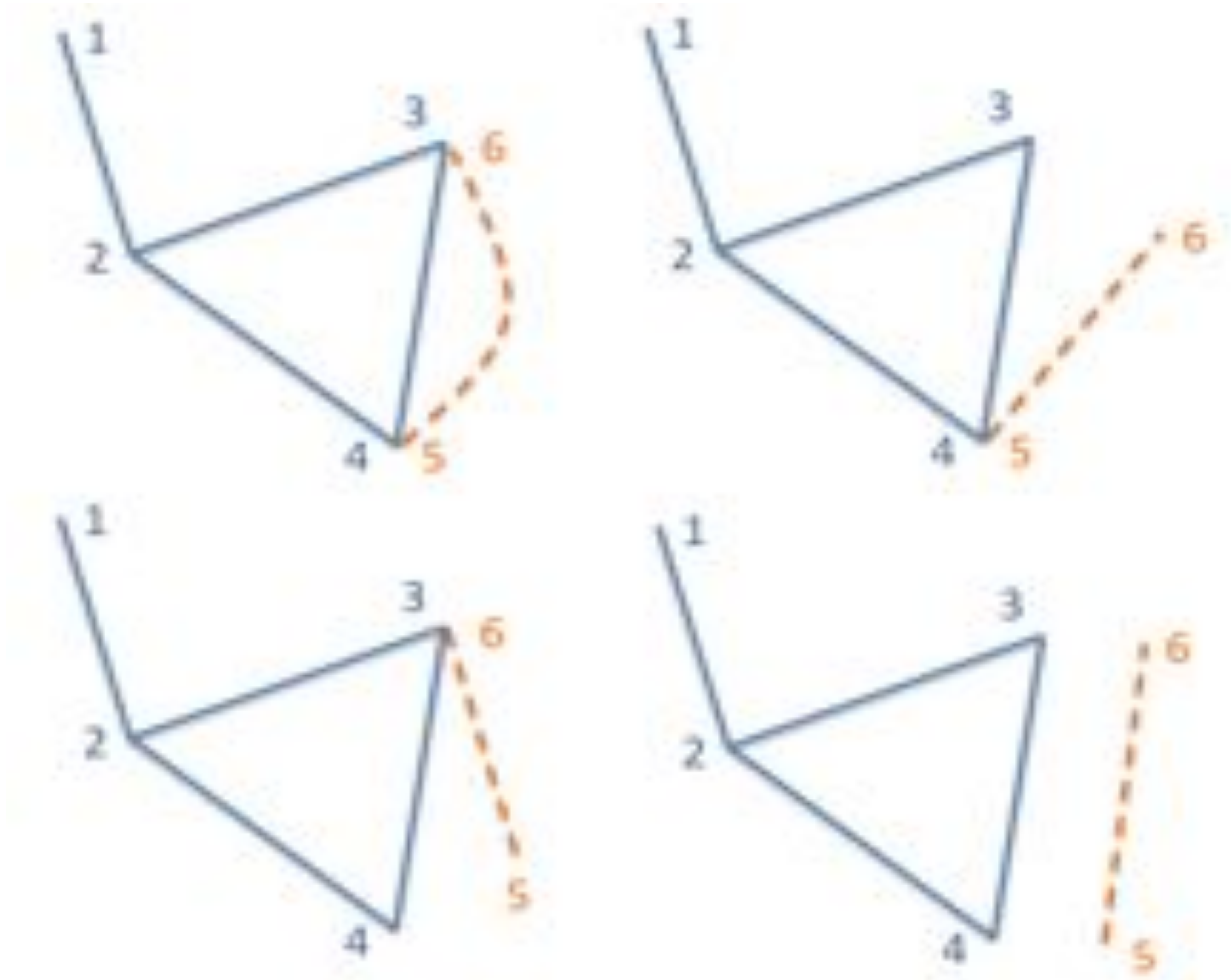
ε-neighbor graph:
All points in the ε-neighbor are identified as a single node in a graph





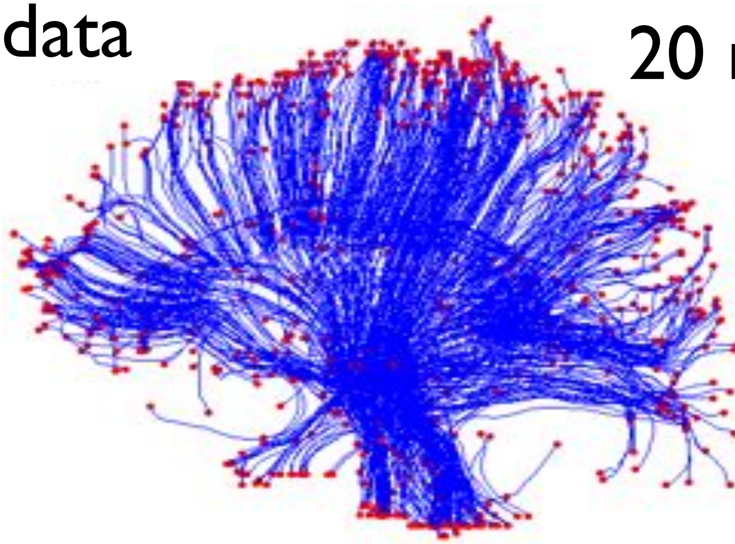
Surface: white matter boundary

Iterative graph construction

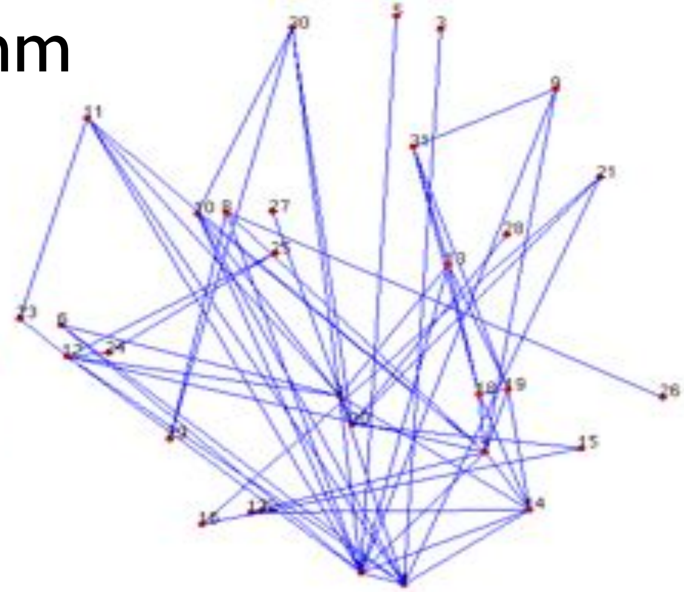


ε-neighbor graphs with different ε

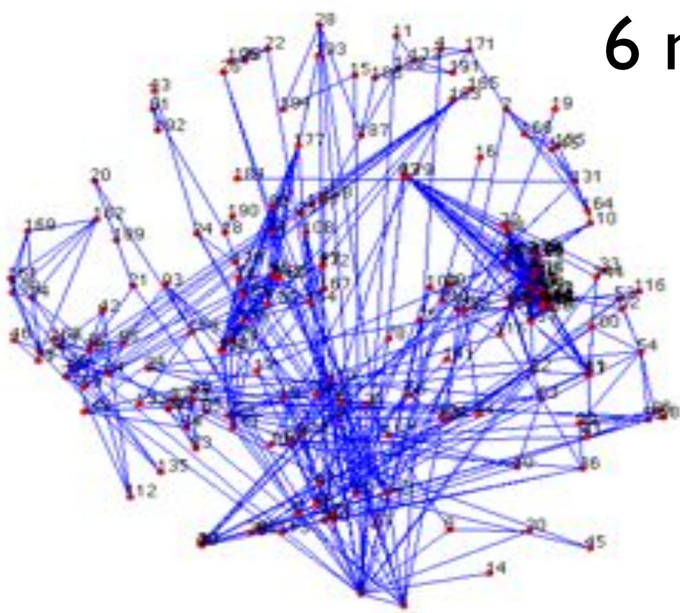
original data



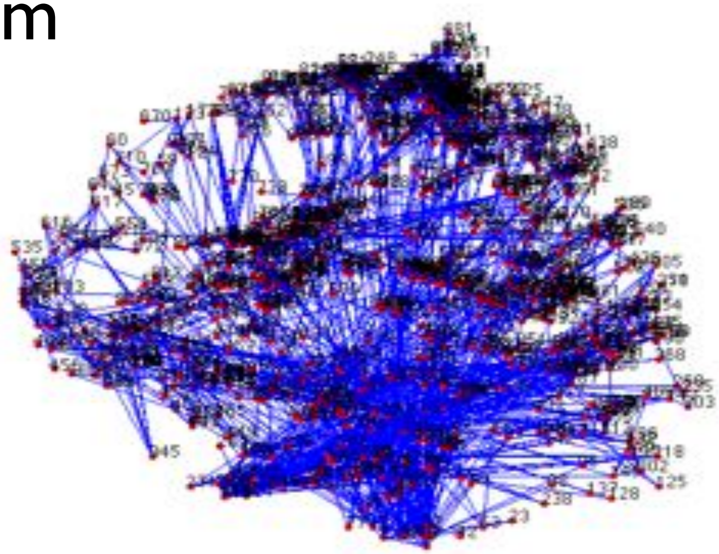
20 mm



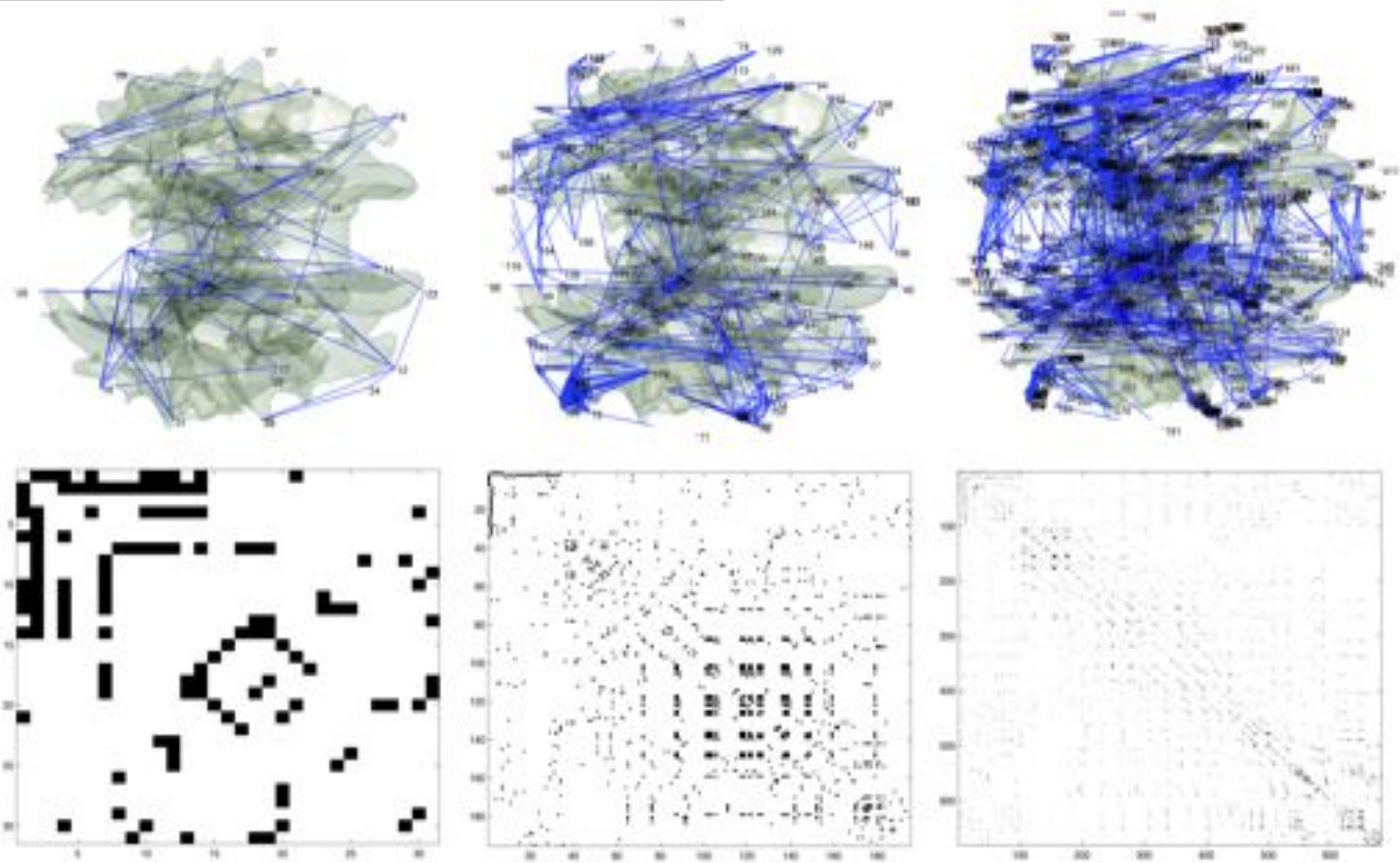
10 mm



6 mm



Adjacency matrix



Selected as one of the best abstracts in HBM2010 meeting (e-poster)

Main contribution: *the first data-driven DTI structural network construction framework without parcellation.*

MATLAB demonstration

Lecture 7

More on topology &
brain network modeling
Sparse modeling
Compressed sensing

Read

chung.2009.miccai…
horak.2009……
lee.2010.tmi….