Computational Methods in NeuroImage Analysis

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Lecture 5
Curvilinear Structure Modeling

October 1, 2010

Curvilinear brain structures

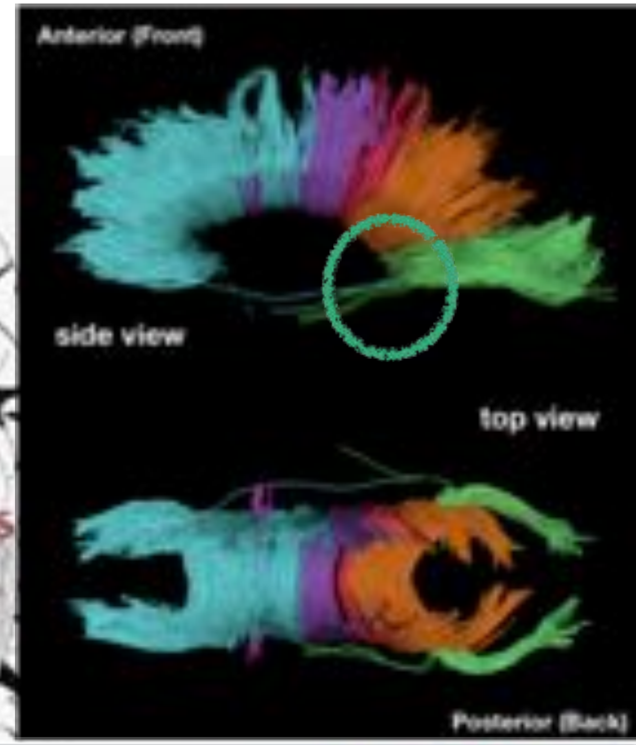
White matter fiber tracts

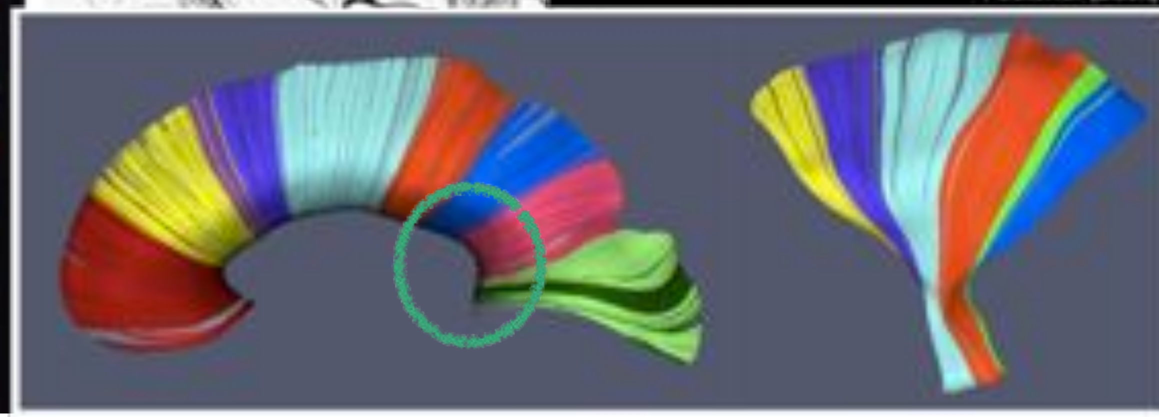
Corpus callosum boundary

Major sulcal lines

White matter fibers

James Gee



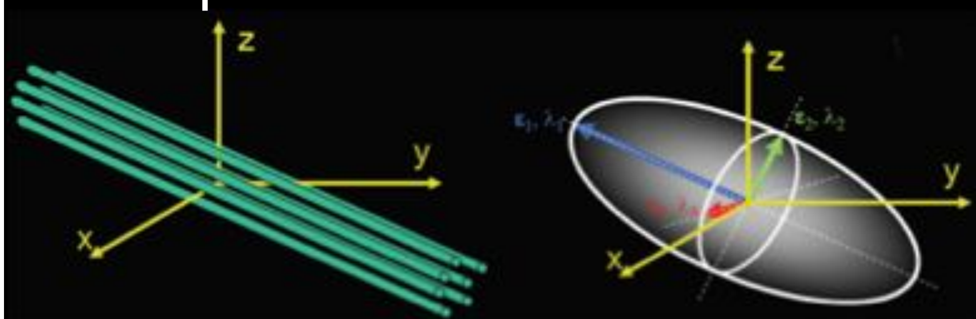


Fibers passing through the splenium of the corpus callosum

Diffusion Tensor Imaging

Mori and van Zijl NMR Biomed 2002

isotropic diffusion



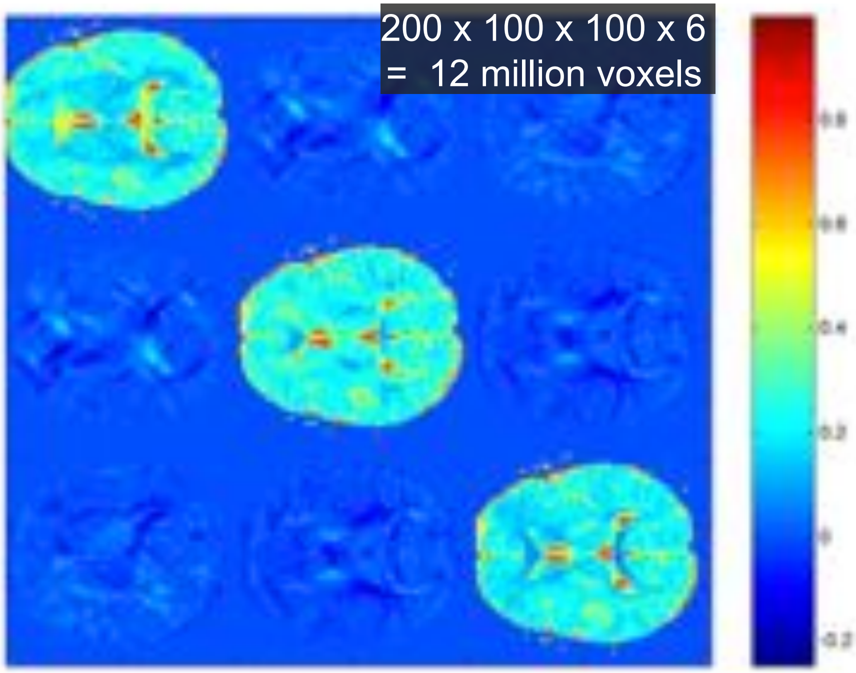
anisotropic diffusion

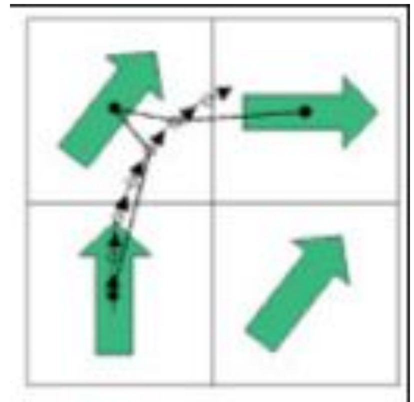
diffusion tensor

$$p(x \mid x_o, \tau) = \frac{1}{\sqrt{(4\pi\tau)^3 |\underline{D}|}} \exp\left(\frac{-(x - x_0)^T \underline{D}^{-1} (x - x_0)}{4\tau} \right)$$

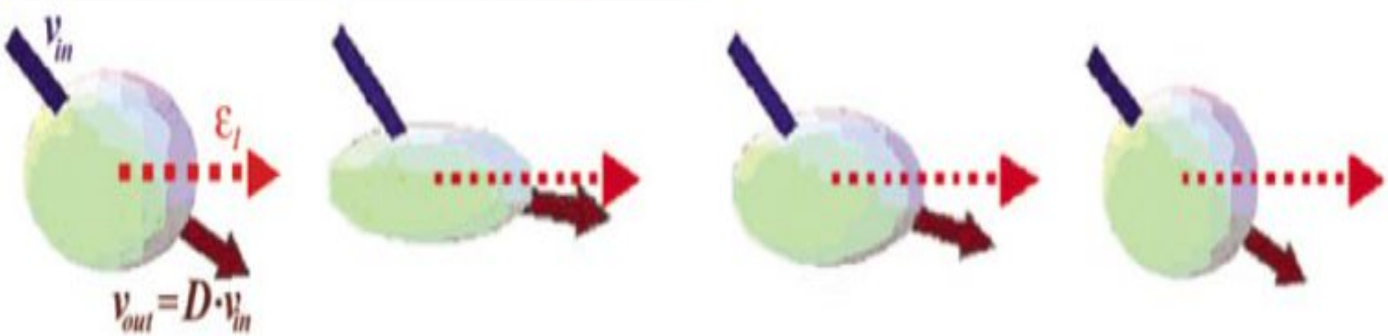
transition probability from x_0 to x

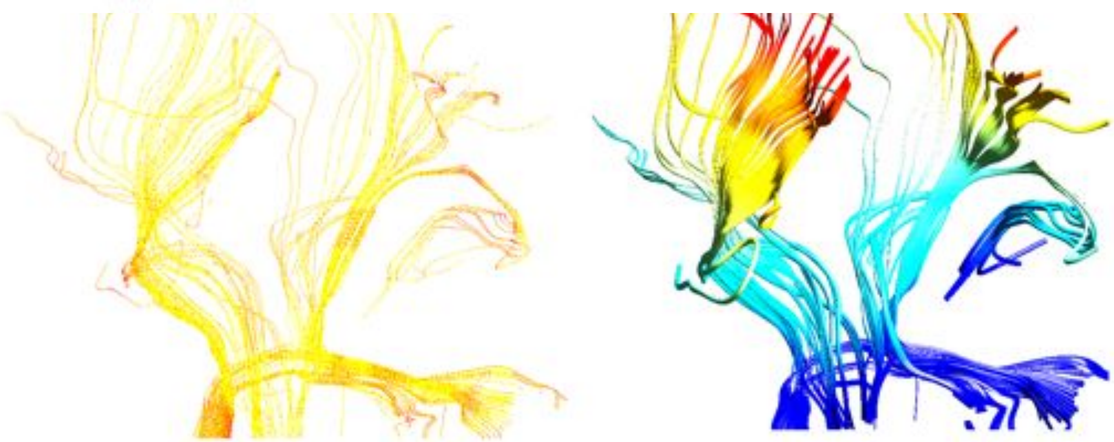
Tractography





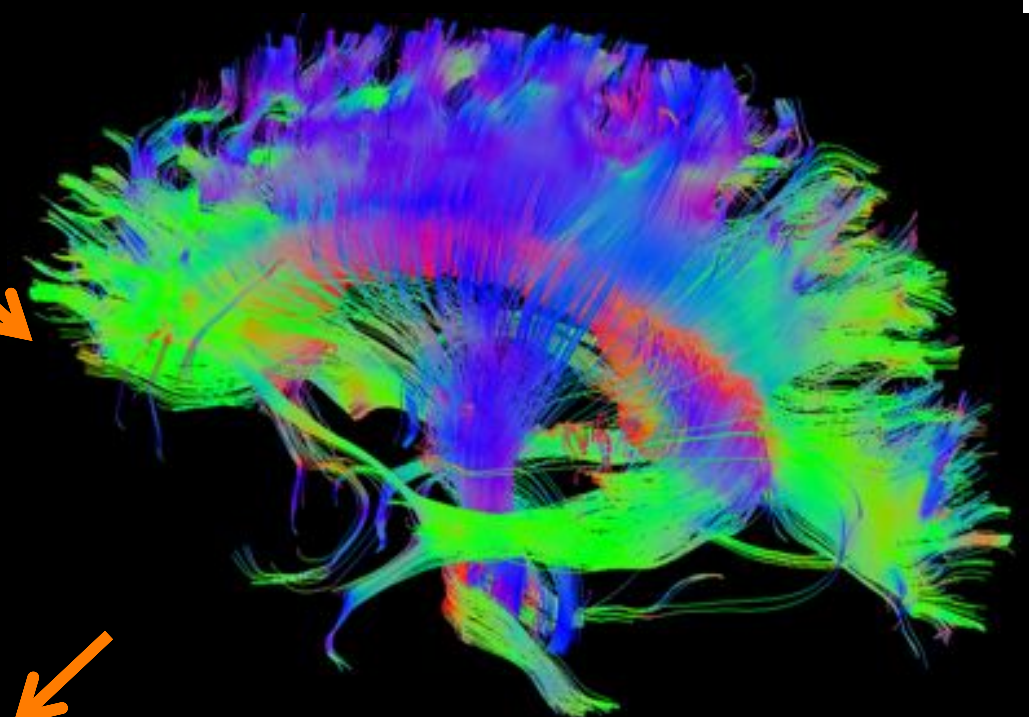
TENsor Deflection (TEND) algorithm

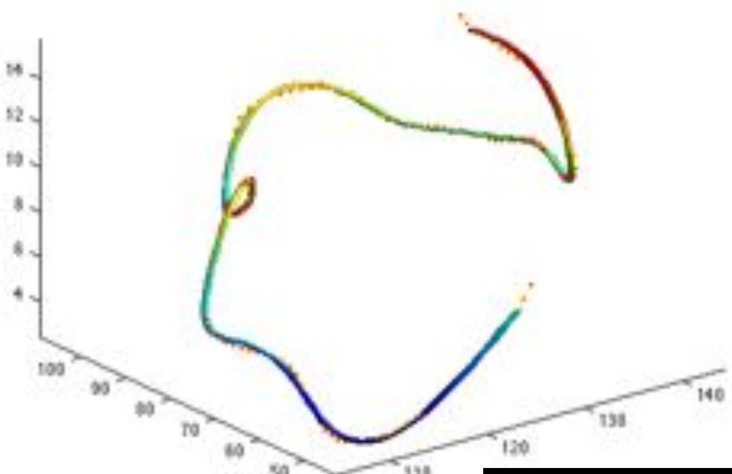




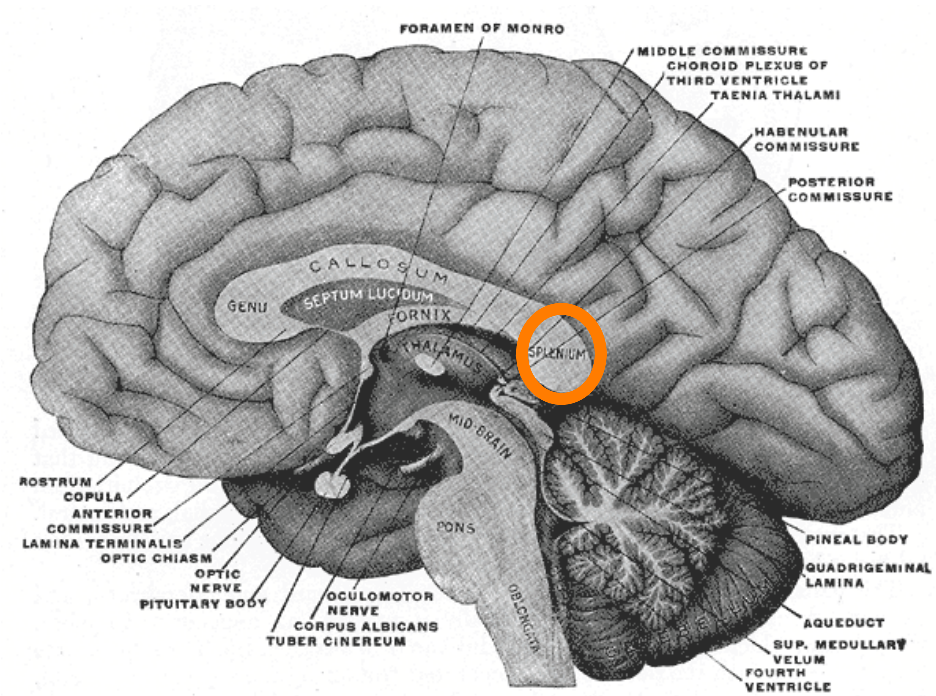
Second order Runge-Kutta algorithm with TEND (Lazar et al., HBM 2003)

White matter fiber tractography

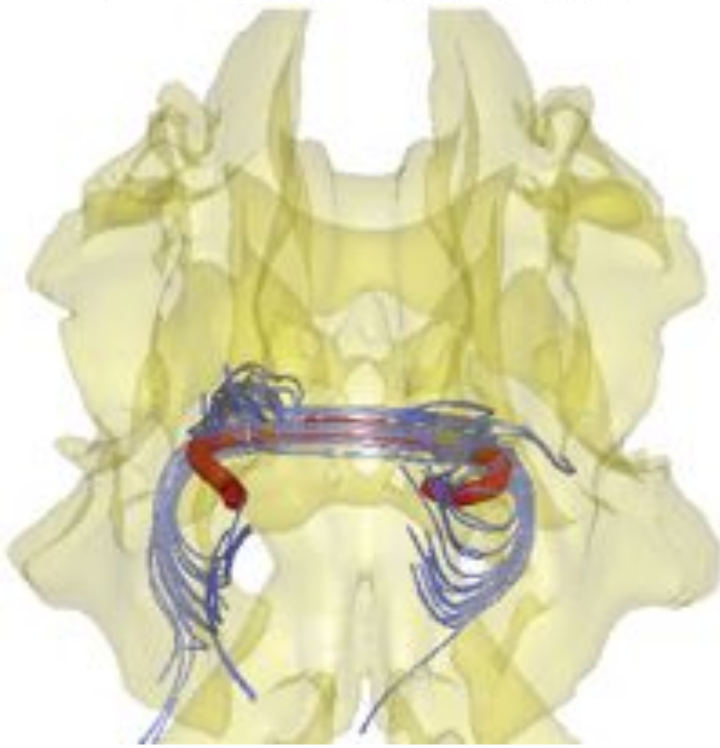


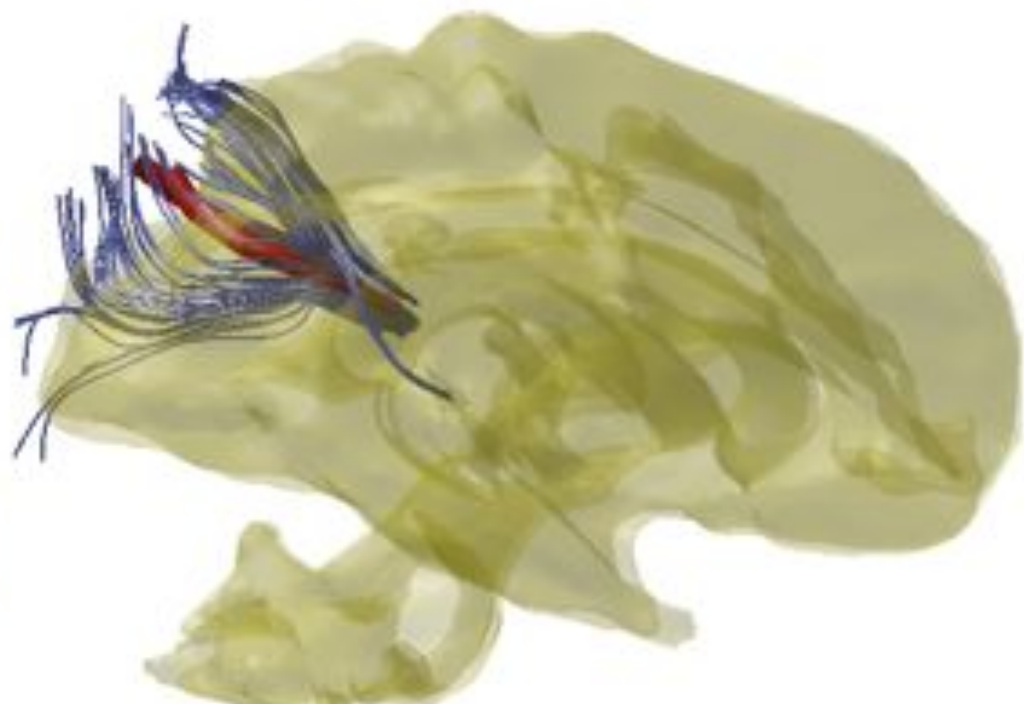


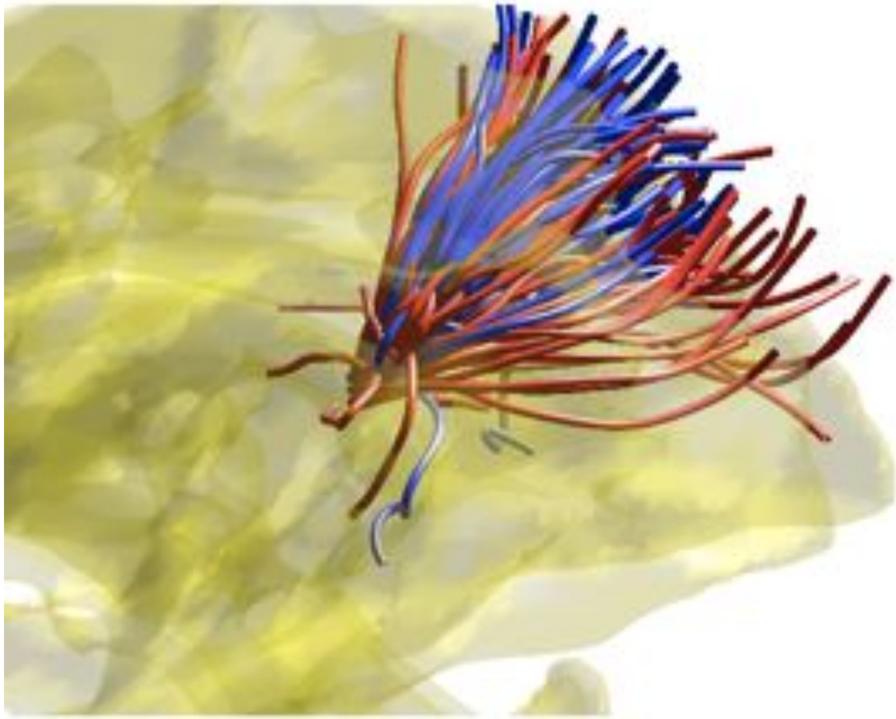
single tract → parameterization

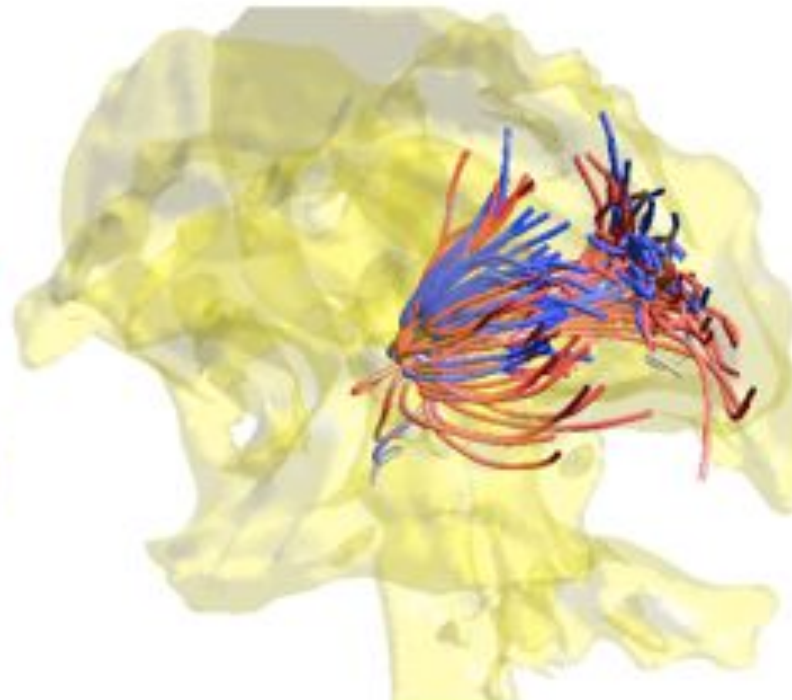


Tracts passing through the splenium of the corpus callosum



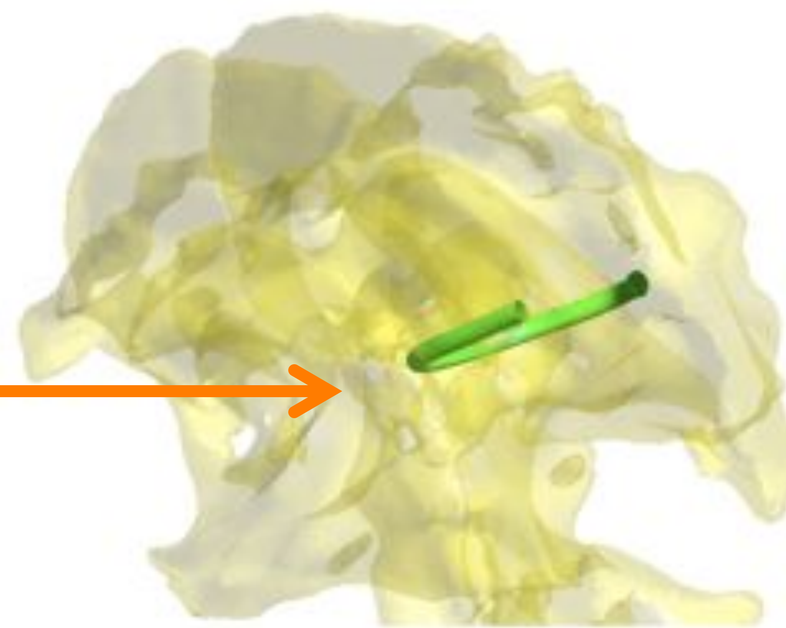






Average tracts across 74 subjects
(42 autistic 32 control)

Average of average



Read section 8.3

Limited number of literature on parametric model of white fiber tracts

Clayden et al. IEEE TMI 2007

Cubic B-spline is used to model and match tracts.

:computational nightmare

Batchelor et al. MRM 2006

Sine and cosine Fourier descriptors are used to extract global shape features for classification

: cosine basis is sufficient

Orthonormal basis in [0,1]

$$\Delta f + \lambda f = 0$$

Eigenfunctions form orthonormal basis

With periodic constraint

$$f(t+2) = f(t)$$

$$\sin(l\pi t), \cos(l\pi t)$$

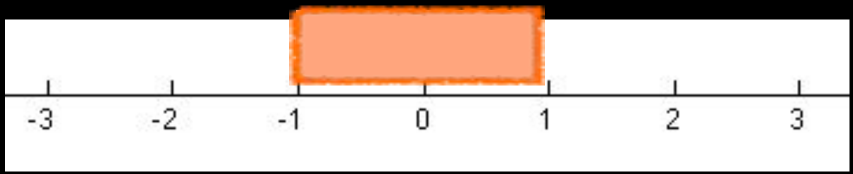
Additional symmetric constraint

$$f(t) = f(-t)$$

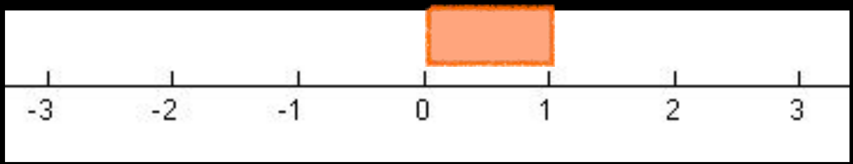
$$\lambda_l = -l^2\pi^2$$

$$\psi_0 = 1, \psi_l = \sqrt{2}\cos(l\pi t)$$

Make it only
valid in [-1,1]



Make it only
valid in [0,1]



Fourier analysis in [0,1]

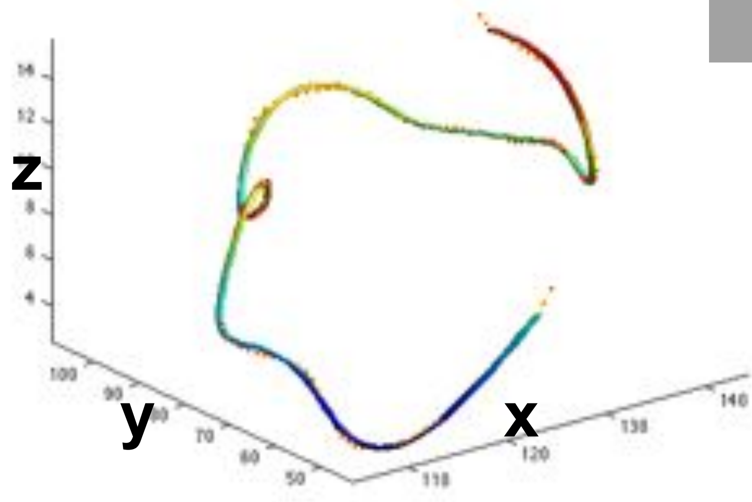
$$\psi_0 = 1, \psi_l = \sqrt{2}\cos(l\pi t)$$

$$\sum_{l=0}^{k} f_l \psi_l(t) \to f$$

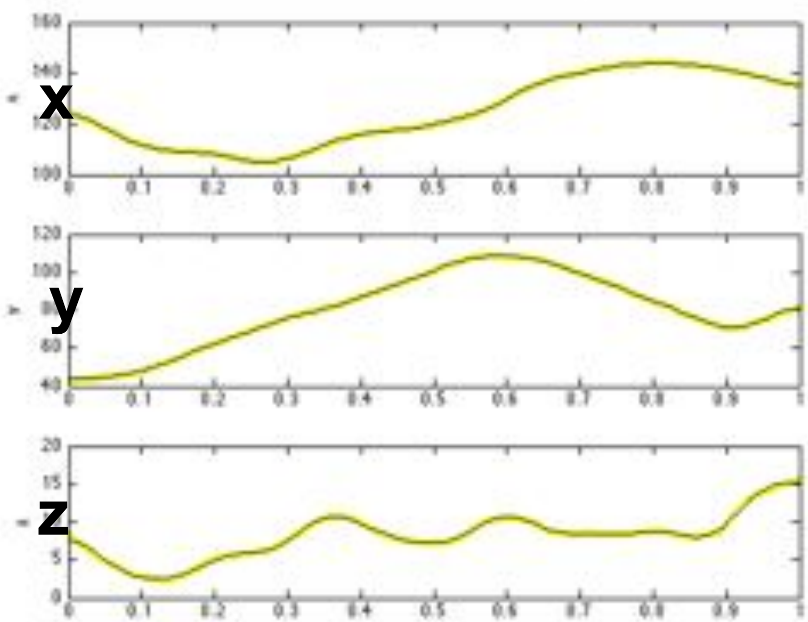
$$f_l = \langle f, \psi_l \rangle = \int_0^1 f(t)\psi_l(t)\, dt$$

This integral can be computed
by matrix inversion

White matter fiber tract model



parameterization



88.1799	56.6336	5.7367
-12.4775	-11.2552	-2.0791
2.4336	-15.4428	-0.4021
4.3956	2.2733	-0.9354
-0.0106	-0.0674	0.6999
2.1773	-2.4194	-0.1176
0.5808	0.8390	1.2942
0.0615	-0.1893	0.1188
-0.2629	0.7524	0.1089
0.7909	-0.7276	-0.1901
0.5458	0.6236	0.6939
0.4295	-0.4337	0.2185
0.2150	0.4157	0.0254
0.1584	-0.1973	0.0762
-0.1557	0.2466	-0.1086
0.0632	-0.0978	-0.0208
0.0389	-0.0143	-0.0284
-0.0014	-0.1193	0.1970
0.0004	0.0129	-0.0198
0.1342	0.0002	0.0260

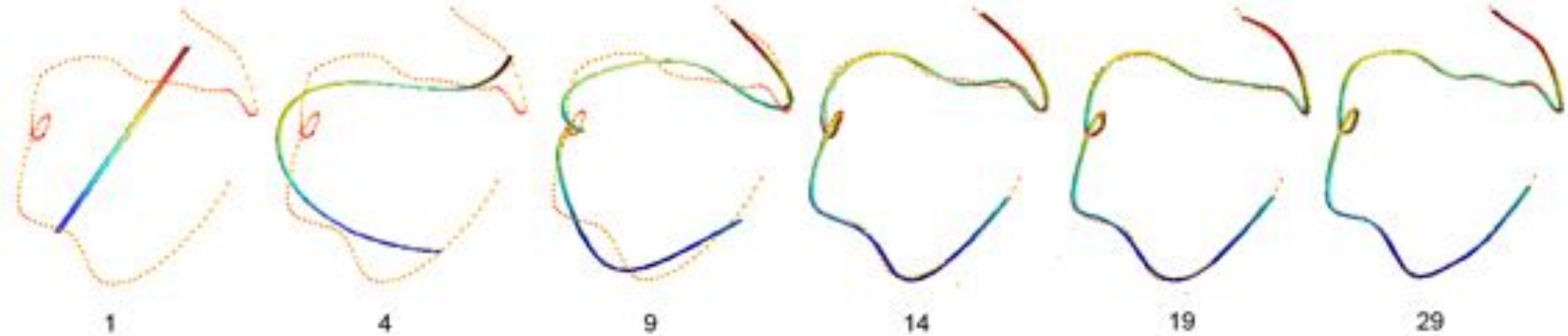
Any tract can be compactly parameterized with only 60 coefficients.

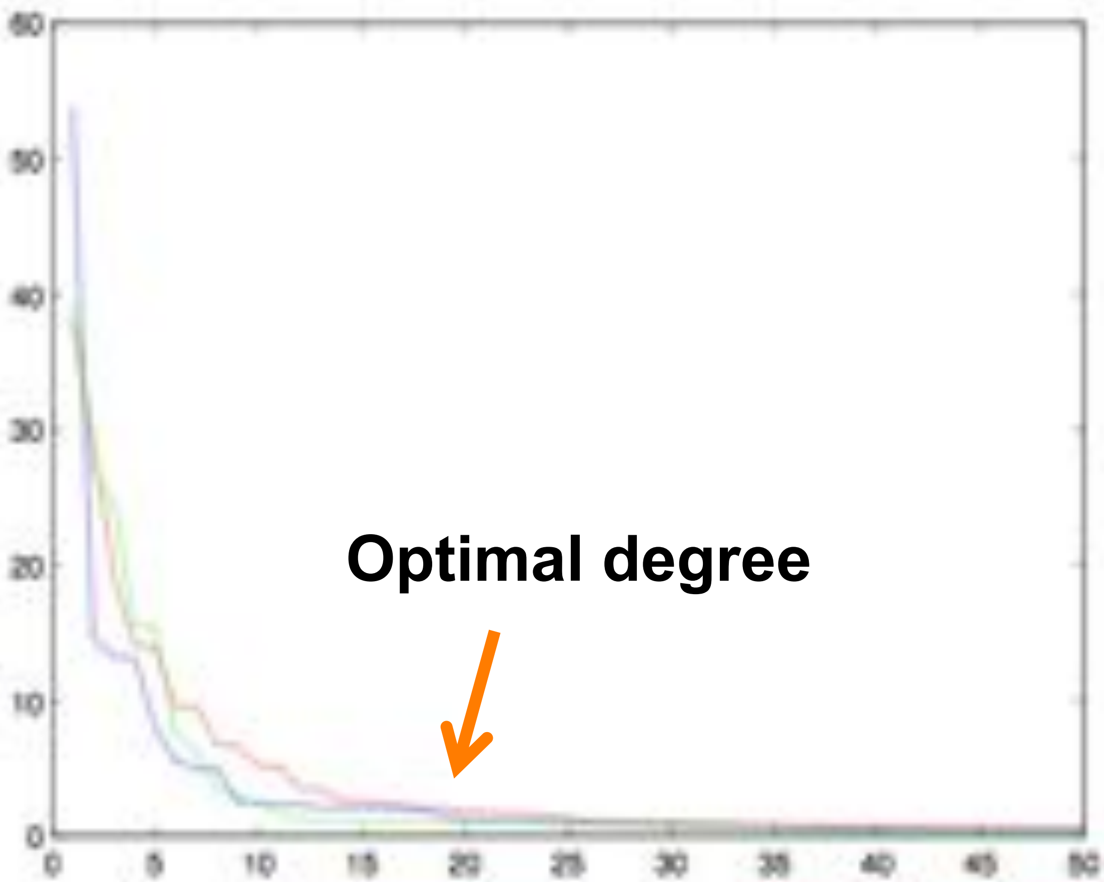
Tract registration is done by matching these parameters.

basis expansion

$$(x, y, z)' = \sum_{l=0}^{19} \beta_l \cos(l\pi t)$$

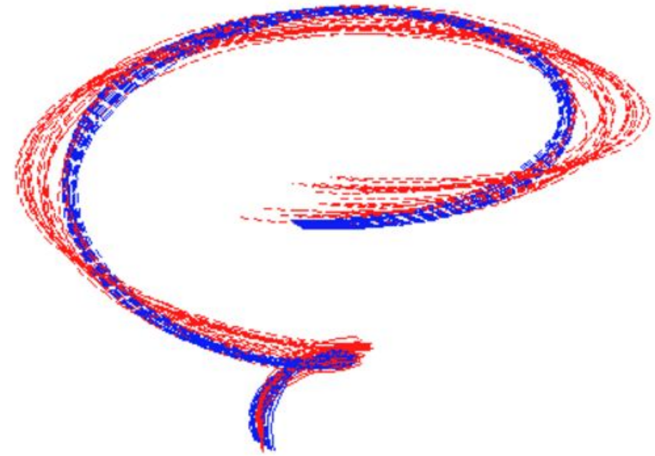
Cosine series representation





Optimal degree chosen using the forward model selection method.

Random Simulation of curves



We have performed a simulation study to
proposed framework can detect small tract
between two collection of similarly shaped
the parametric curve

$$(18) \qquad (x, y, z) = (s \sin s, s \cos s, s), s \in [0, 10]$$

as a basis for simulation, we have generated two groups of random curves. This gives a shape of a spiral with increasing radius along the z-axis. The first group consists of 20 curves generated by

$$(x, y, z) = (s \sin(s + e_1), s \cos(s + e_2), s + e_3),$$

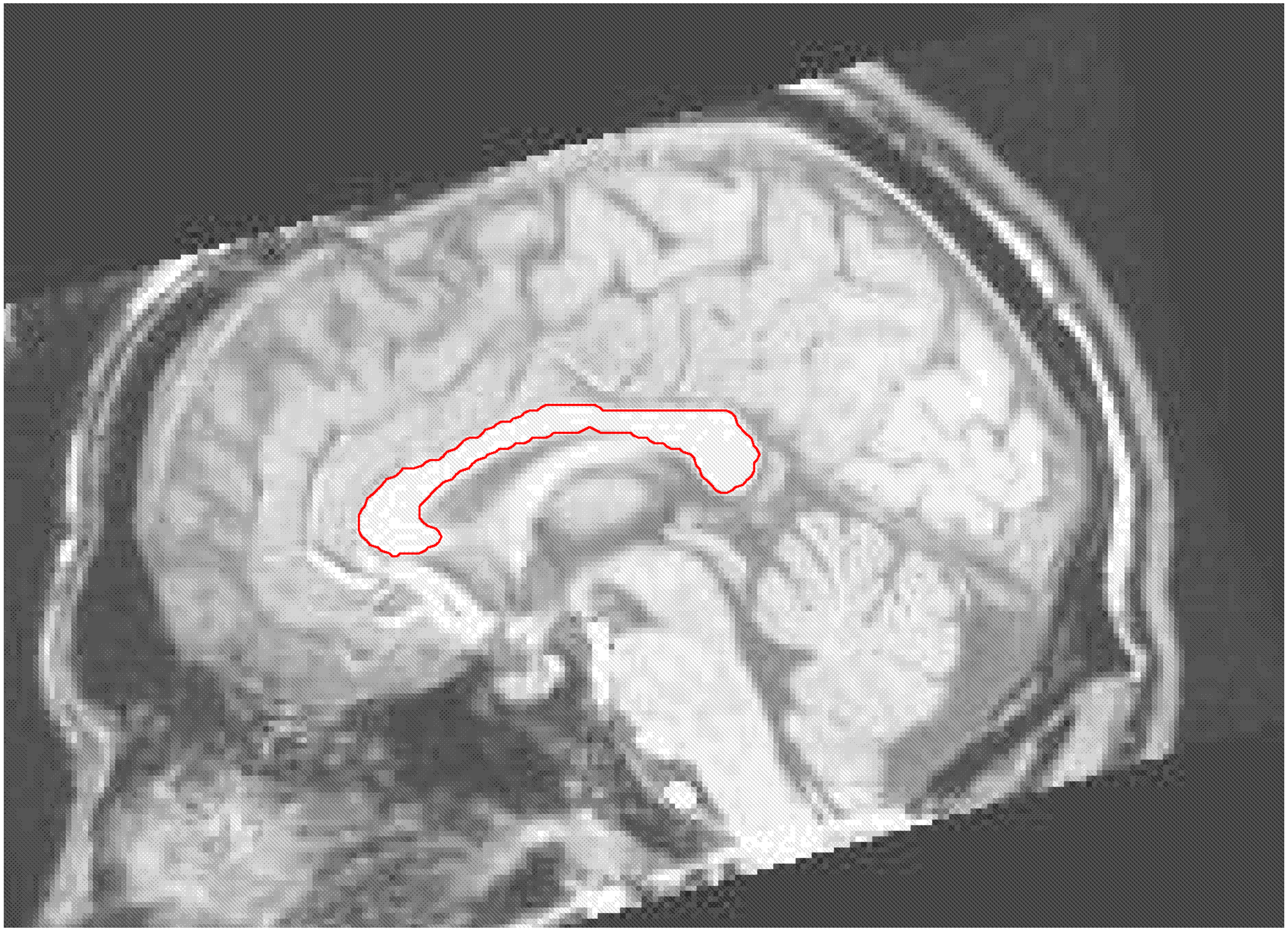
where $e_1, e_2, e_3 \sim N(0, 1)$. The second group consists of 20 curves generated by

$$(x, y, z) = ((s + e_4) \sin(s + 0.1), (s + e_5) \cos(s - 0.1), s - 0.1),$$

where $e_4, e_5 \sim N(0, 0.2^2)$. The non-additive noise is given

MATLAB Demonstration

Corpus callosum boundary



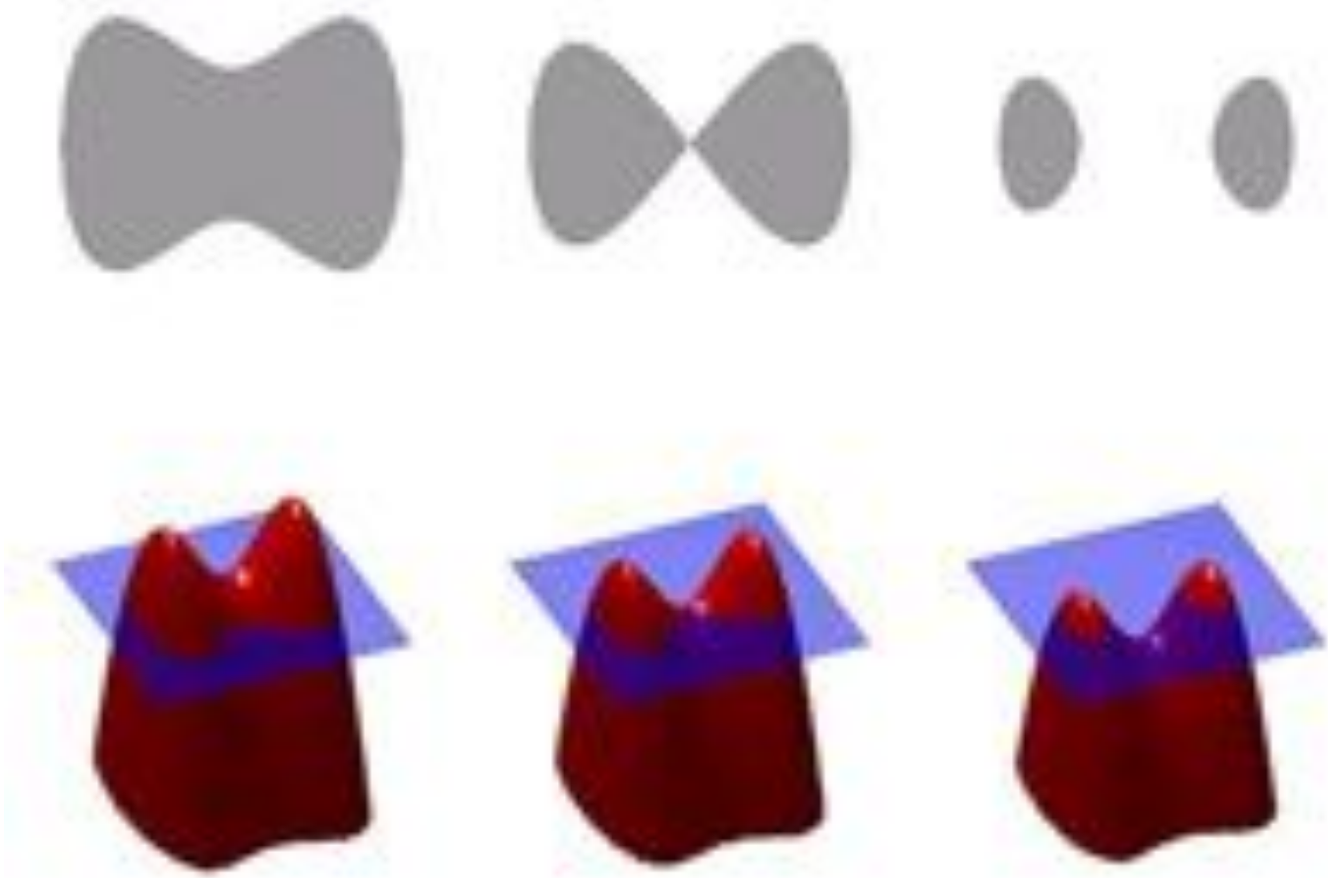
Partial Volume Effect & discretization error

Level Set Method

- Developed by Osher and Sethian in 1980's.
- It is a numerical method for propagating the boundary of an object.
- Mainly used in segmenting objects in an image.
- Idea: Start with an initial closed curve in a region, and represent it as an expanding front satisfying Hamilton-Jacobi equation.

Read section 2.3

wiki

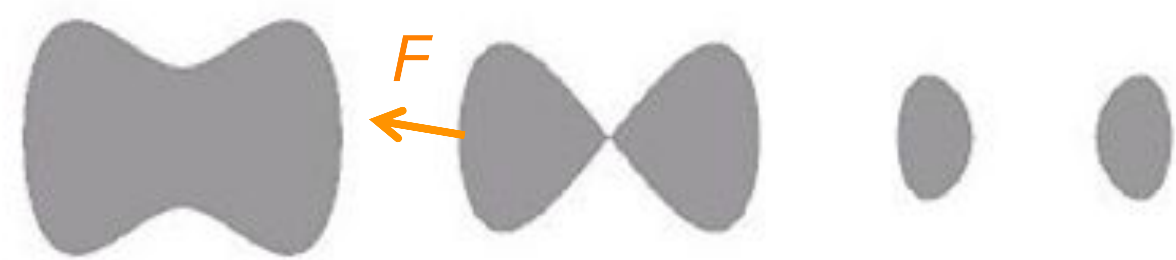


Zero level set of a function defines a closed boundary

$$\psi(x, y) = 0$$

Hamilton-Jacobi equation

The boundary moves in the normal direction with speed function *F*:

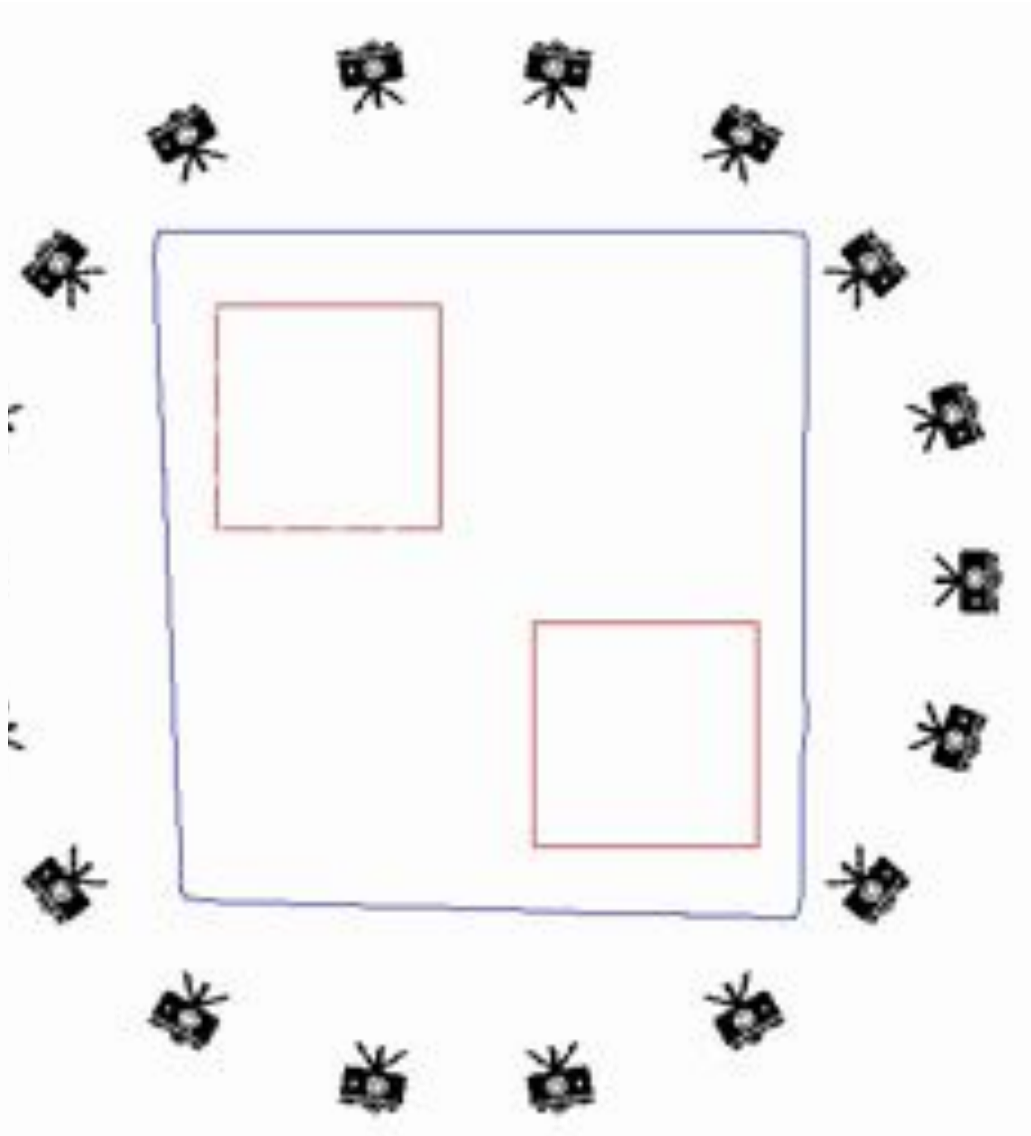


$$\psi_t + F(x, y)\left|\nabla \psi\right| = 0$$

signed distance function to the boundary

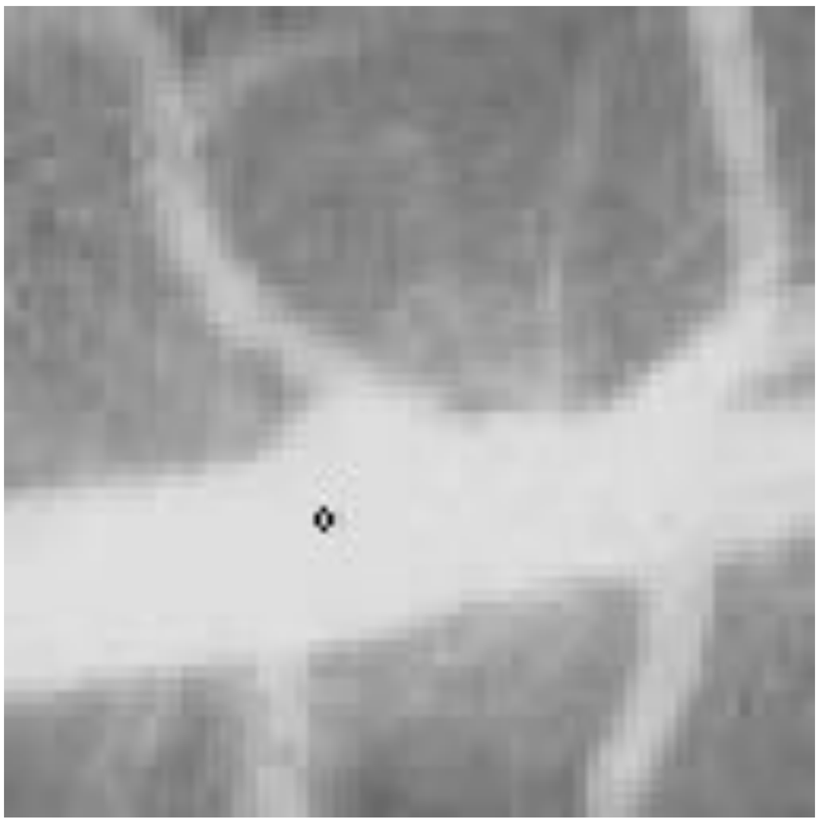
Propagation speed of the boundary

MATLAB: http://barissumengen.com/level_set_methods/



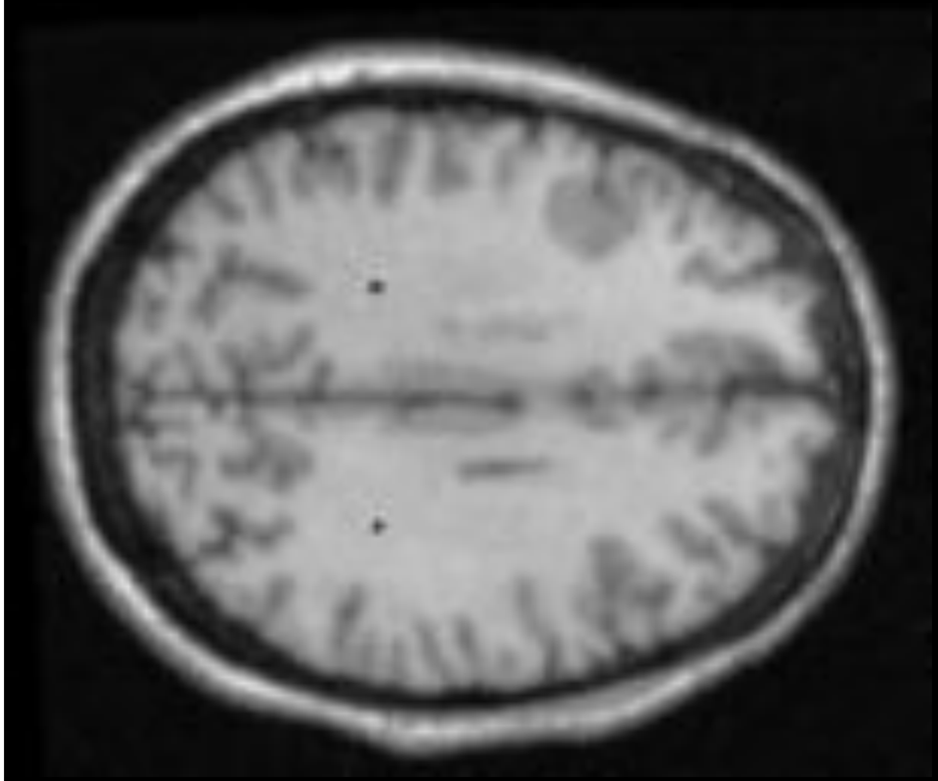
Fan Ding

Vein segmentation



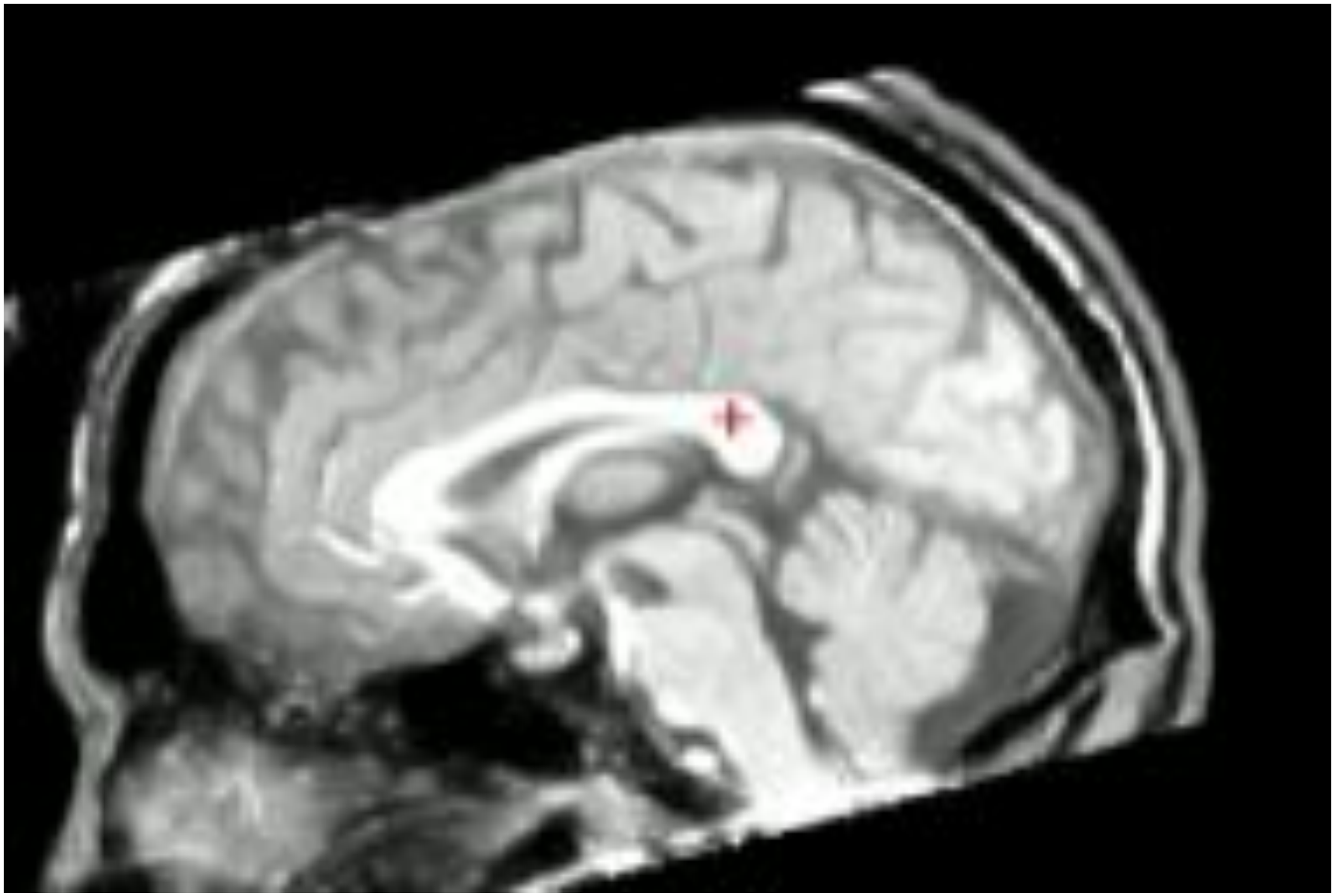
Sethian's result

White matter segmentation



Inner surface extraction

Corpus callosum boundary segmentation



Tomas Hoffman

Taubin's surface & curve fairing approach

$$x' = x + \lambda \Delta x \qquad 0 < \lambda < 1$$

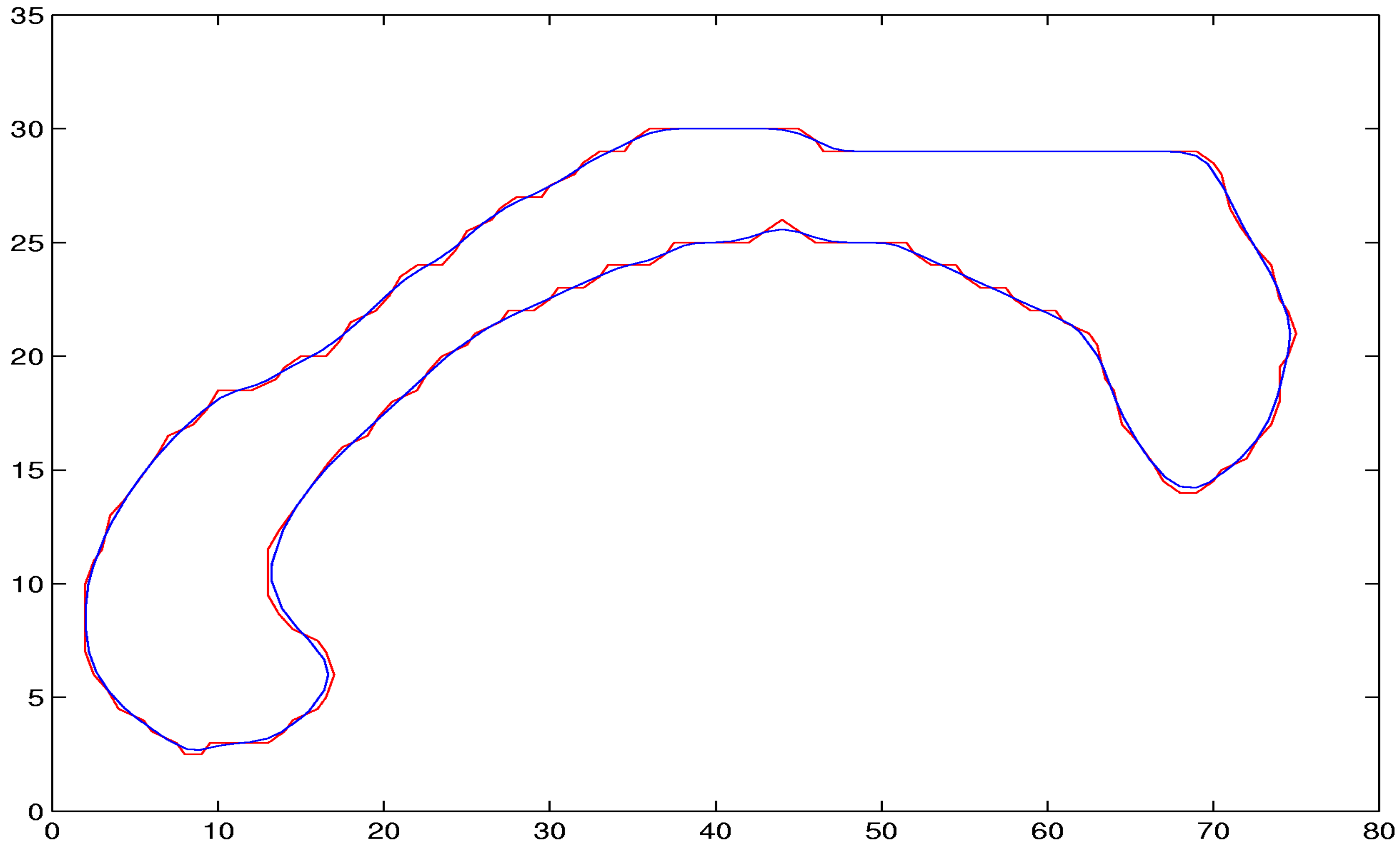
Laplacian

$$\Delta x = -Kx \qquad K = \frac{1}{2}\begin{pmatrix} 2 & -1 & & -1 \\ -1 & 2 & -1 & \\ & \cdots & \cdots & \cdots \\ -1 & & -1 & 2 \end{pmatrix}$$

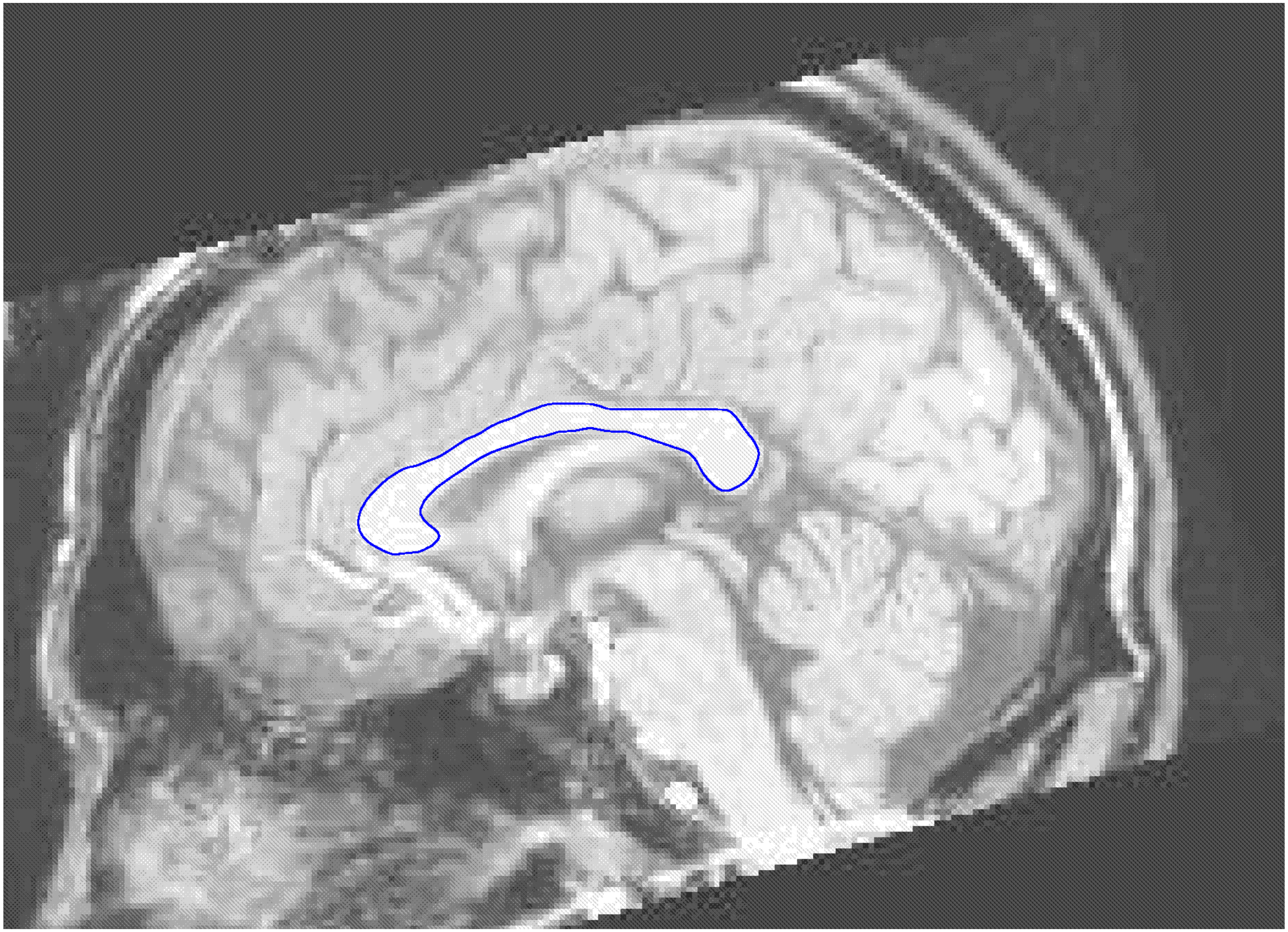
$$x' = (I - \lambda K)x$$

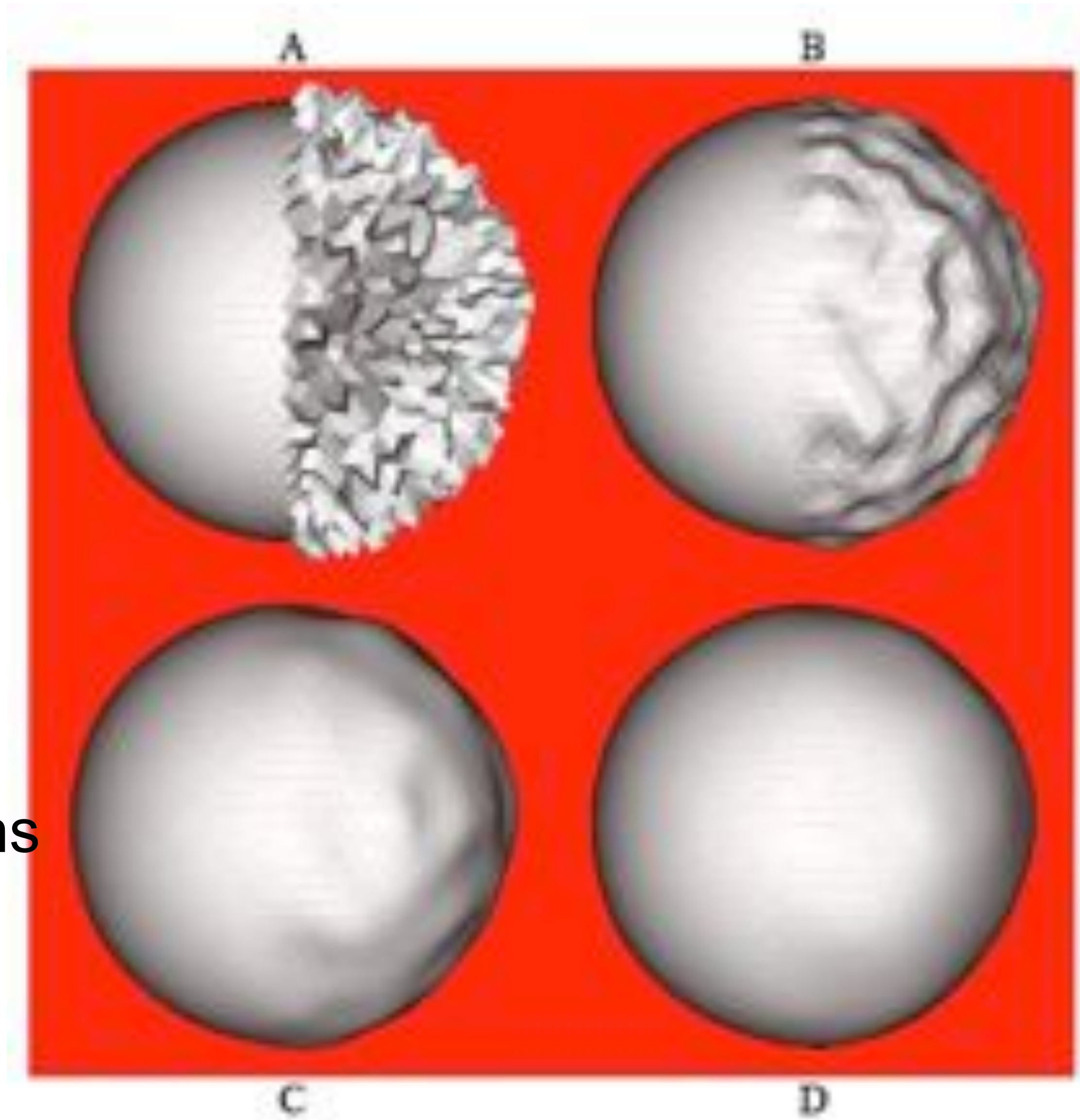
Read taubin.1995.pdf

Smoothing Results



Taubin's curve fairing

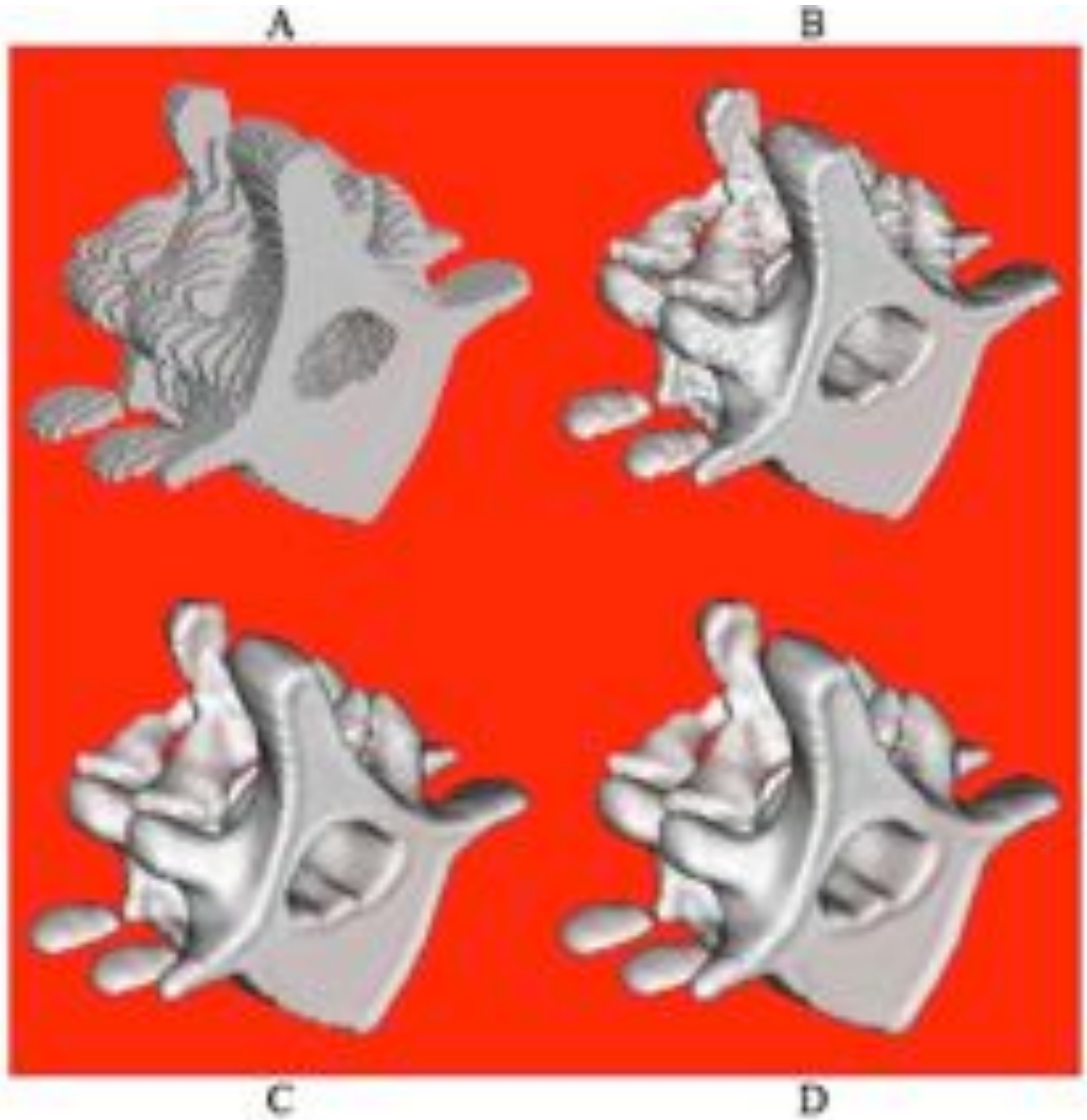


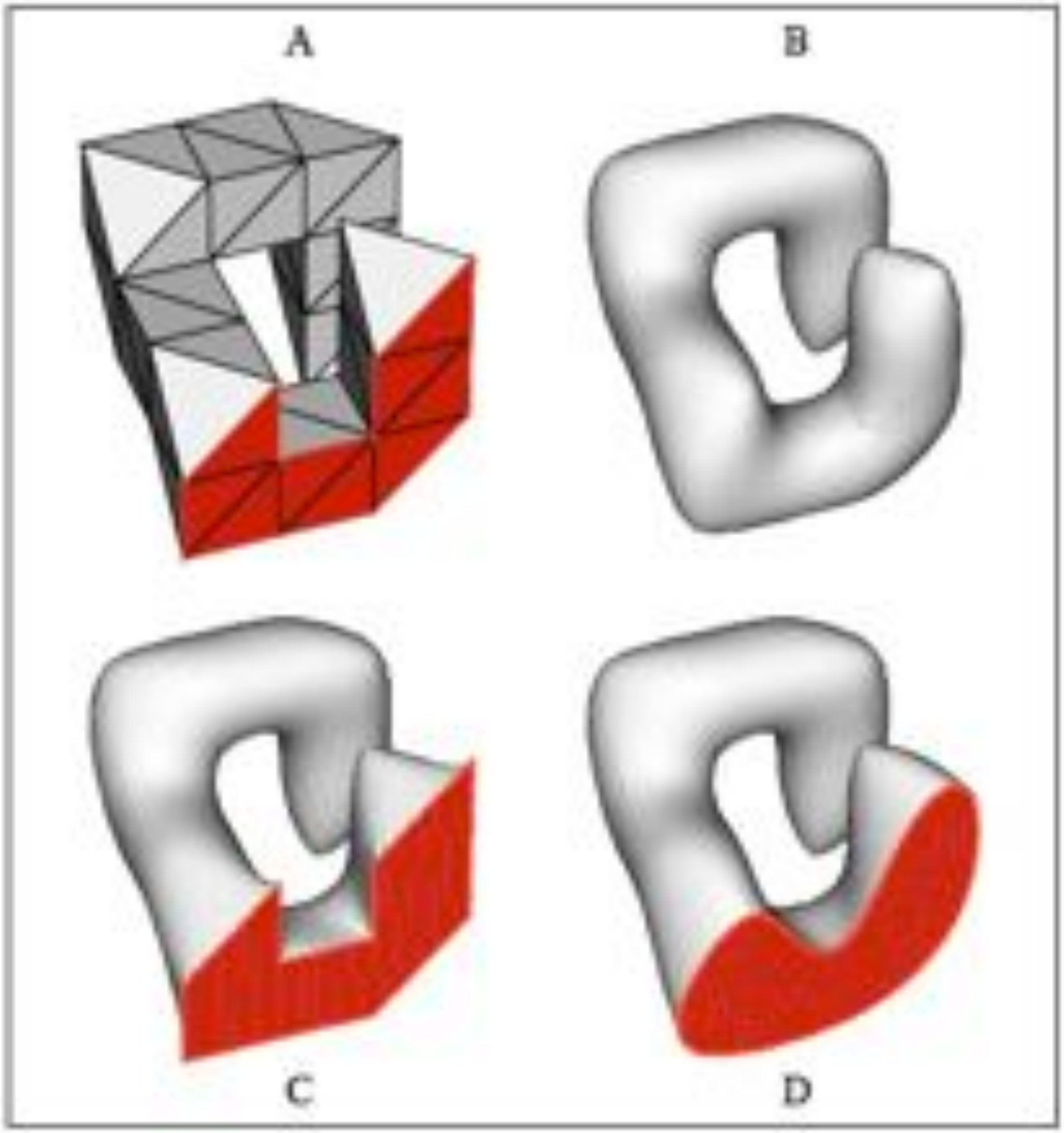


10 iterations

200 iterations

50 iterations


A
B
C
D


A
B
C
D

MATLAB Demonstration

Snake (active contour)

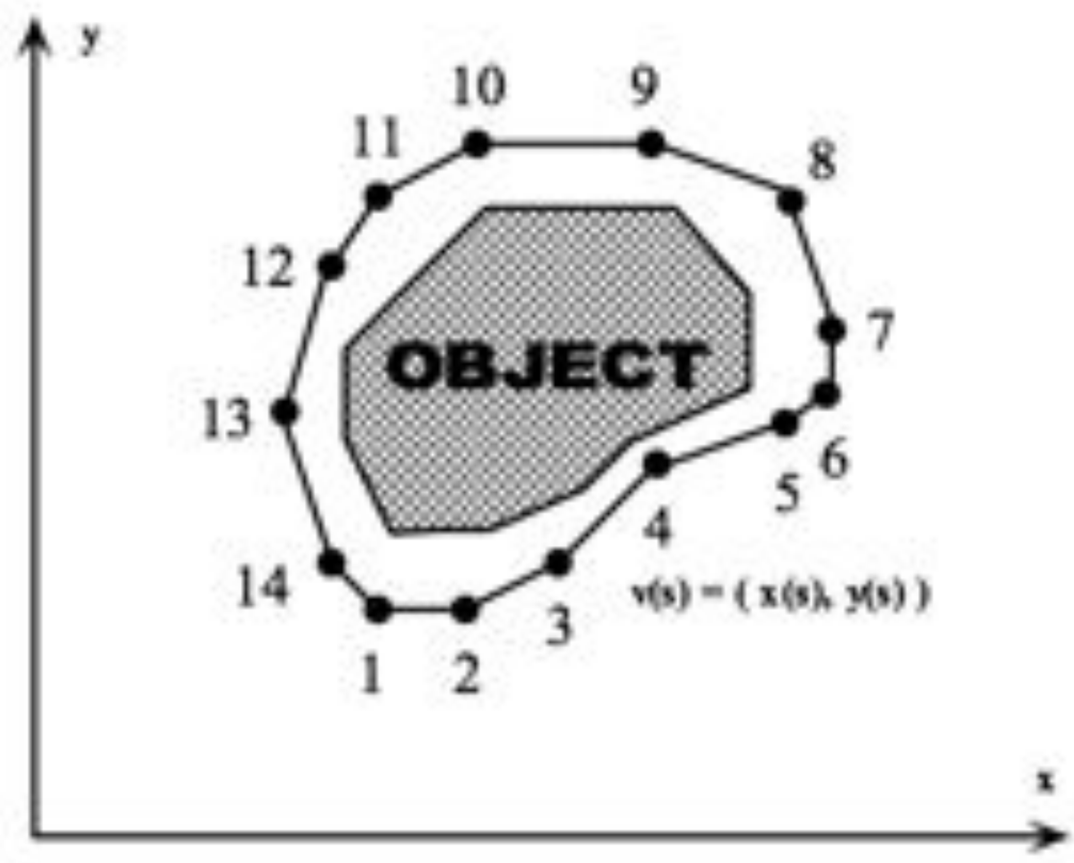
Active Contour (snake): Kass et al. in 1987

An ordered control points p_1, p_2,… p_n that will capture the boundary of an anatomical object

We move these control points such that they minimizes a certain energy

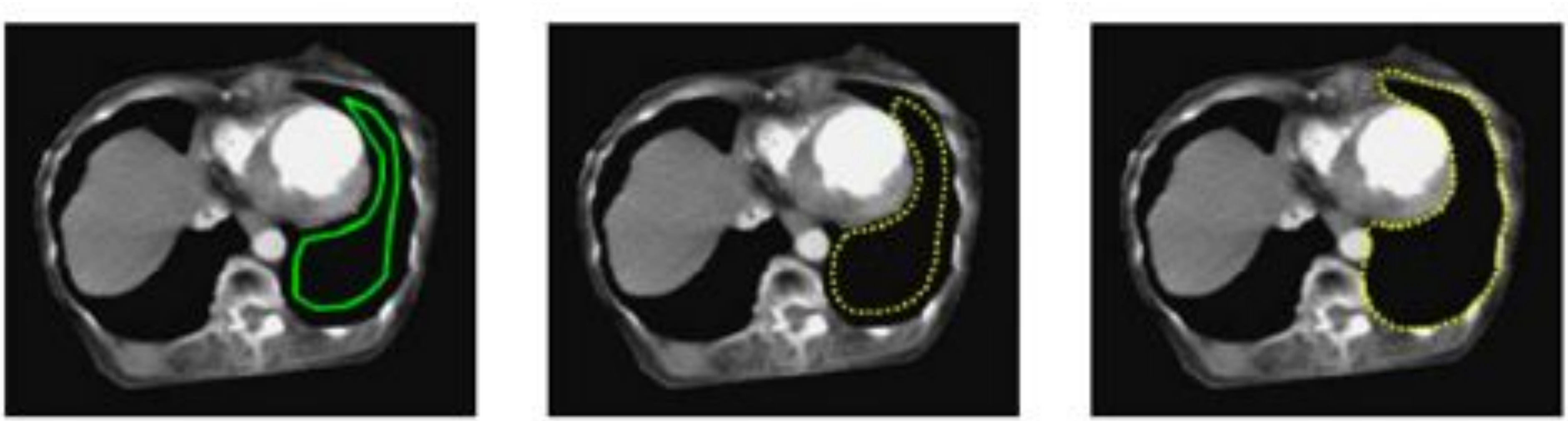
Read section 2.3
MATLAB http://www.iacl.ece.jhu.edu/static/gvf/

Basic form of active contour



Snake movement

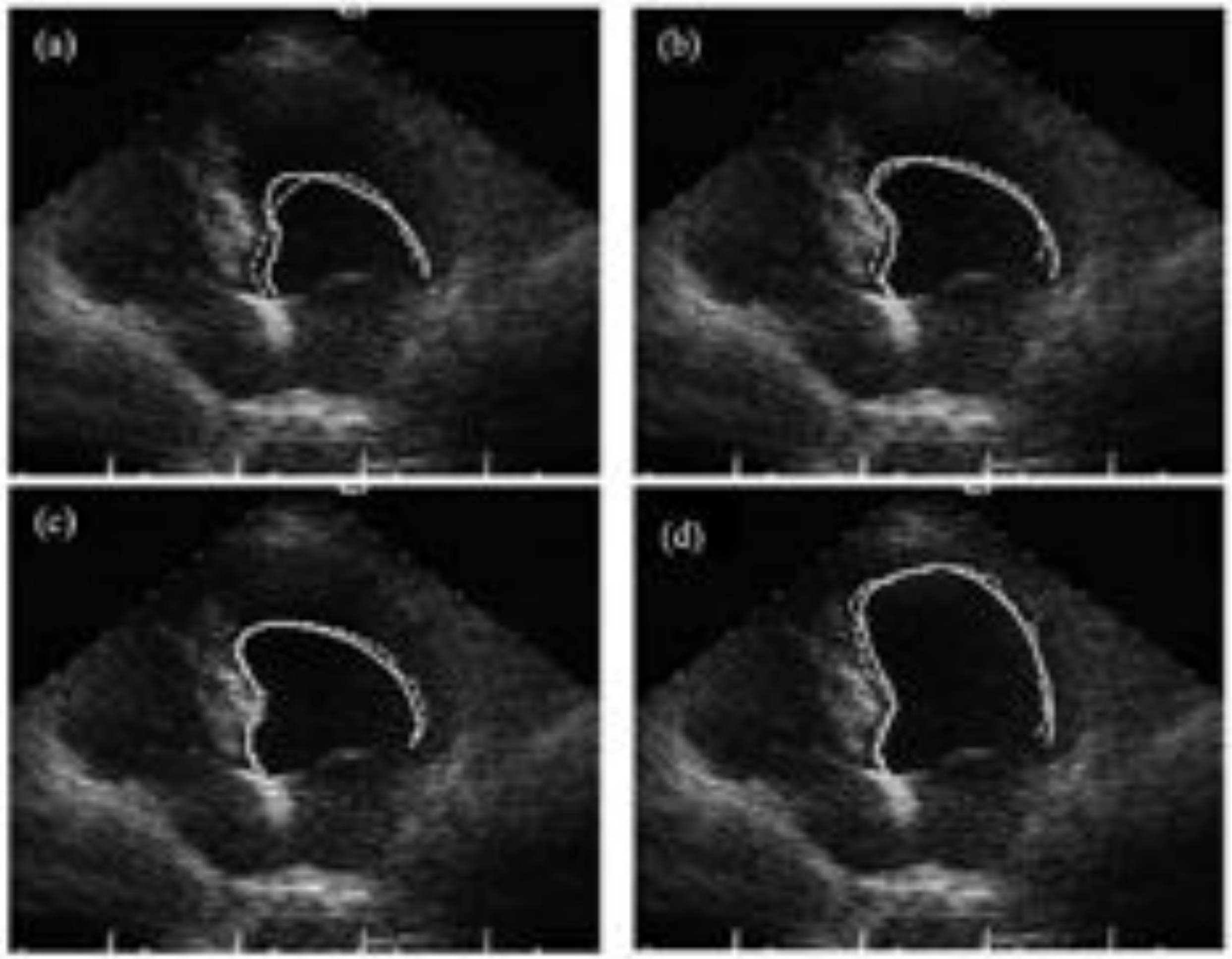
Active contour in action



Initialization **5th iteration** **15th iteration**

Bakos

Ultrasound image example



Hamarneh and Gustavsson

Minimization of energy

Parameterization of boundary C

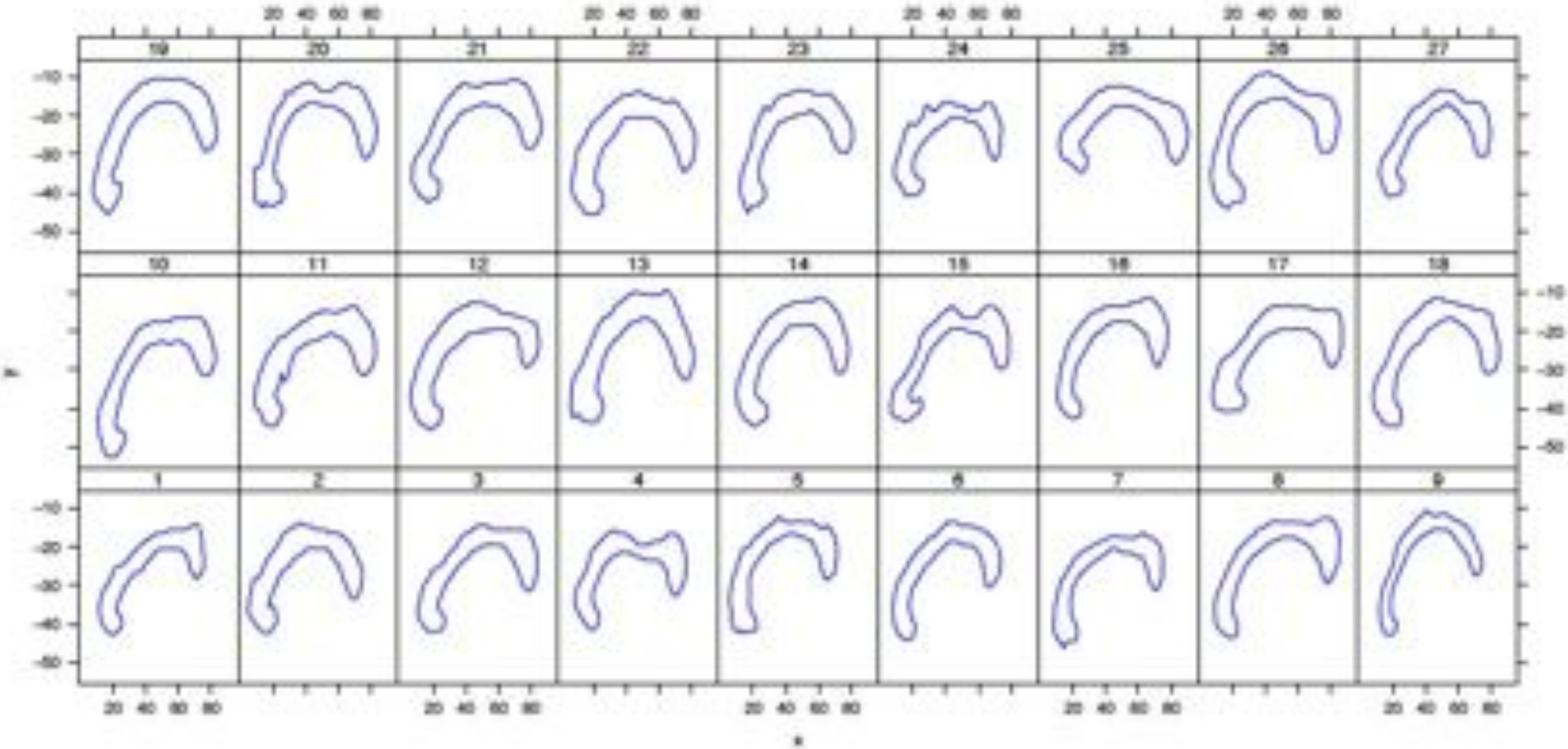
$$E(C) = \alpha \int |X'(p)|^2 + \beta \int |X''(p)| - \lambda \int |\nabla I(X(p))|$$

Internal Energy External Energy

Euler-Lagrange equation:

$$-\frac{d}{dp}(\alpha X') + \frac{d^2}{dp^2}(\beta X'') - \lambda \Delta I(X) = 0.$$

The equation is solved numerically
by the finite difference scheme.


1
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22
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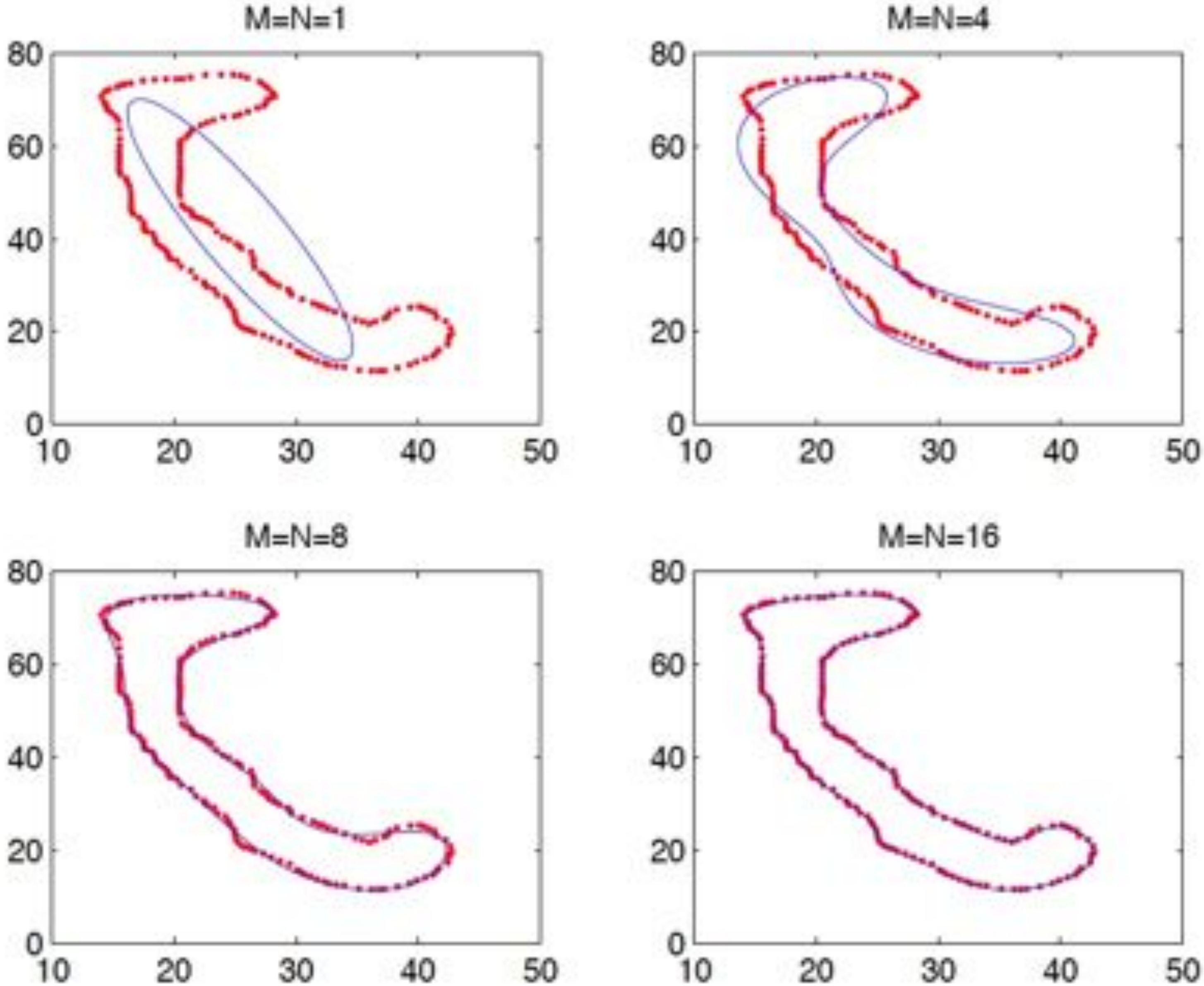
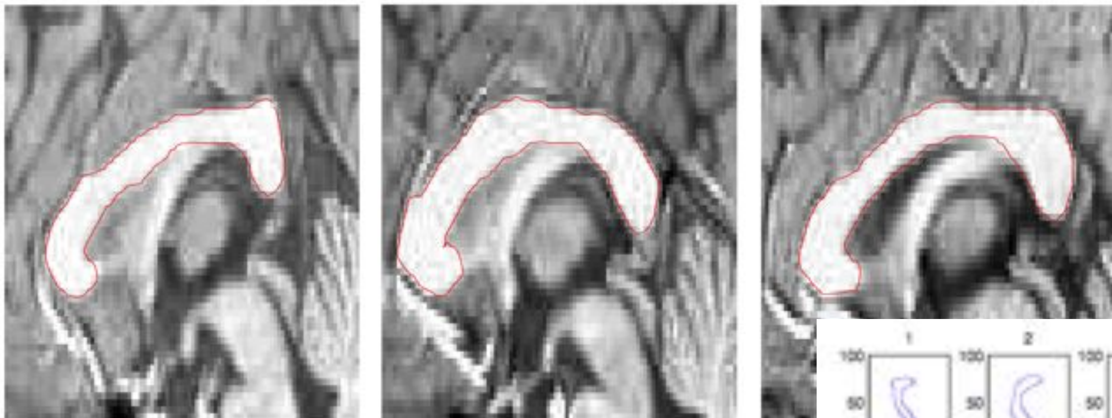
24
25
26
27
x
y

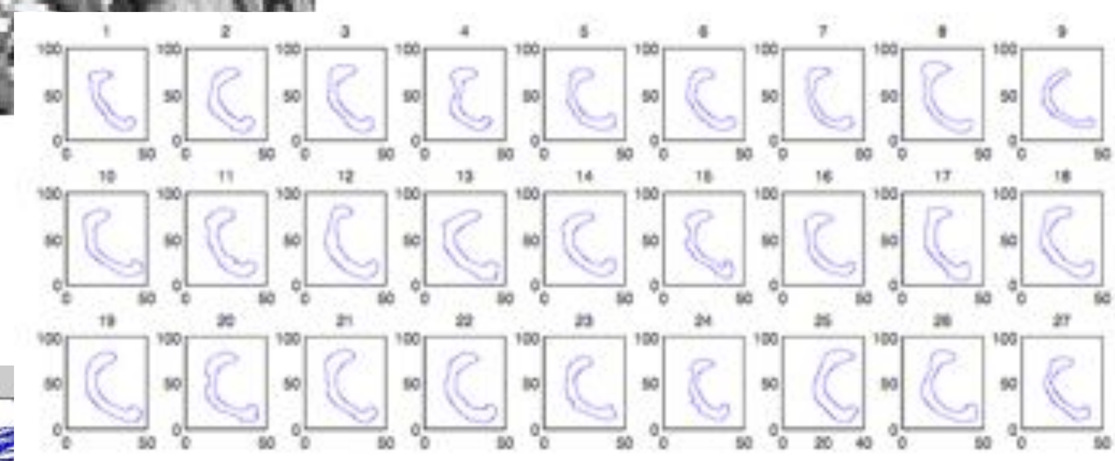
Fourier series representation of closed curves



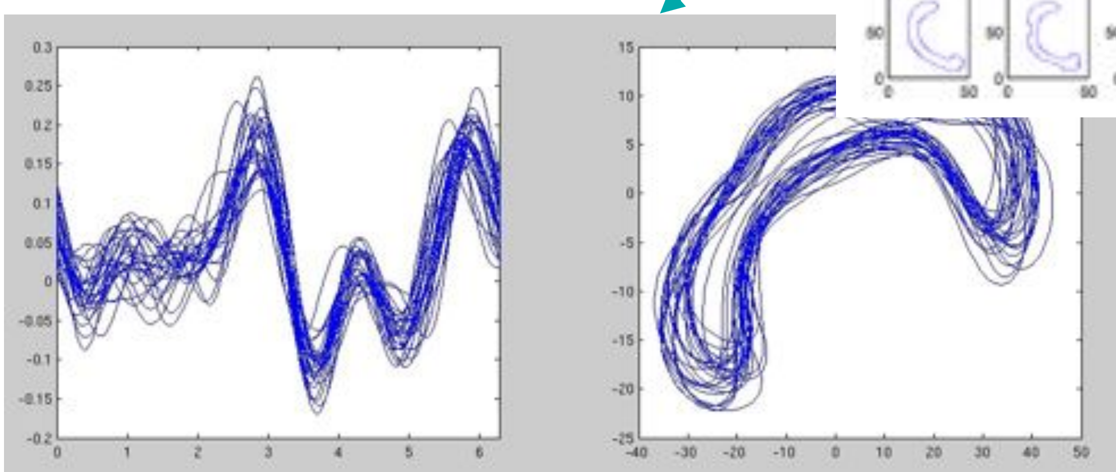
Application:corpus callosum shape classification

Shubing Wang





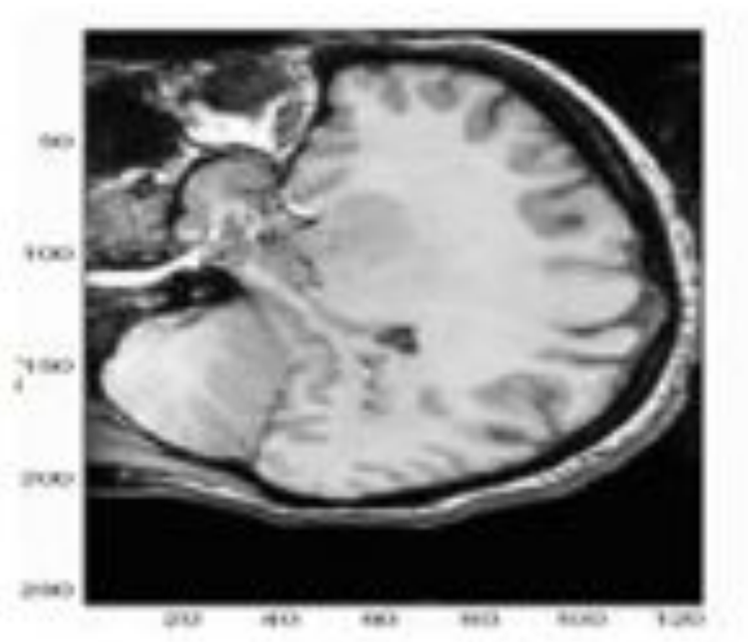
Parameterization
normalization

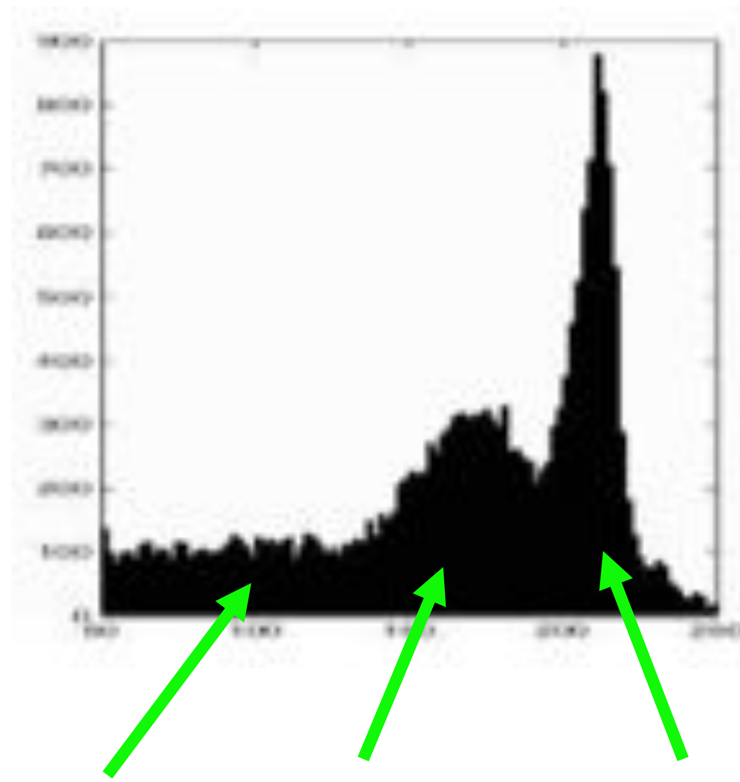


Classification
Data analysis

Other segmentation techniques

Segmentation by image intensity histogram





CSF   gray matter   white matter

Gaussian mixture model

$$f(y) = pf_1(y) + (1-p)f_2(y)$$

$$f_1(y) \approx N(\mu_1, \sigma_1^{\ 2})$$

$$f_2(y) \approx N(\mu_2, \sigma_2^{\ 2})$$

p = mixing proportion

Parameters are estimated by MLE

Maximum likelihood function

$$L = \prod_{i=1}^{n} \left[p f_1(y_i) + (1-p) f_2(y_i) \right]$$

$$\log L = \sum_{i=1}^{n} \log\left[p f_1(y_i) + (1-p) f_2(y_i) \right]$$

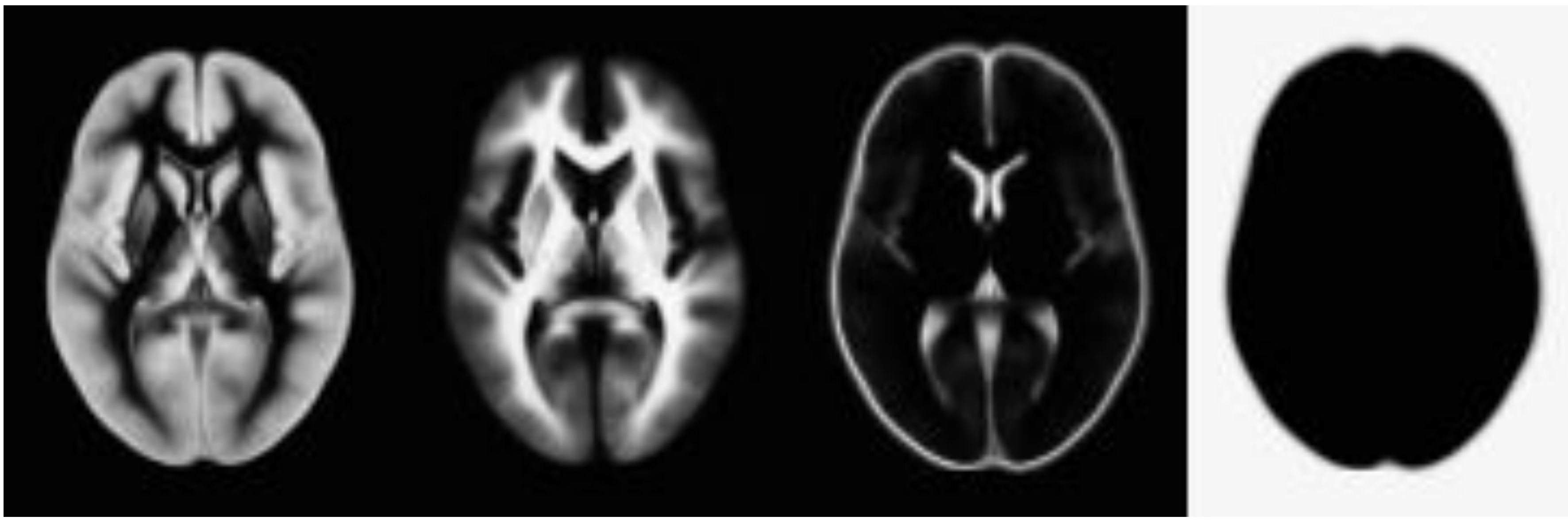
Solve $\dfrac{\partial \log L}{\partial p} = 0$ numerically

Bayesian framework

- Once we obtained all parameters of the Gaussian mixture model, we can compute the posterior tissue probability map

Bayesian framework

Prior tissue probability maps



ICBM Tissue Probabilistic Atlases
Source: C. Phillips

Bayes' theorem

- Posterior probability can be obtained from a prior probability

$$P(class \mid \text{intensity}) = \frac{P(\text{intensity} \mid class)P(class)}{\sum_{class} P(\text{intensity} \mid class)P(class)}$$

Likelihood

Prior map

: the probability obtaining image intensity given class.
This can be obtained from our Gaussian mixture model

Lecture 6

Graph theory & brain network analysis

Topological structure of point cloud data

Read

chung.2010.SPIE…

singh.2007.topology….