Neuroimage Processing

Instructor: Moo K. Chung

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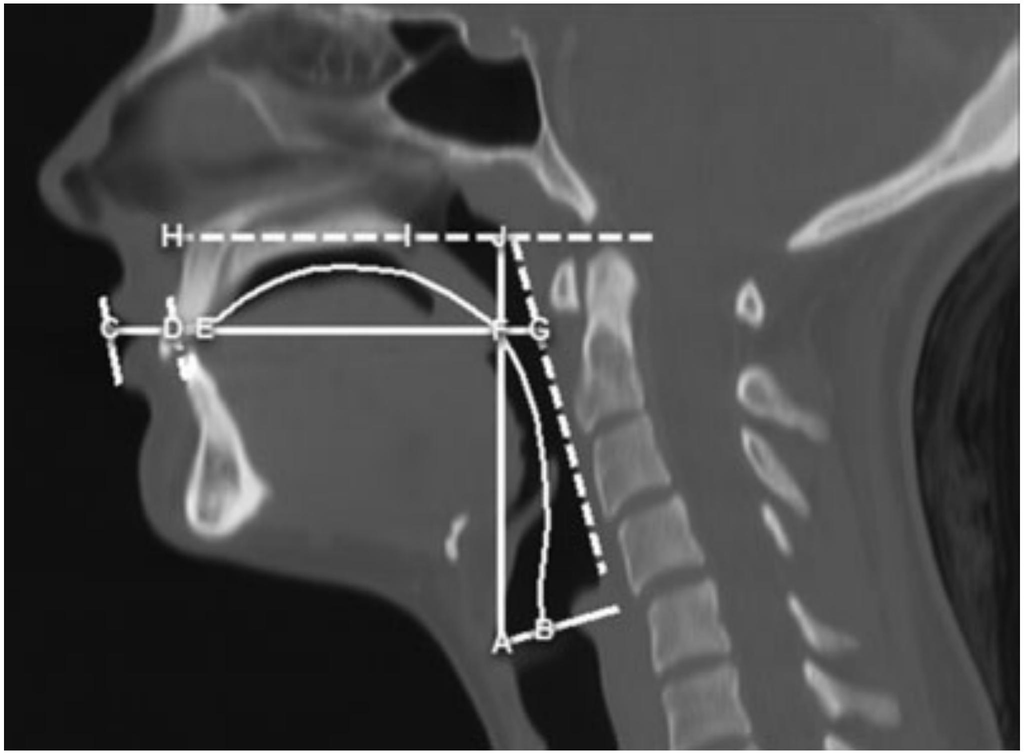
Lecture 12

Additional Issues

December 4, 2009

Analysis of landmarks



Landmark based approaches are fairly old and somewhat outdated method but still widely used for identifying ROIs across images.

Midsagittal CT image displaying the variables used to assess the anthropometric differences of race. Variables include the following: Vocal Tract Length (VT Length): the curvilinear line extending from points B to C. Vocal Tract-Vertical (VT-Vertical): vertical distance from points A to J. Nasopharyngeal Length (Nasopharynx): vertical distance from points F to J. Vocal Tract-Horizontal (VT-Horizontal): horizontal distance from points C to G. Oro-hypopharyngeal width (Oropharynx): points F to G. *Durtschi et al., 2009. Clinical Anatomy*

Euclidean distance matrix analysis (EDMA)

S. R. Lele and J. T. Richtsmeier. An Invariant Approach to Statistical Analysis of Shapes. Chapman and Hall/CRC Press, 1st edition, 2001.

Given k-landmarks, EDMA computes the k(k-1)/2 pairwise inter-landmark distances.

Invariant to rotations and translations.

k x k form matrix can be used as a representation.

Two set of landmarks are compared using form difference matrix (FDM), where entries are the ratio of form matrices.

Statistic = ratio of the largest to the smallest entry

This is invariant what landmark configuration is used in the numerator.

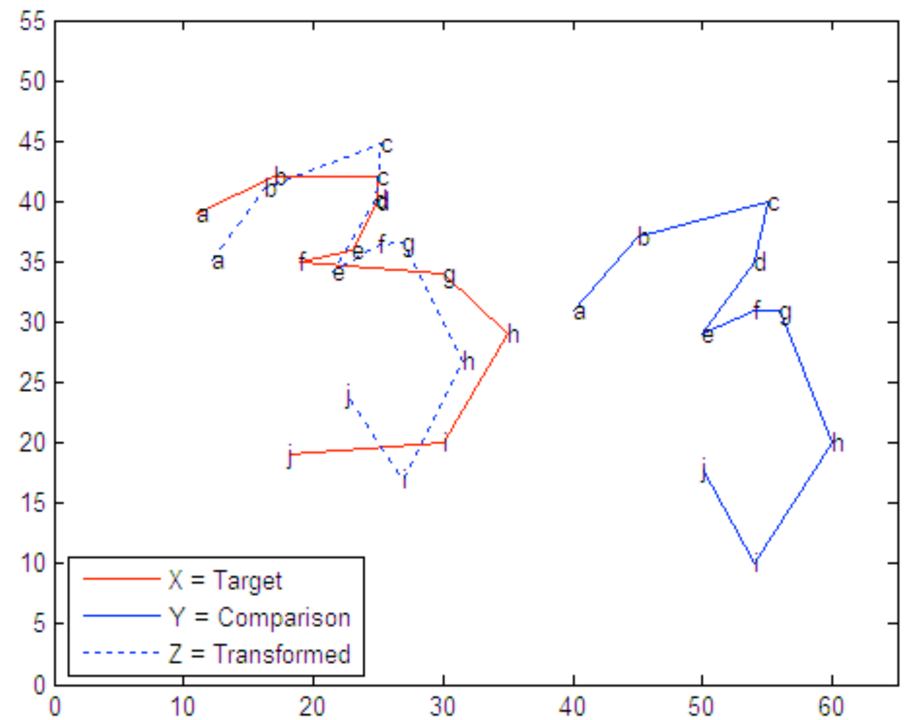
Bootstrap methods are mainly used for inference on FDM entries.

Shape Analysis by Dryden and Mardia

I. L. Dryden and K. V. Mardia. Statistical Shape Analysis. John Wiley and Sons, 1st edition, 1998.

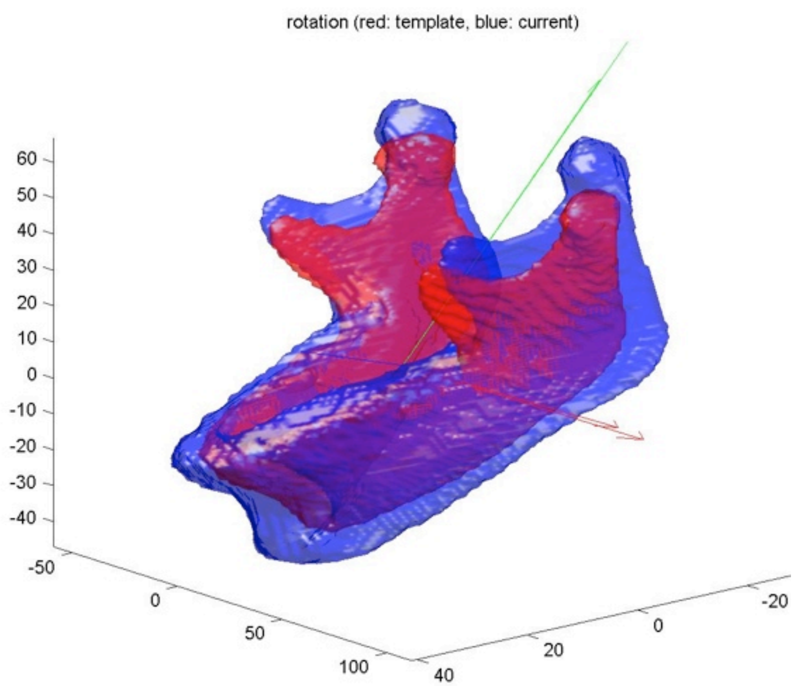
k landmarks are represented as a single point in a high-dimensional shape space

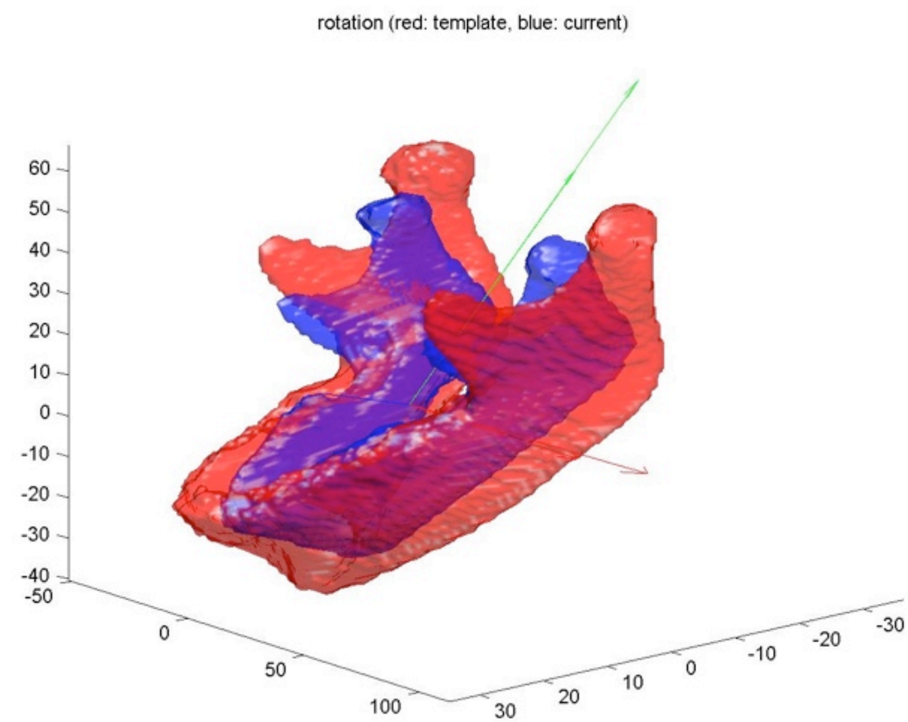
Procrustes method: It involves estimating translation and rotation in such a ways that the sum of squared distances between corresponding landmarks is minimized. The parameters of translation and rotations are estimated iteratively.



MATLAB built-in function **`procrustes`**

Mandible surface alignment





Red: template
Blue: subjects

Dongjun Chung, University of Wisconsin-Madison

Multidi-mensional scaling (MDS)

I. Borg and J. F. Croenen. Modern Multidimensional Scaling. Springer, 2nd edition, 2005

For extremely large k landmarks, dimensionality reduction methods such as MDS and ISOMAP can be used to embed the points in lower dimensions.

MDS mainly use the pairwise distances between the points that approximate the pairwise dissimilarities of the objects.

$$X_{n \times p}$$

Matrix of p-variables of n-subjects

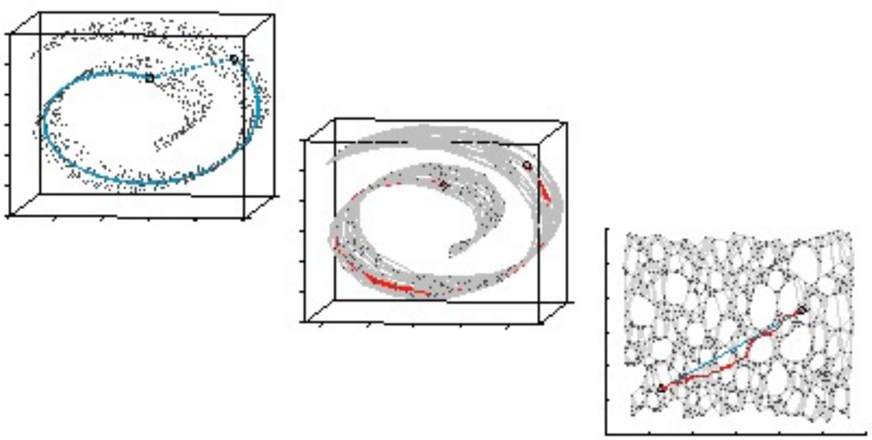
$$X'_{p \times n} X_{n \times p}$$

Compute eigenvalues

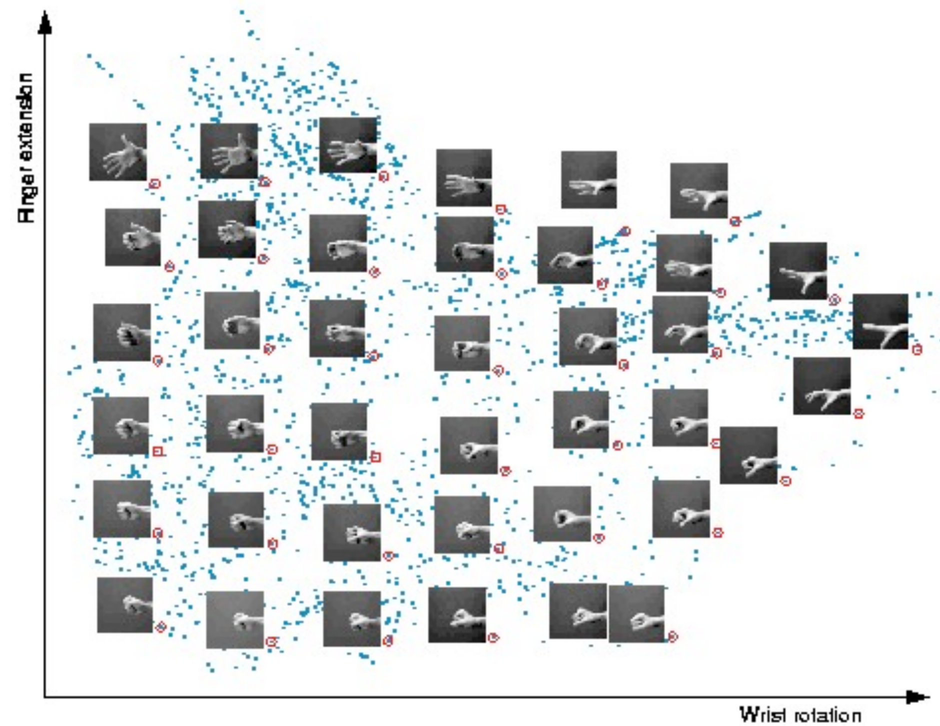
k largest eigenvalues correspond to the dimension to embed

ISOMAP

J. B. Tenenbaum, V. de Silva, and J. C. Langford. A global geometric framework for nonlinear dimensionality reduction. Science, 290(5500):2319–2323, 2000



http://waldron.stanford.edu/~isomap/



To be covered in computational methods class

Small-n and large-p problem

When sample size is small while we have large number of parameters to estimate, the usual least squares estimation technique fails.

Example: In constructing the Hotelling's T-square statistic, we need to construct the covariance matrix of size p x p using only n samples. The constructed covariance matrix is of rank n. So it can't be inverted.

Solution 1: Perform Moore-Penrose generalized inverse of the rank deficient covariance matrix.

Solution 2: Reduce dimension by performing PCA.

Cortical thickness analysis using FreeSurfer

To improve statistical power, cortical thickness needs to be obtained in the native MRI space rather than in the Talairach space (Ad-Dab'bagh et al., 2005).

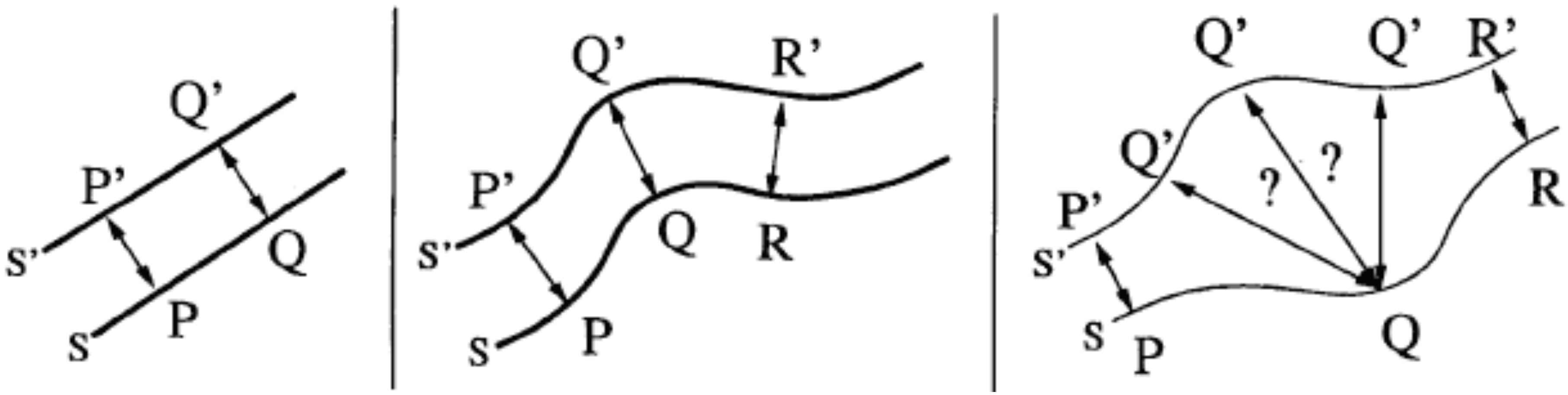
Ad-Dab'bagh, Y., Singh, V., Robbins, S., Lerch, J., Lyttelton, O., Fombonne, E., Evans, A.C., 2005. Native space cortical thickness measurement and the absence of correlation to cerebral volume. In: Proceedings of the 11th Annual Meeting of the Organization for Human Brain Mapping, Toronto

Every thickness is resampled to the average surface using the correspondence obtained from aligning each individual cortical folding pattern with the average folding pattern on a sphere.

Fischl, B., Sereno, M.I., Tootell, R.B.H., Dale, A.M., 1999. High-resolution intersubject averaging and a coordinate system for the cortical surface. Hum. Brain. Mapp. 8, 272-284

Cortical thickness using 3D gray matter segmentation

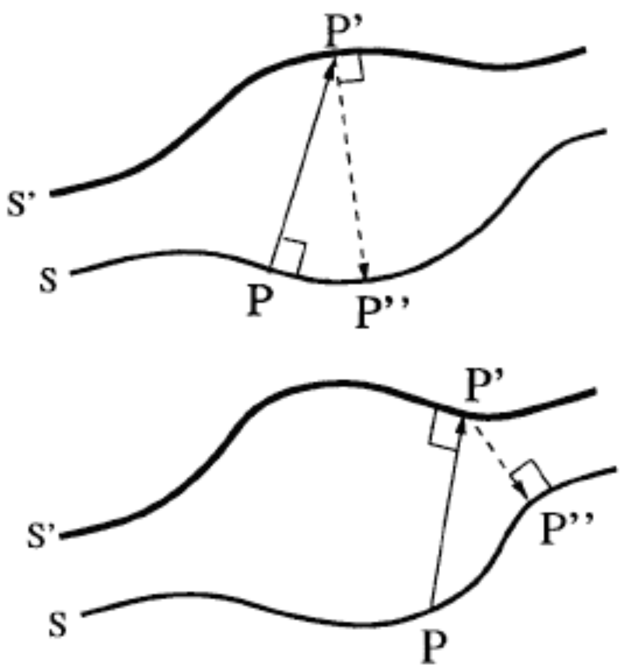
There are many numerical techniques for measuring the cortical thickness. The minimum Euclidean distance method in Fischl and Dale (2000), Laplace equation method in Jones *et al.* (2000), Bayesian construction in Miller *et al.* (2000) and the automatic linkage method in MacDonald *et al.* (2000) are available. The automatic linkage method of MacDonald *et al.* (2000) has been validated in Kabani *et al.* (2000) and used in Chung *et al.* (2003) for quantifying normal cortical development.

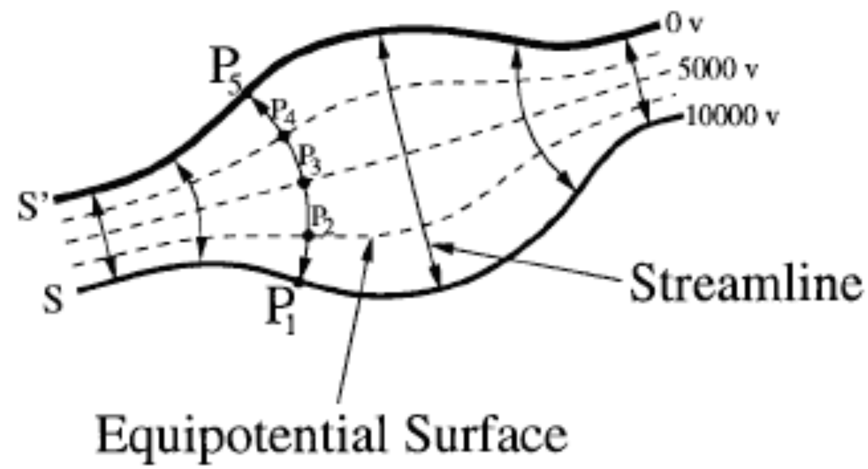


page 68-69 of textbook

A fairly popular method of Jones *et al.* (2000) computes thickness directly from the volumetric data by assuming the gray matter to be inside of two conducting boundaries. The distribution of fictional charges within the two boundaries sets up a scalar potential field Ψ, which satisfies the Poisson equation

$$\Delta\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} = \frac{\rho}{\epsilon_0},$$

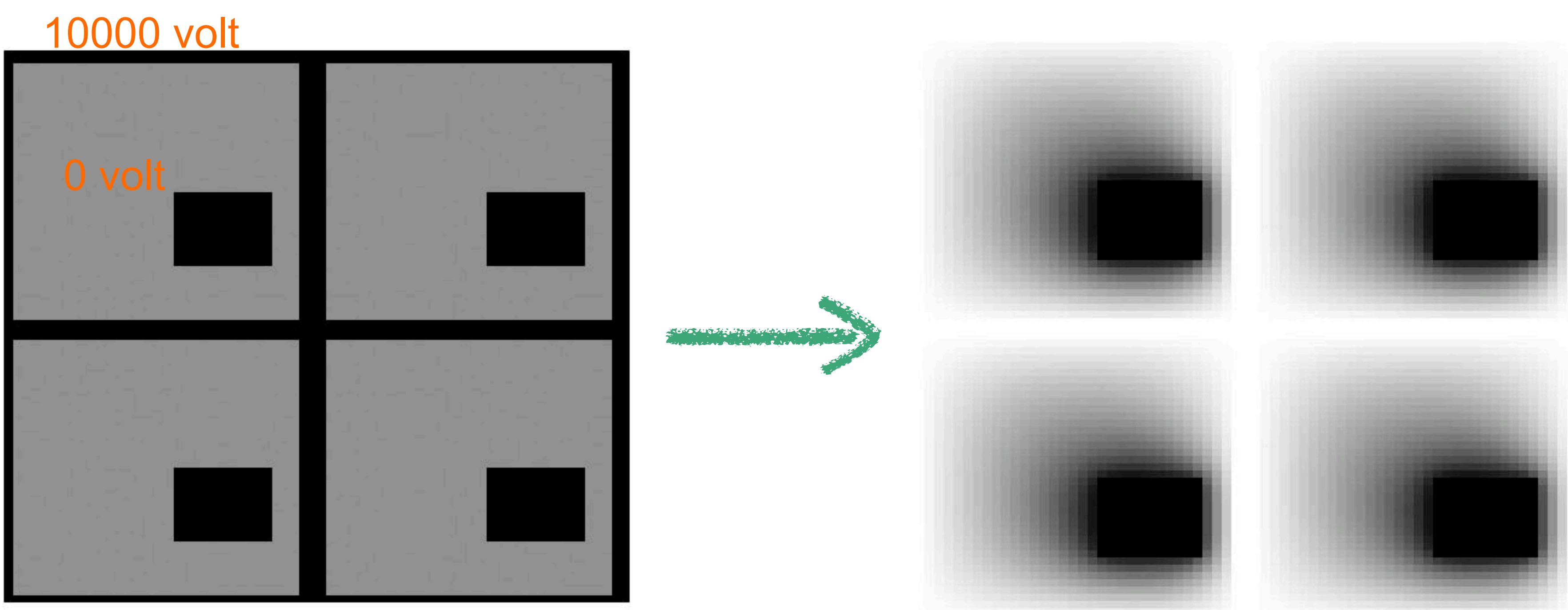




where ρ is the total charge within the boundaries. If we set up the two boundaries at different potential, say at Ψ_0 and Ψ_1, without any enclosing charge, we have the Laplace equation

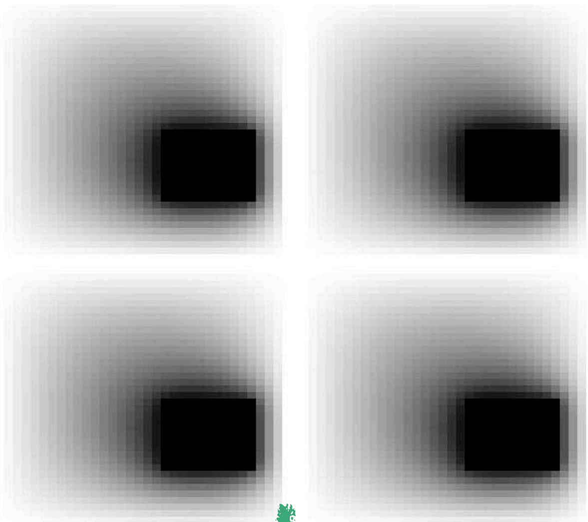
$$\Delta\Psi = 0.$$

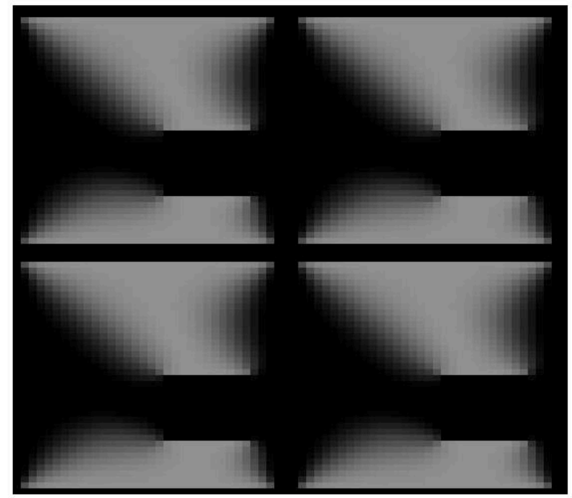
By solving the Laplace equation with the two boundary condition, we obtain the potential field Ψ. Then the electric field perpendicular to the isopotential surfaces is given by $-\nabla\Psi$. The electric field lines radiate from one conducting surface to the other without crossing each other.



Brianna Schuyler, Univ. of Wisconsin-Madison

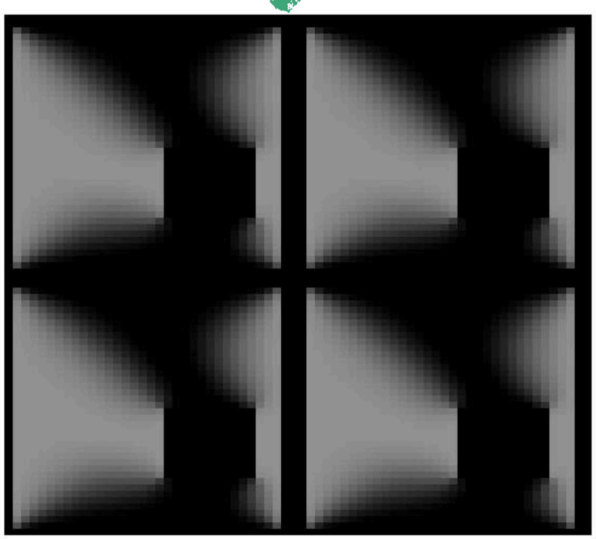
Computing field line (stream line)



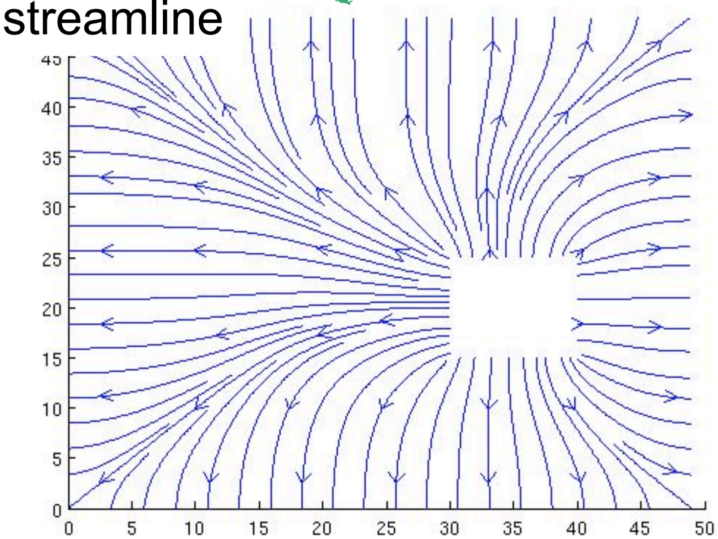


d Ψ/dx

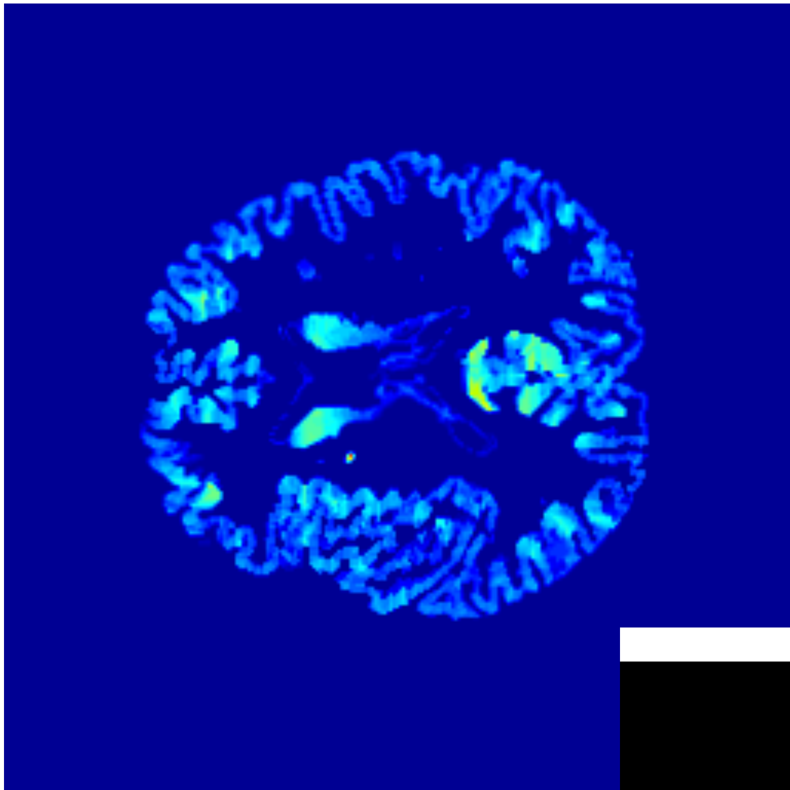
d Ψ/dy

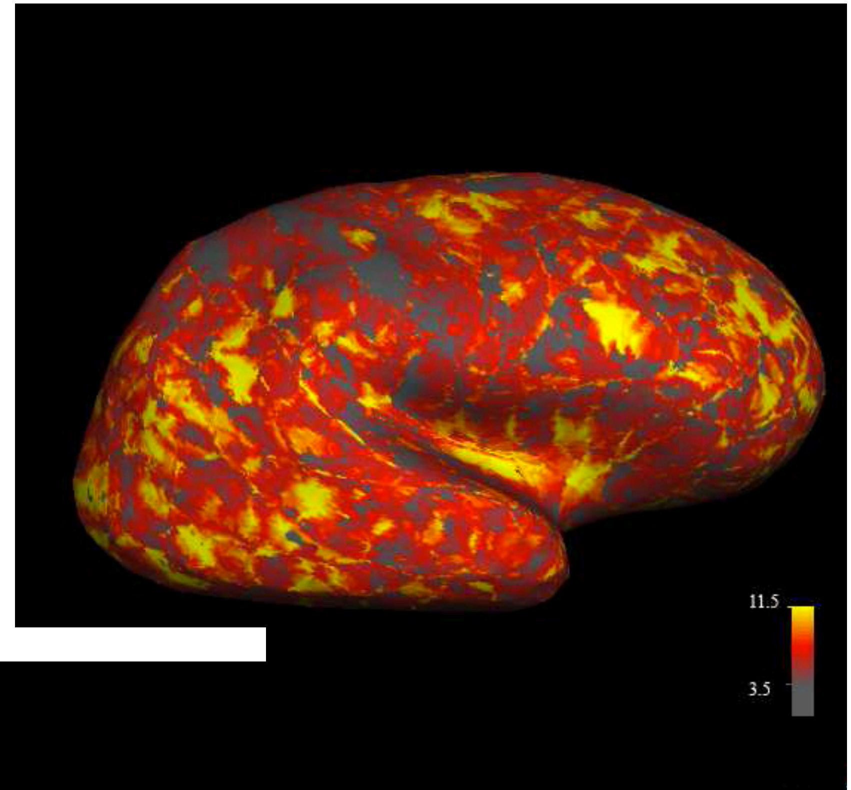


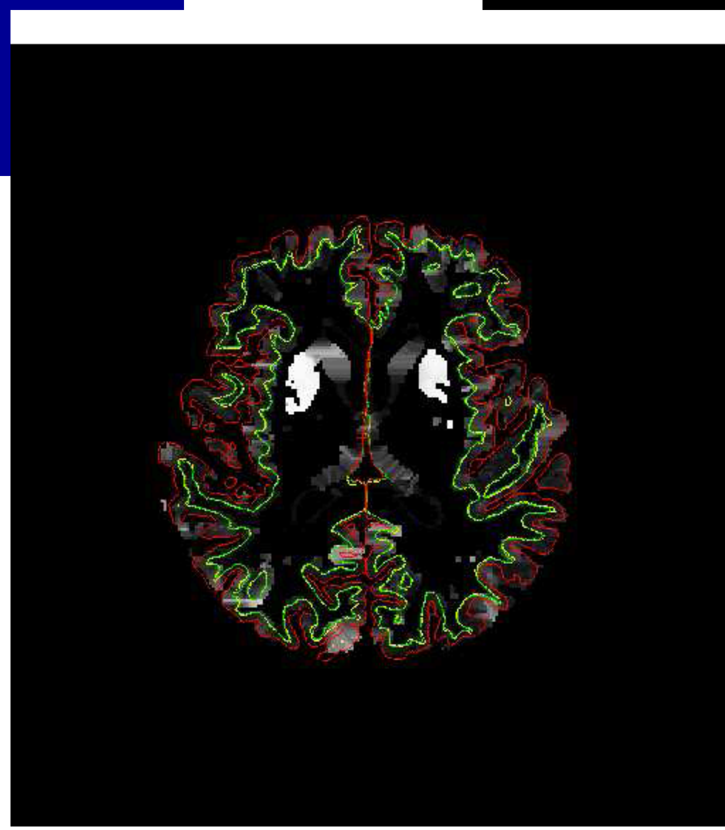
solve for streamline



Result



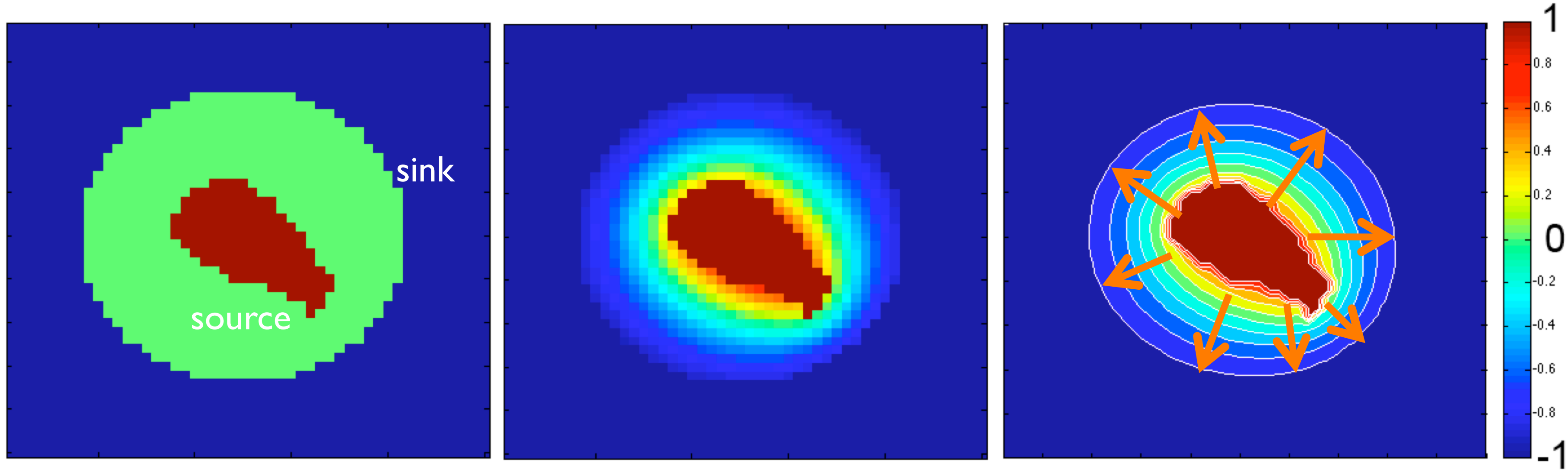




Surface flattening using Laplace equation

Read textbook section 6.2 page 68-70

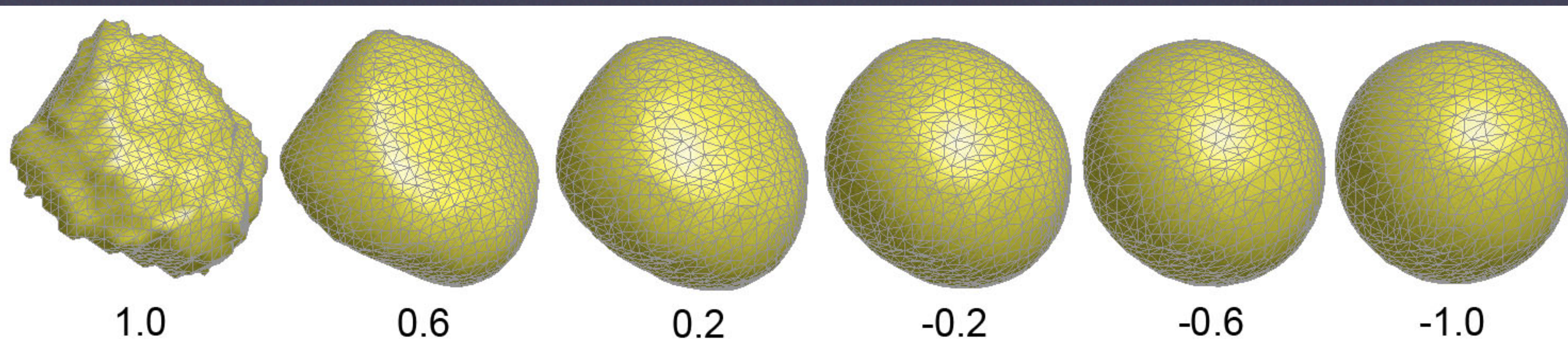
Amygdala surface flattening



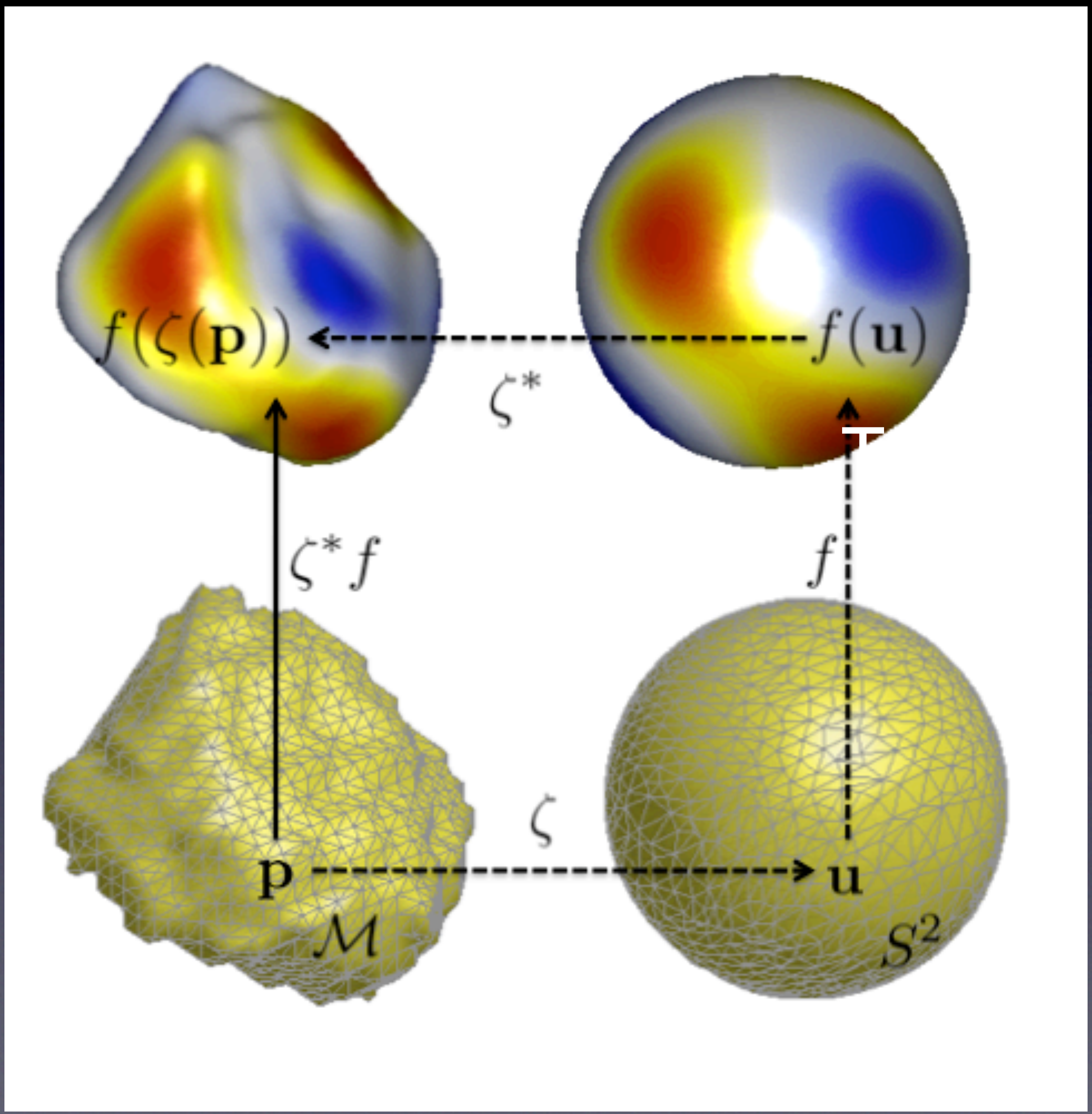
Boundary condition　　Equilibrium state　　Geodesic contour

$$\frac{\partial f}{\partial \sigma} = \Delta f \xrightarrow{\text{Equilibrium state}} \Delta f = 0$$

$$f(\texttt{source}, \sigma) = 1, f(\texttt{sink}, \sigma) = -1$$



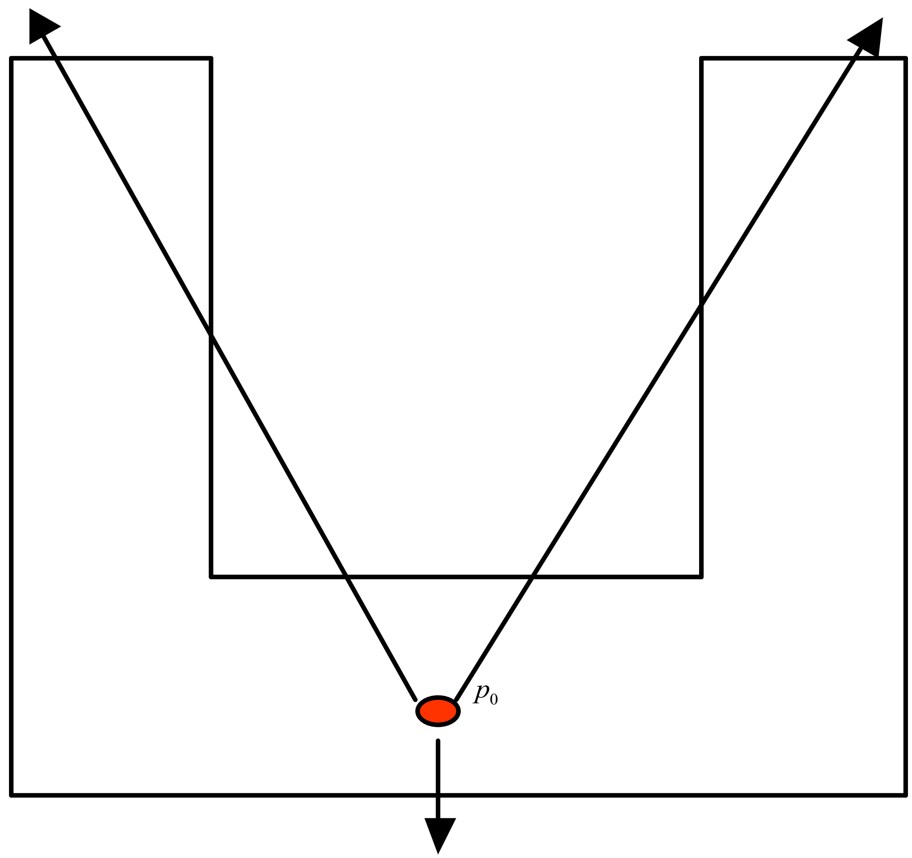
Shape analysis using spherical harmonics



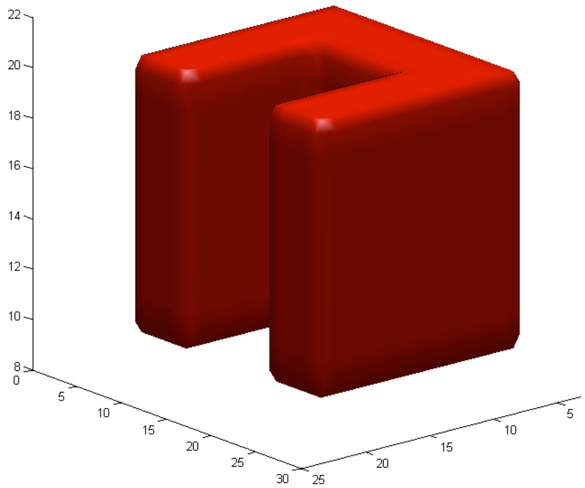
Why we wish to flatten an amygdala surface to a sphere?

It provides a uniform coordinate system to represent the (x,y,z) coordinates of the surface.

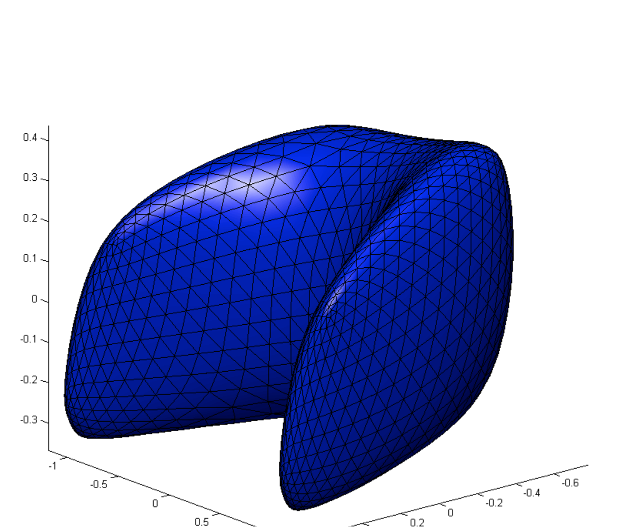
Mandible surface flattening



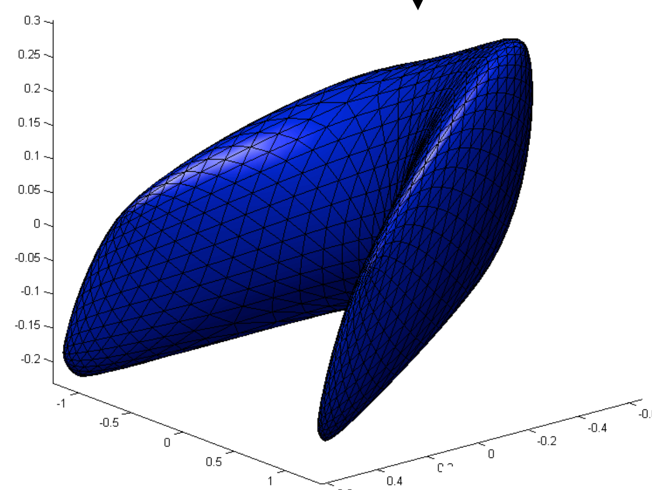
Will heat kernel smoothing on coordinates will flatten the surface?



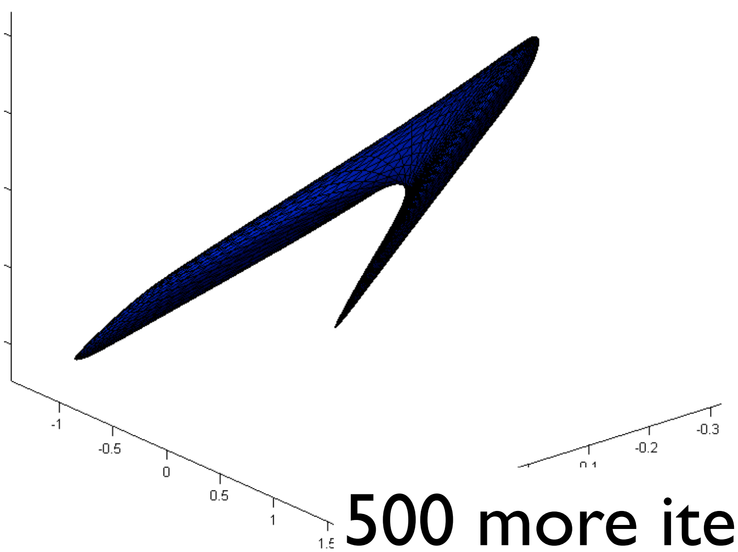
After heat kernel smoothing on coordinates



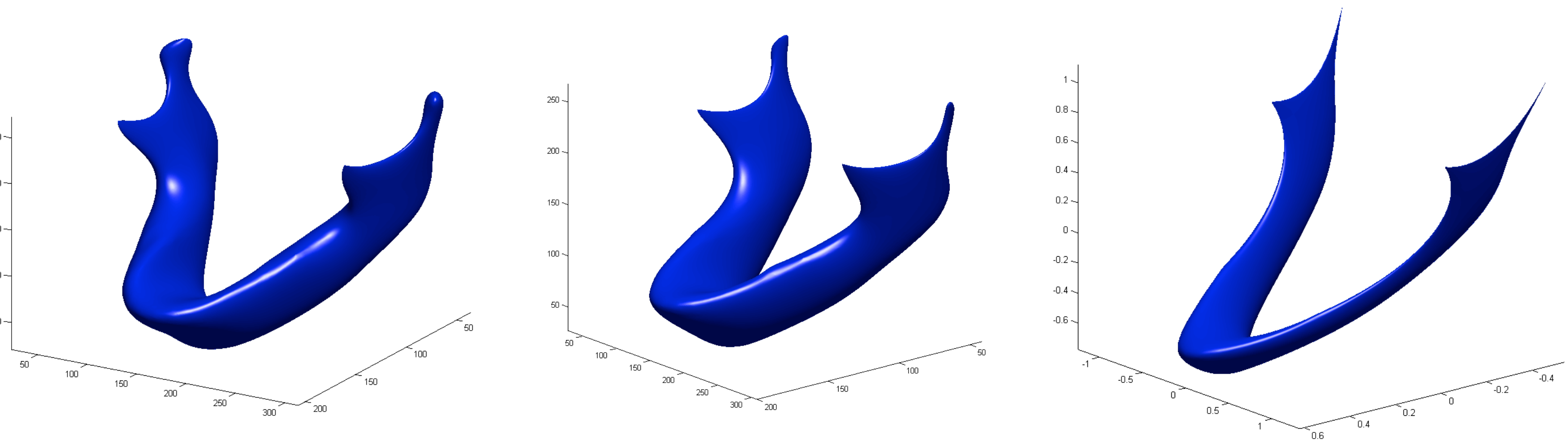
200 iterations



300 iterations

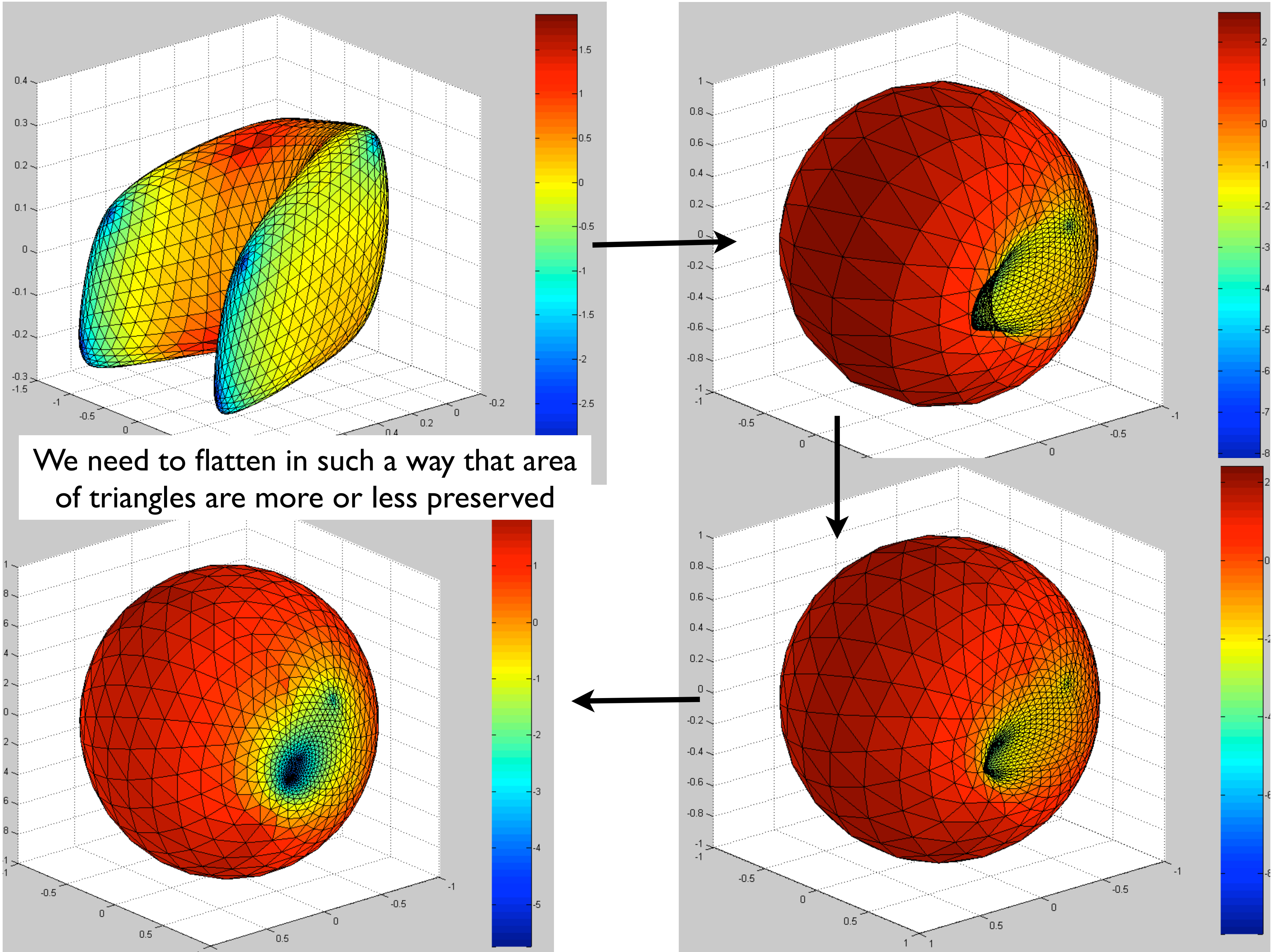


500 more iterations

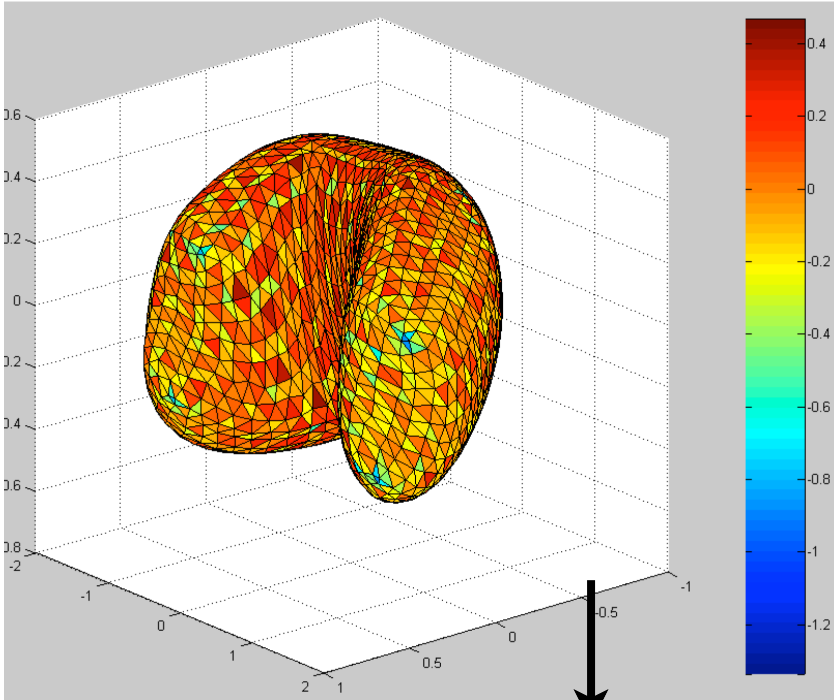


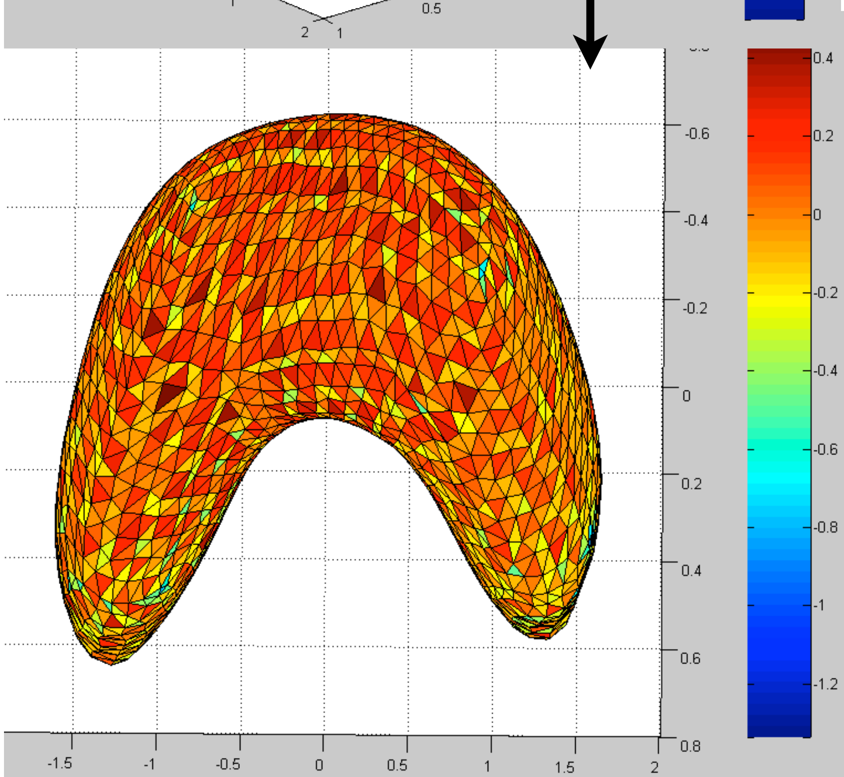
250
200
150
100
50
50 100 150 200 250 300
50 100 150 200
250
200
150
100
50
50 100 150 200 250 300
200 150 100 50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-1 -0.5 0 0.5 1
0.6 0.4 0.2 0 -0.2 -0.4

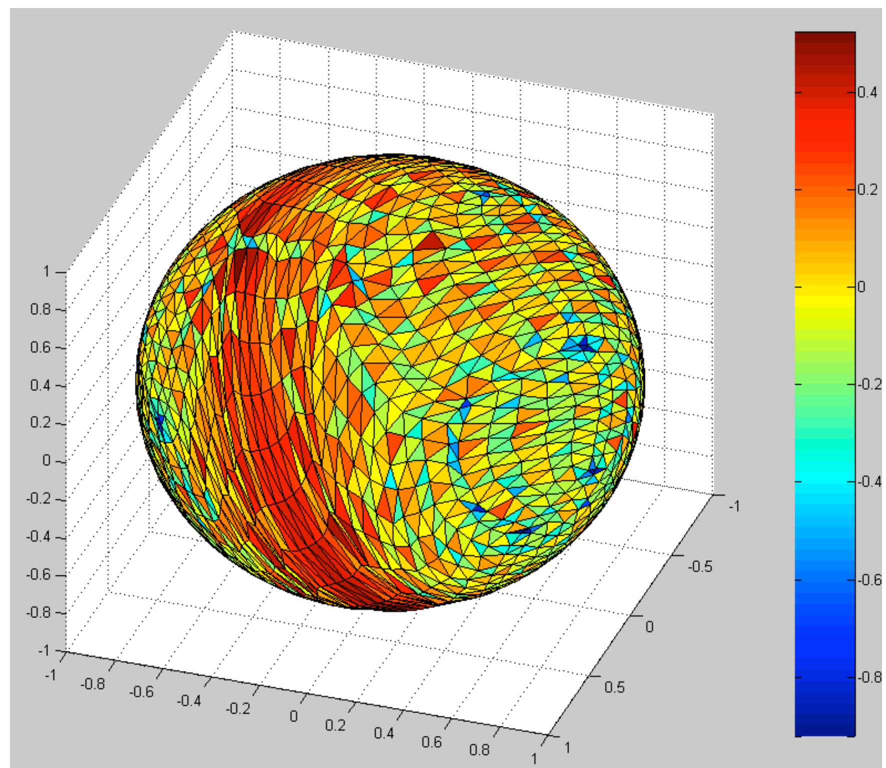
We need to flatten in such a way that area of triangles are more or less preserved



Better result
but not good enough







Intrinsic approach: spectral geometry

Mark Kac, 1966.
Can one hear the shape of a drum?
American Mathematical Monthly

↓

This formulation can be used to obtain basis functions.
Why you want basis functions?

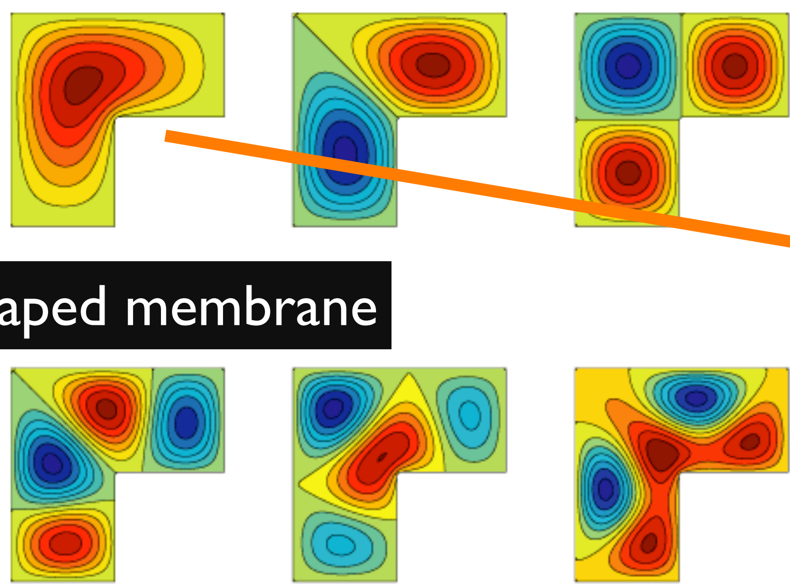
Well why you even need Fourier analysis in science in general?

It'a all about representing data using finite number of values.

Orthonormal basis

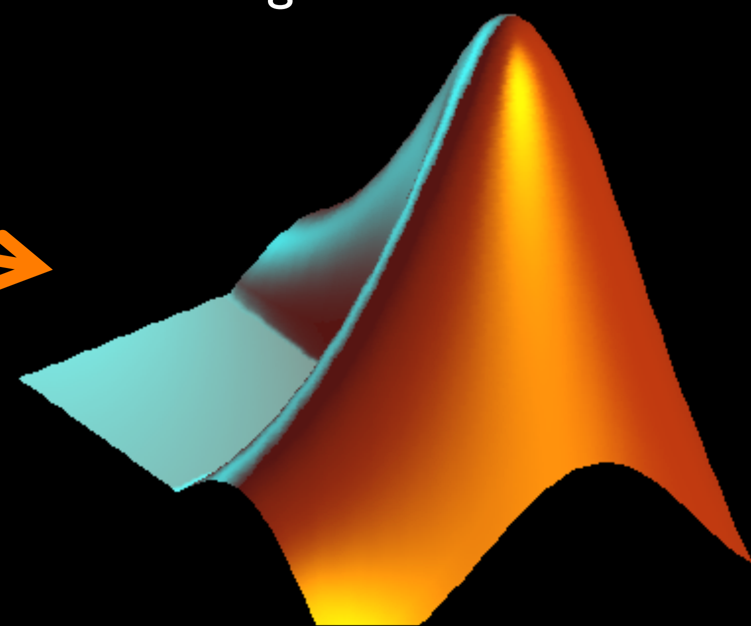
Steady-state oscillations in wave equation

Helmholtz equation $$\Delta_X F = \lambda F$$



L-shaped membrane

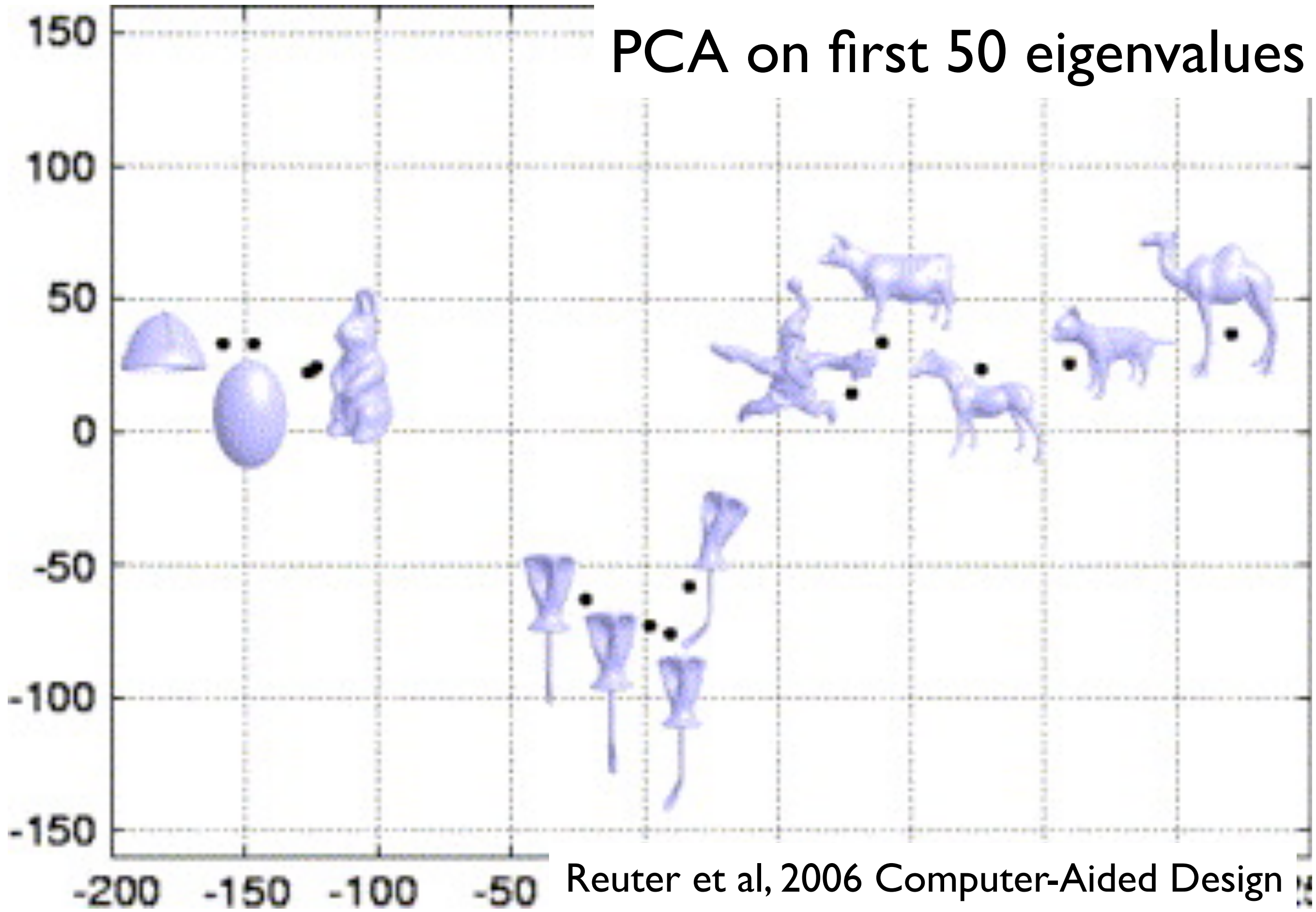
MATLAB logo



http://www.mathworks.com/company/newsletters/news_notes/clevescorner/win03_cleve.html

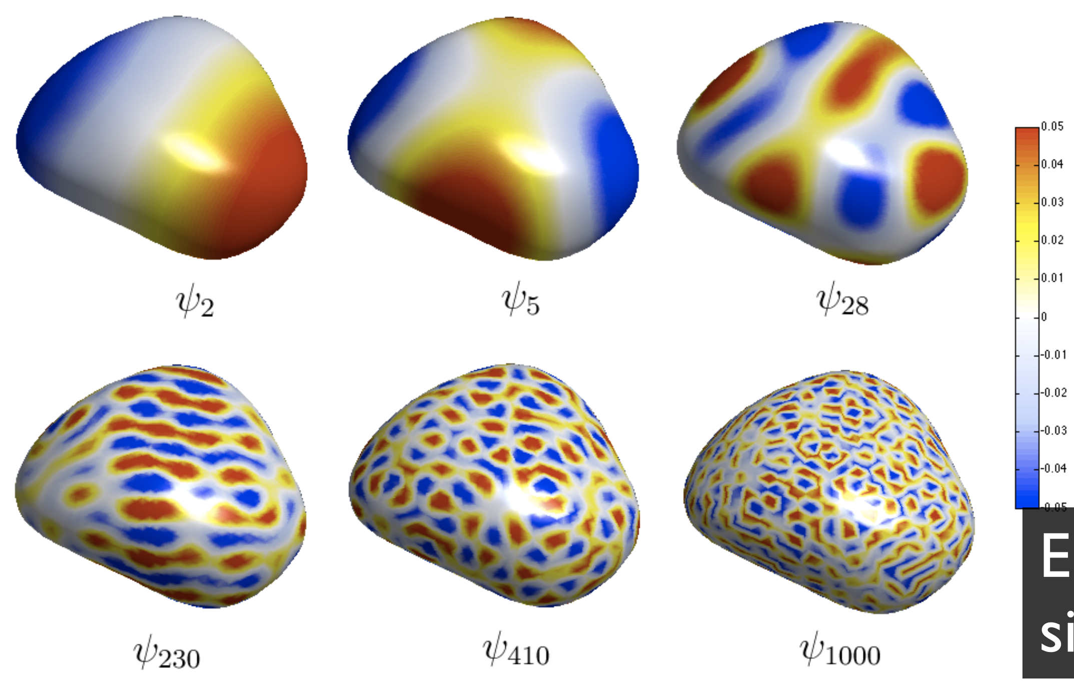
Shape spectrum

PCA on first 50 eigenvalues



Reuter et al, 2006 Computer-Aided Design

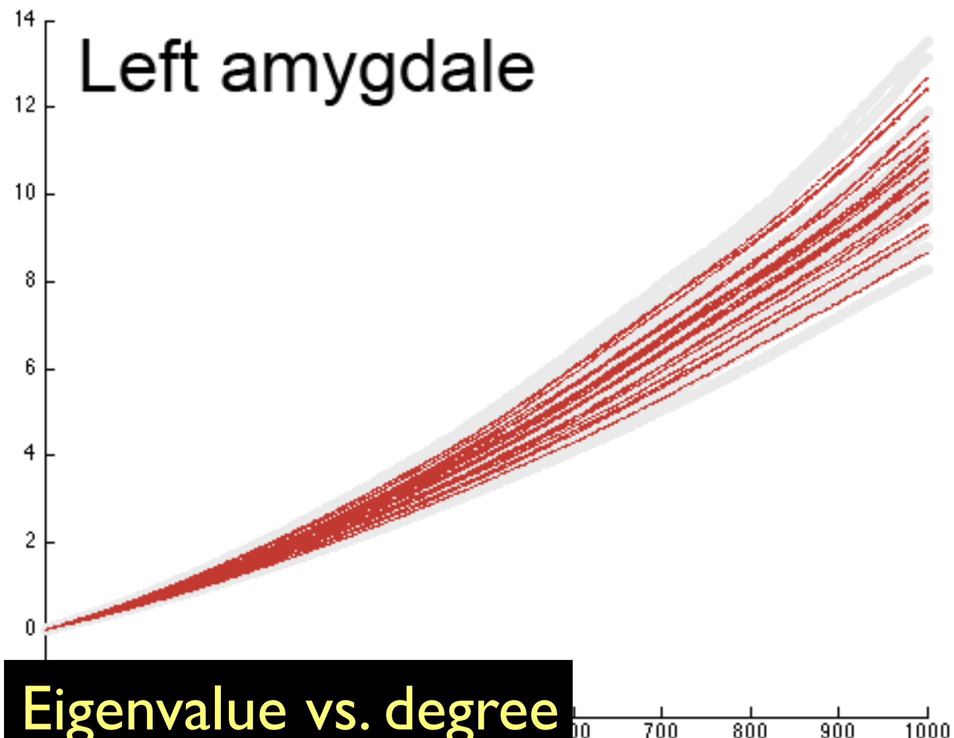
Eigenvalues (frequency of shapes) can characterize shape



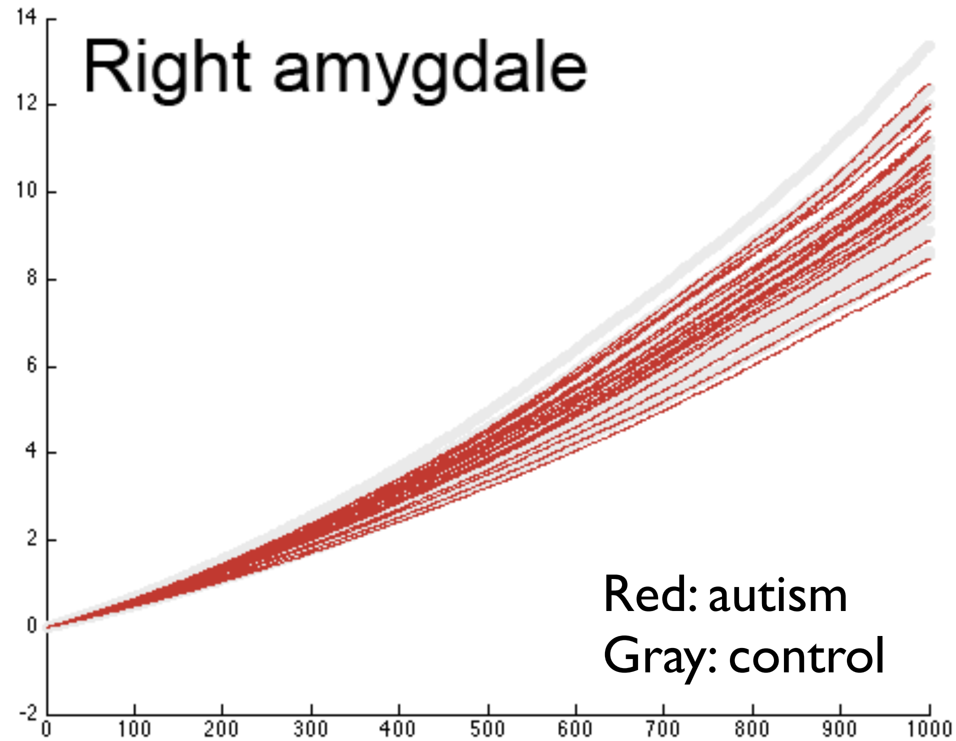
Weyl's formula

$$\lambda_k \to \frac{4\pi k}{\mu(\mathcal{M})}$$

Eigenvalues can't discriminate similarly shaped objects.

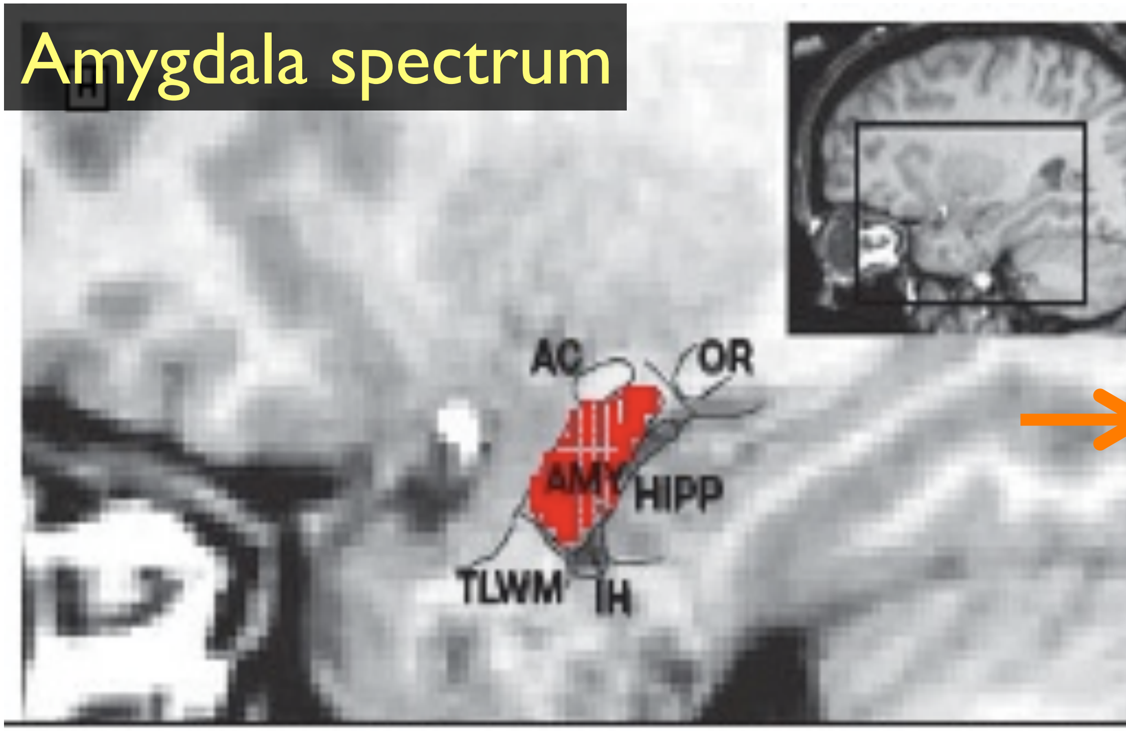


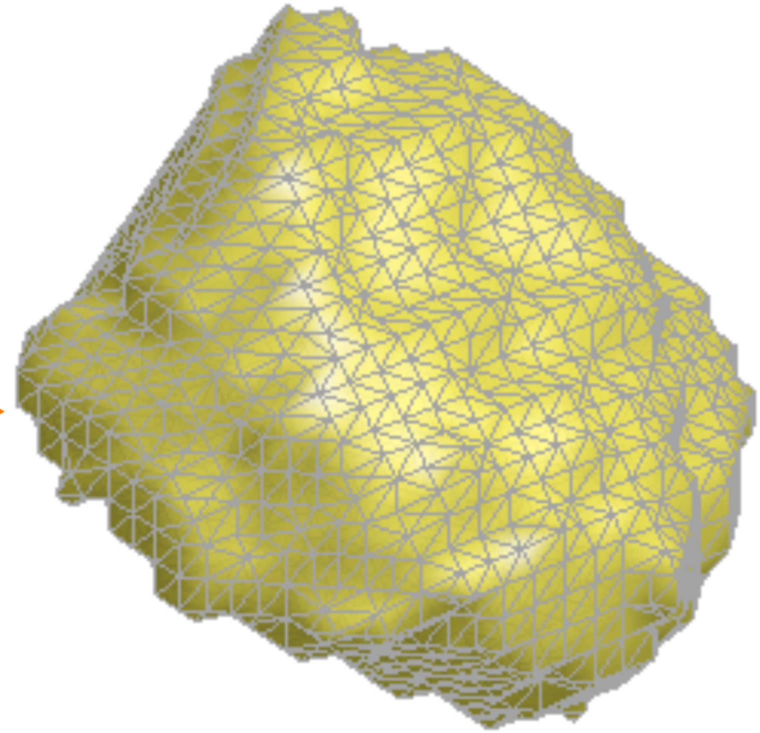
Eigenvalue vs. degree



Red: autism
Gray: control

Amygdala spectrum





Generalized eigenvalue problem

$$\Delta_X F = \lambda F$$

discretization

$$\lambda A\psi = C\psi$$

$$a_{ii} = \frac{1}{12} \sum_{p_j \in N(p_i)} T_{ij}^{+} + T_{ij}^{-}$$

$$c_{ii} = \frac{1}{2} \sum_{p_j \in N(p_i)} (\cot\theta_{ij} + \cot\phi_{ij})$$

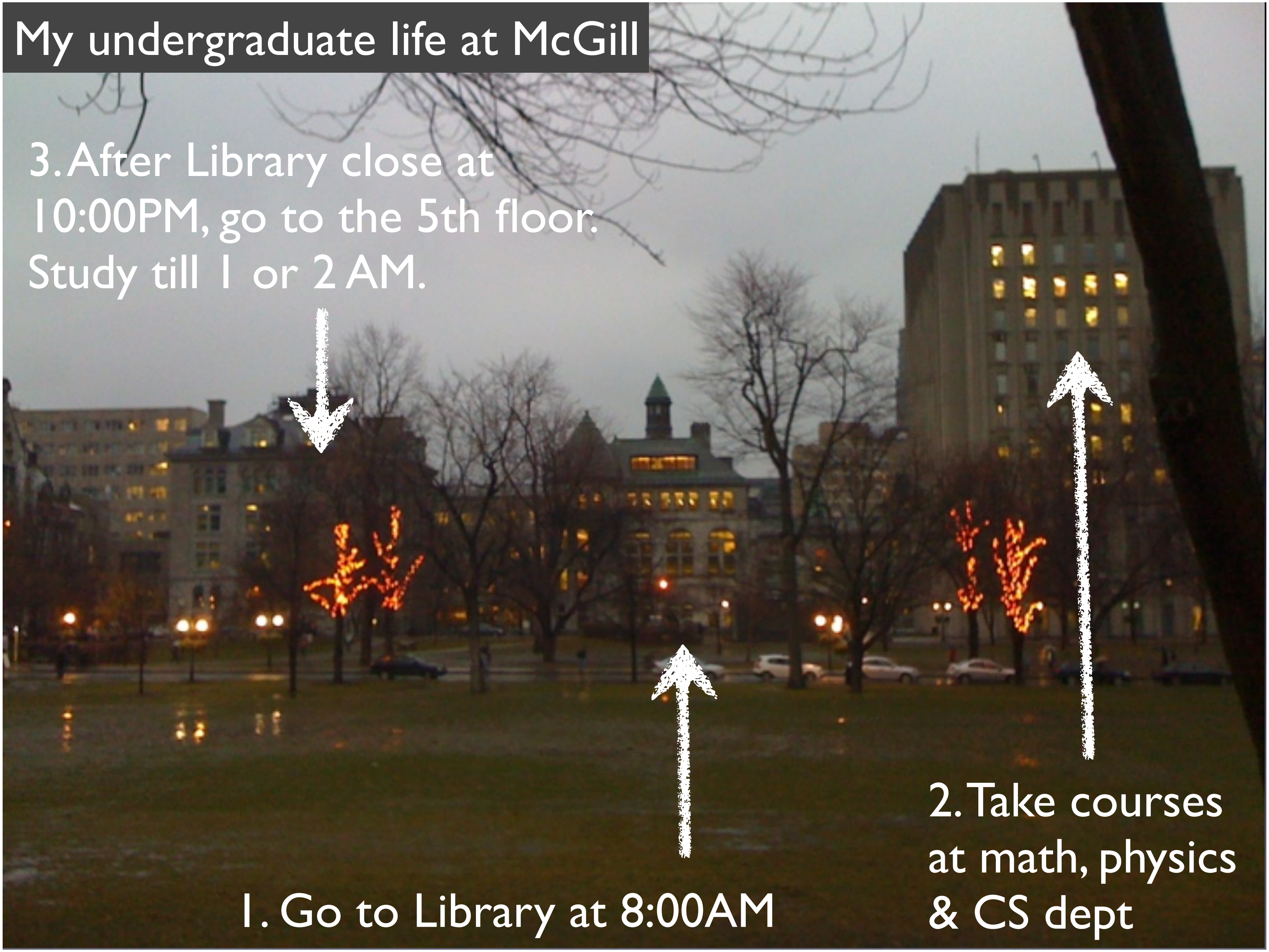
Read Textbook Chapter 6

About your project report... :)

Oral presentation (10%)
For the next week's presentation, send PDF of the presentation to me by December 11 11:00AM. We will use only one computer to present.

The final project report (30%) is due December 16 1:00PM. Late report will be penalized by 10% per day. After 3 days, you get 0%.

Thanks for taking this course. Please do not take the 2nd and 3rd course from me unless you like torturing yourself.
The courses will be twice harder.


My undergraduate life at McGill
3. After Library close at 10:00PM, go to the 5th floor. Study till 1 or 2 AM.
2. Take courses at math, physics & CS dept
1. Go to Library at 8:00AM