Neuroimage Processing Final Exam

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Instructions:

1. For all problems, clearly write down step-by-step derivations. No point will be given for problems that does not drive or explain clearly how you get the answer.
2. Simply stating “MATLAB gave the answer” will result in zero credit. You have to compute by hand. All problems can be easily solved by hand. But MATLAB can possibly be used for validation if necessary.
3. Not allowed to talk to other students.
4. No point given to illegible handwriting.
5. Plagiarism of any form is not allowed.

Problem 1. Group A consists of 3 people and group B consists of 2 people. Cortical thickness for group A is 1, 2 and 3 mm for some cortical region. Cortical thickness for group B is 1 and 2 mm. Estimate the thickness of group A using the least squares method. What is the sum of squared error (SSE) of your estimation? Set up a general linear model testing for equality of cortical thickness between the two groups, and compute the actual F-statistic value.

Problem 2. Given a T1-weighted MRI of brain, the proportions of gray matter and white matter in the image are 0.7 and 0.3 respectively. Then explain why we can model image intensity values as the mixture of two Gaussian components with mixing proportions 0.7 and 0.3.

Problem 3. Consider a 2D image represented as a matrix \( I = \begin{pmatrix} 1 & 0 & 2 & 0 \\ -1 & -2 & 1 & -1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \). Given a kernel \( K = \begin{pmatrix} 1/16 & 1/8 & 1/16 \\ 1/8 & 1/4 & 1/8 \\ 1/16 & 1/8 & 1/16 \end{pmatrix} \), evaluate kernel smoothing \( K \ast I \) at the pixel position (2,2). Note that at (2,2), image intensity value is -2. Why heat kernel smoothing is used for cortical thickness data instead of Gaussian kernel smoothing?
**Problem 4.** Given diffusion tensor coefficients $D = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ in a voxel, determine the fractional anisotropy (FA) index. The following MATLAB code is provided. What does FA value measure?

```
>> D=[3 0 1
    0 2 0
    1 0 1]
[A B]=eig(D)
D =
  3     0     1
  0     2     0
  1     0     1
A =
  0.3827         0    0.9239
  0    -1.0000         0
-0.9239         0    0.3827
B =
 0.5858         0         0
 0    2.0000         0
 0         0    3.4142
```

Problem 5. Given the following two brain connectivity graphs A and B, write down the adjacency matrices. Write down the degree distributions. Compute the clustering coefficients. Which brain network is more complex?

![Graph A](image1.png)   ![Graph B](image2.png)

**Problem 6.** Given a deformation field $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x - 2y + 1 \\ x + 4y - 1 \end{pmatrix}$ for warping 2D image A to another 2D image B, determine the displacement vector field. Determine the Jacobian determinant at (0,0). Is the pixel at (0,0) in image A shrinking or enlarging under the deformation? What is the main difference between deformation-based morphometry (DBM) and tensor-based morphometry (TBM)?

END OF PROBLEMS