Computational Challenges in Brain Imaging

Moo K. Chung

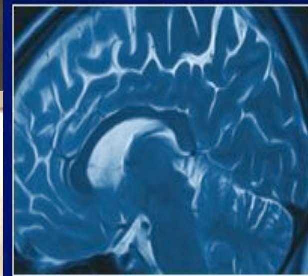
Department of Brain and Cognitive Sciences
Seoul National University

Department of Biostatistics and Medical Informatics
Waisman Laboratory for Brain Imaging and Behavior
University of Wisconsin-Madison

www.stat.wisc.edu/~mchung

SNU BCS DLS
October 14, 2009

University of Wisconsin, Madison



The Waisman Laboratory for Brain Imaging and Behavior

6 faculty members

MRI + PET + MicroPET + MEG+EEG + eye tracking device

5 staff members (2 computer support + 3 administrative/grant)

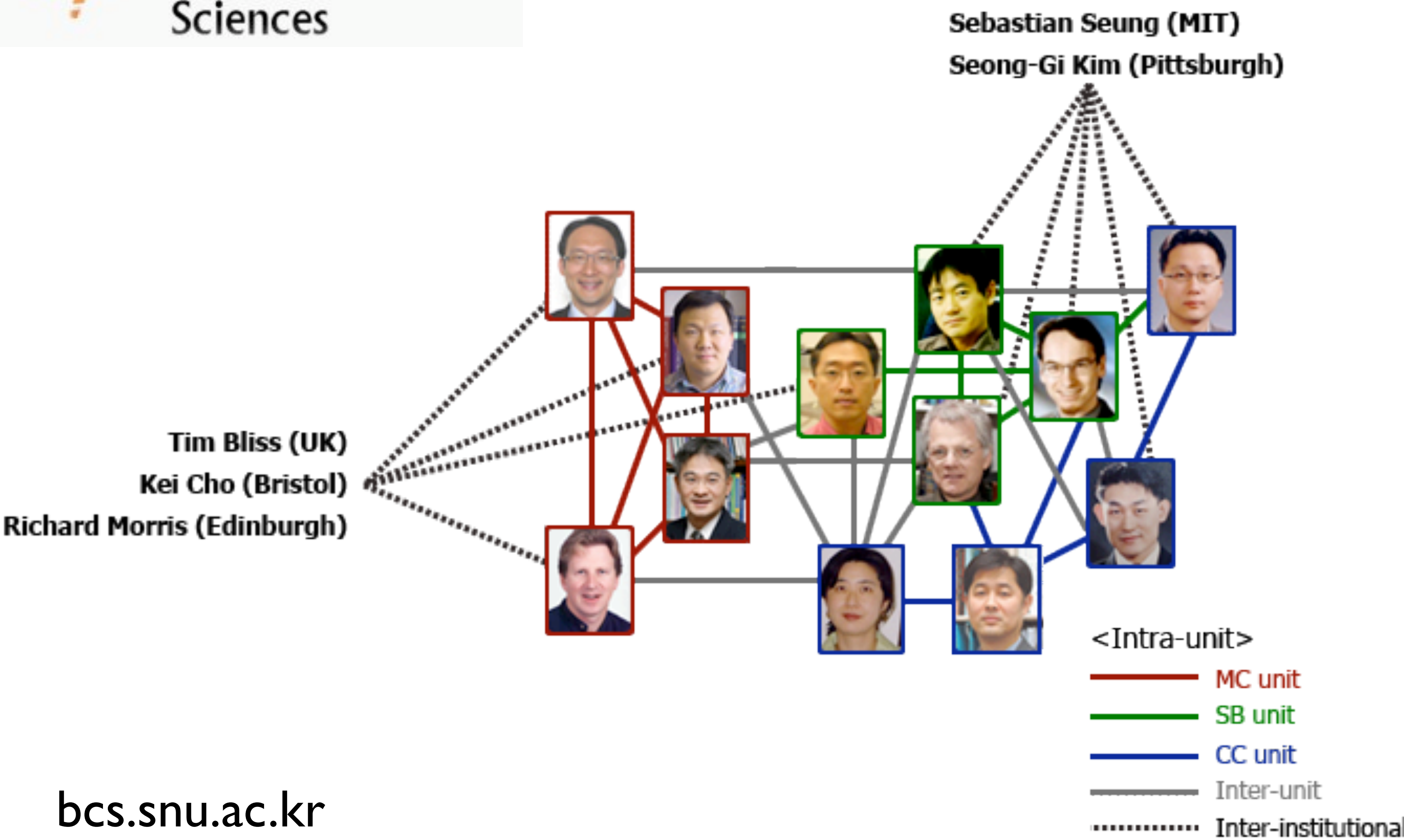
20 PhD level scientists + postdocs

100 graduate + undergraduate students

brainimaging.waisman.wisc.edu

Brain & Cognitive Sciences

Seoul National University



bcs.snu.ac.kr

Job Advertisement

We are looking for graduate students (masters, PhD) and postdoctoral students at BCS who will do brain research in general and computational aspect of brain imaging in particular.

Any student with math, stat, physics, CS and EE will find the field real easy.
Send email to mkchung@wisc.edu

Abstract

Computational neuroanatomy is an emerging field that utilizes various non-invasive brain imaging modalities such as magnetic resonance imaging (MRI) and diffusion tensor imaging (DTI) in quantifying the spatiotemporal dynamics of the human brain structures in both normal and clinical populations in macroscopic level. This discipline emerged about twenty years ago and has made substantial progress in the past decade. It usually deals with computational problems arising from the quantification of within- and between-subject variations associated with the structure and the function of the human brain. Major challenges in the field are caused by the massive amount of nonstandard high dimensional non-Euclidean imaging data that are difficult to analyze using traditional methods. This requires new computational solutions that incorporate geometric and topological nature of brain structures. Overview of various computational issues in neuroanatomy will be presented with example studies on autism.

Outline

1. Brain images & problems

2. Intrinsic method

3. Extrinsic method

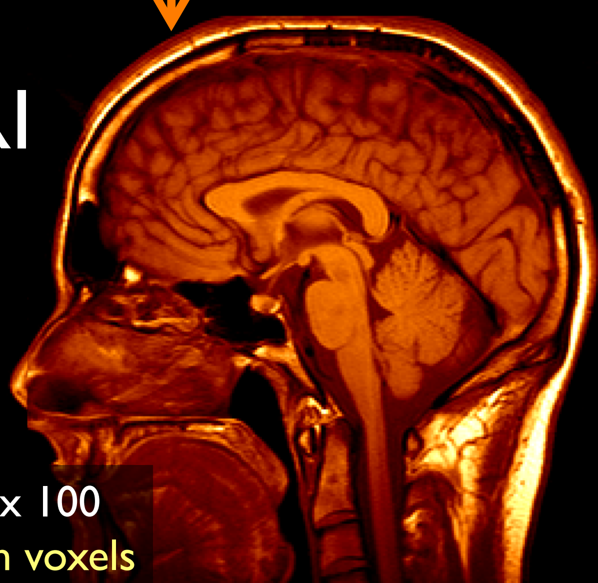
4. Computational Challenge I
(extremely large least squares problem)

5. Computational Challenge II
(extremely large 3D graph model)

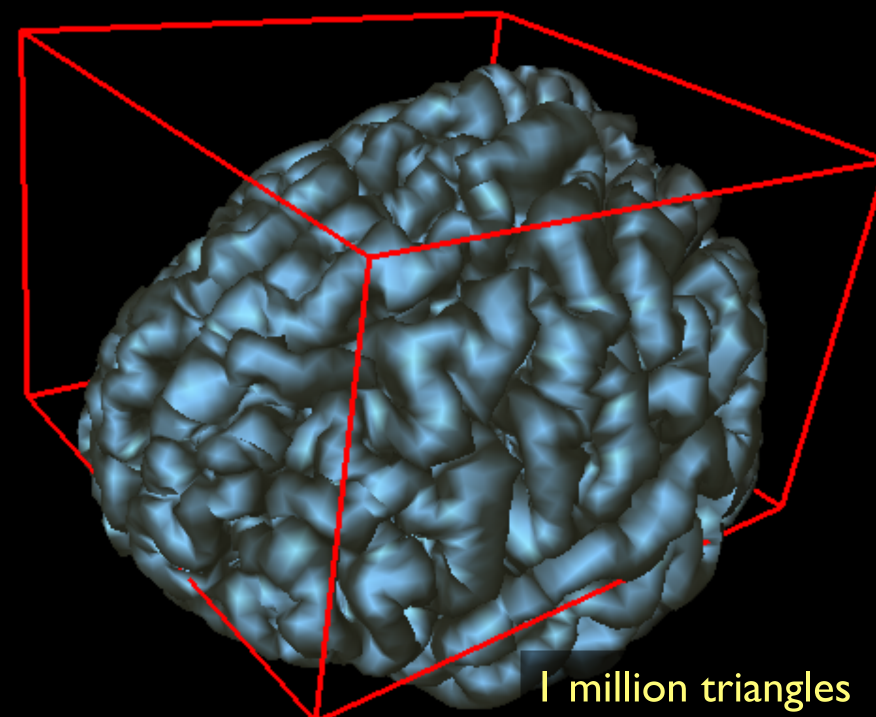
Real brain

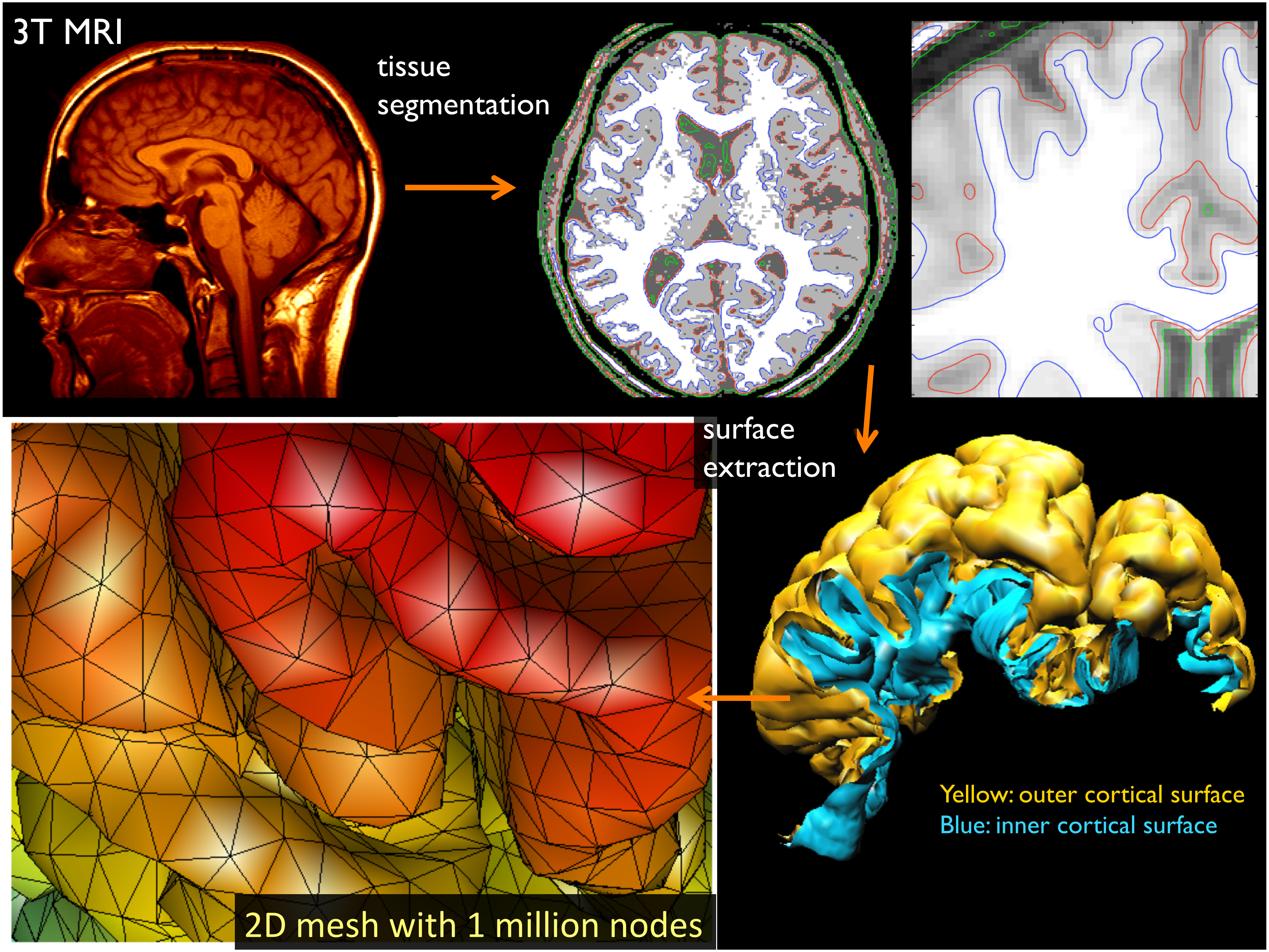
Magnetic Resonance Imaging

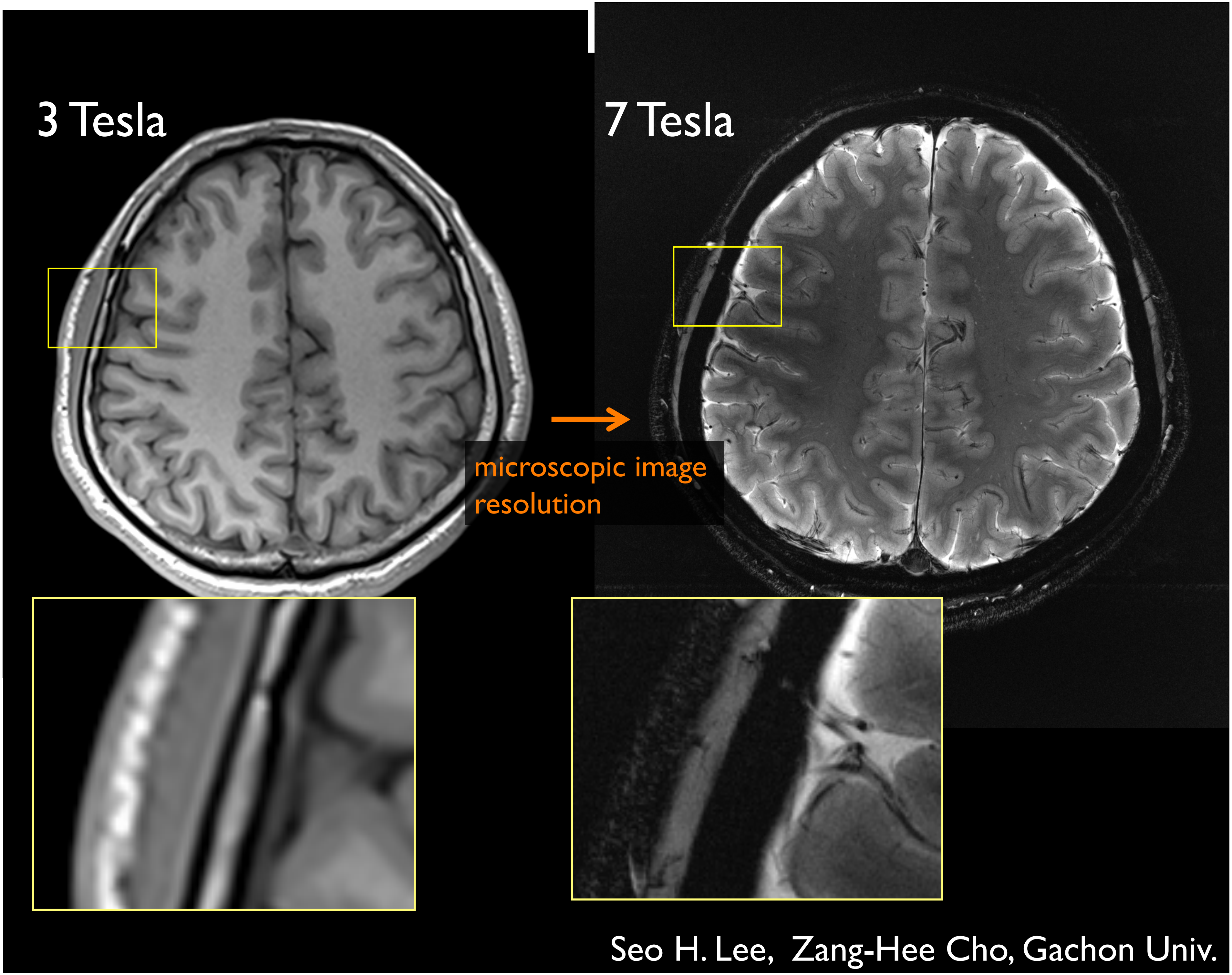
3T MRI

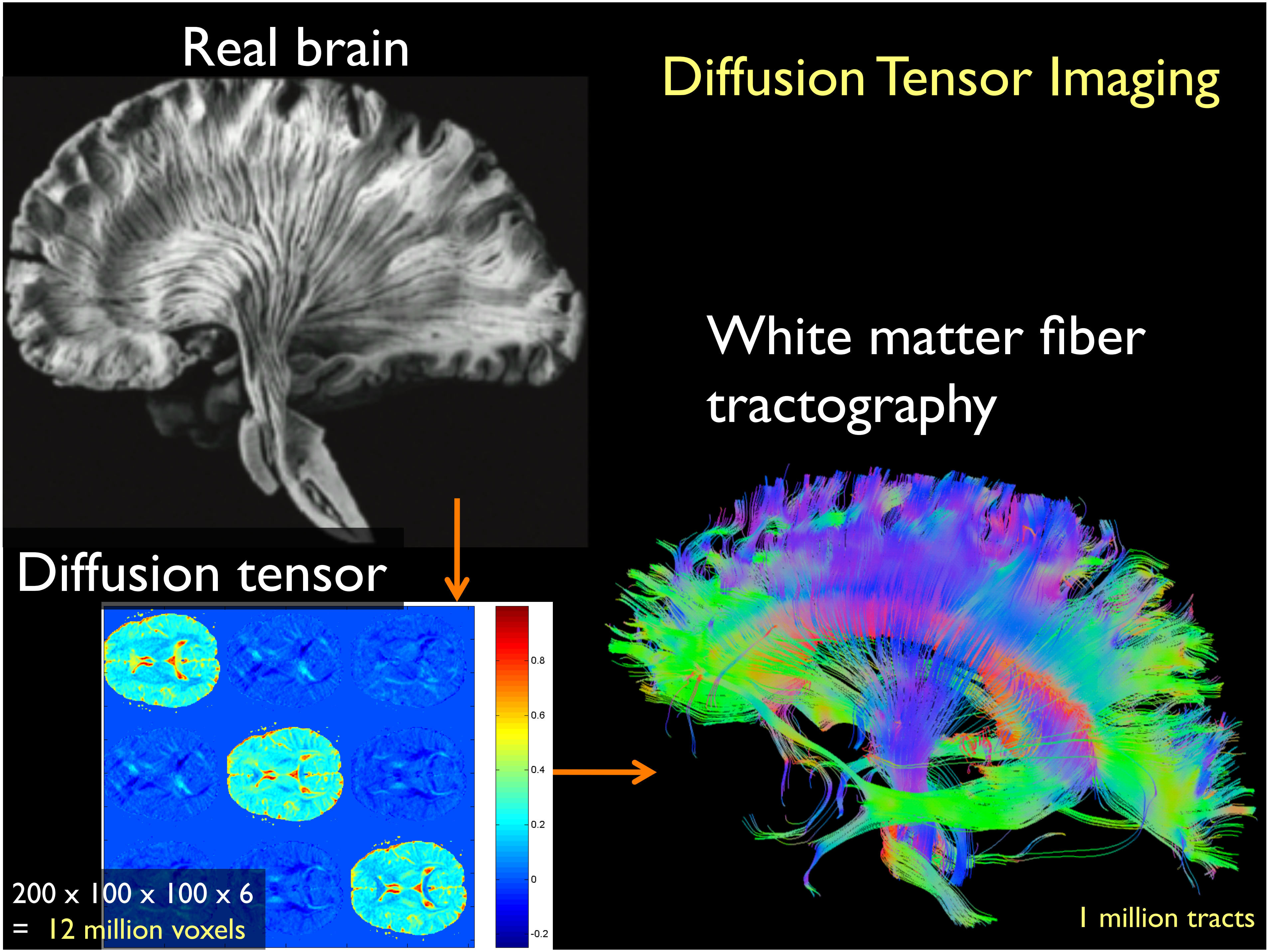


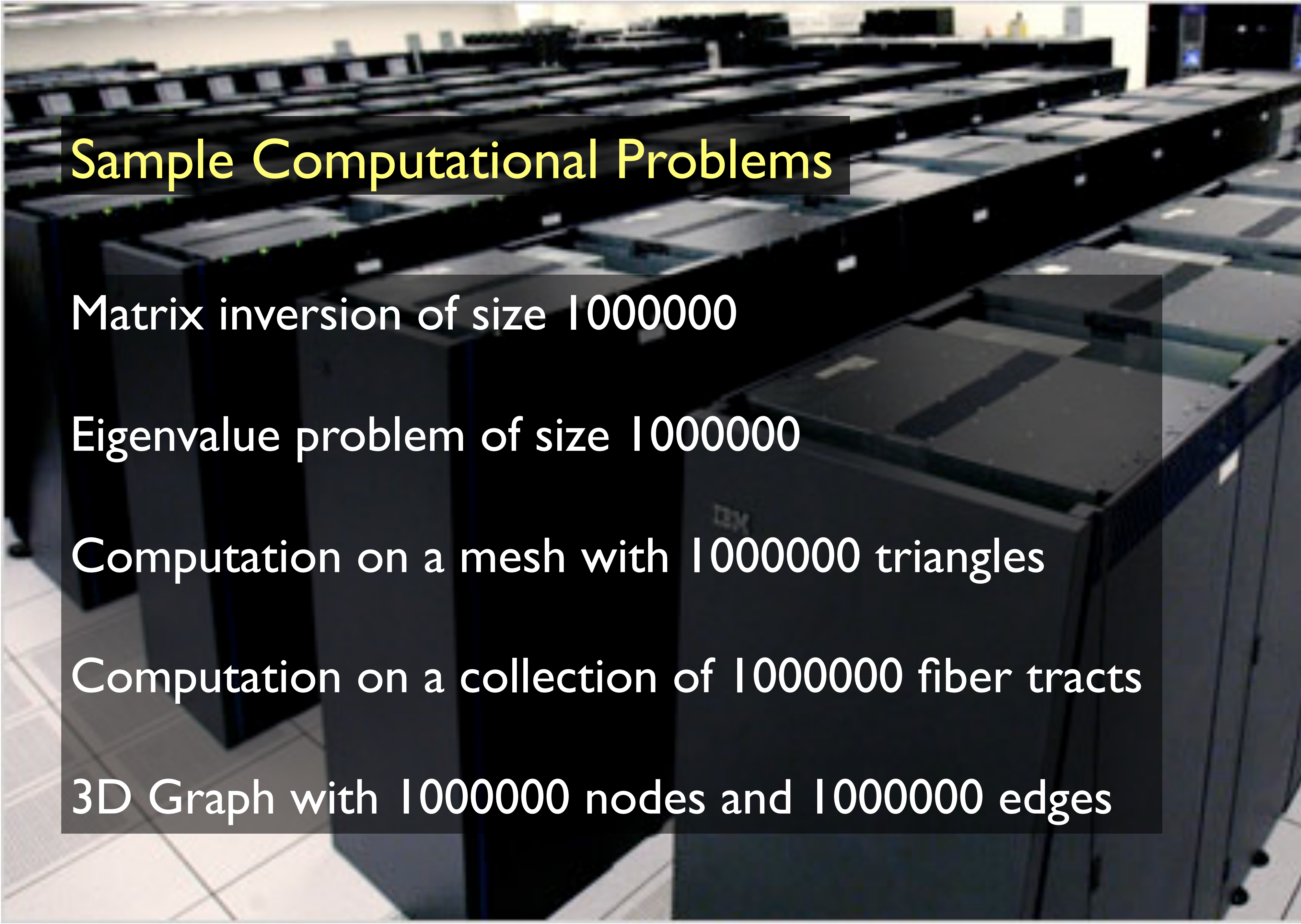
Cortical surface model




3T MRI
tissue segmentation
surface extraction
Yellow: outer cortical surface
Blue: inner cortical surface
2D mesh with 1 million nodes


3 Tesla
7 Tesla
microscopic image
resolution
Seo H. Lee, Zang-Hee Cho, Gachon Univ.


Real brain
Diffusion Tensor Imaging
White matter fiber tractography
Diffusion tensor
0.8
0.6
0.4
0.2
0
-0.2
200 x 100 x 100 x 6
= 12 million voxels
1 million tracts



Sample Computational Problems

Matrix inversion of size 1000000

Eigenvalue problem of size 1000000

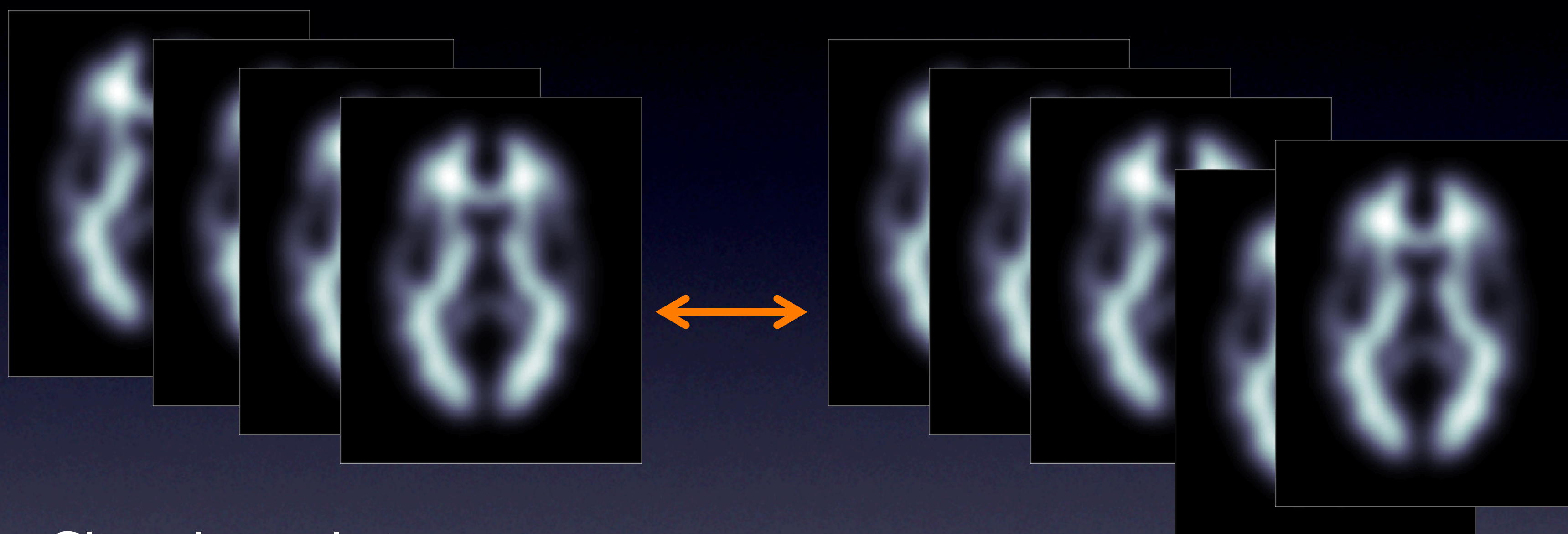
Computation on a mesh with 1000000 triangles

Computation on a collection of 1000000 fiber tracts

3D Graph with 1000000 nodes and 1000000 edges

Typical question in computational neuroanatomy

Given a collection of images



Clinical population:
autism, Parkinson's decease

Normal controls

1. Do brains differ in shape ?
2. How they differ?

Intrinsic approach
Spectral Geometry

Intrinstic approach: spectral geometry

Mark Kac, 1966.
Can one hear the shape of a drum?
American Mathematical Monthly

If we know the shape of a drumhead, we know its frequency. Can we know the shape of the drumhead knowing its frequency?

↓

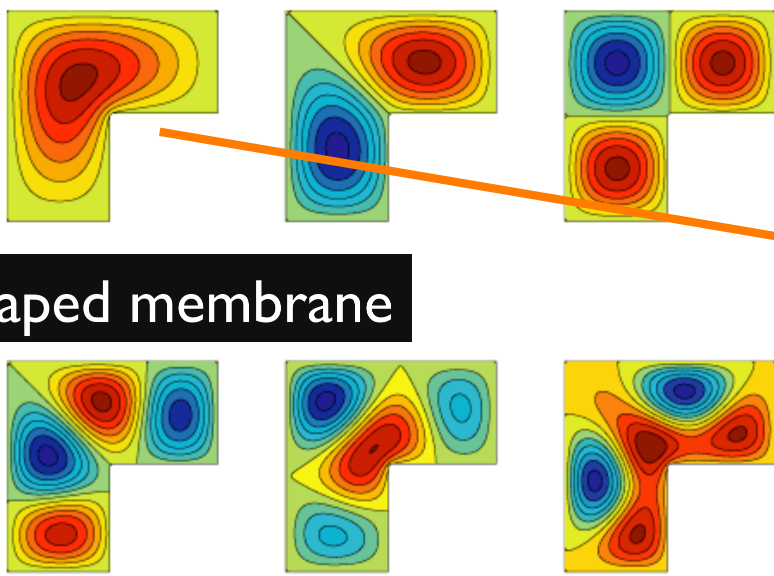
Shape characterization using spectrum

Shape spectrum

Steady-state oscillations in wave equation

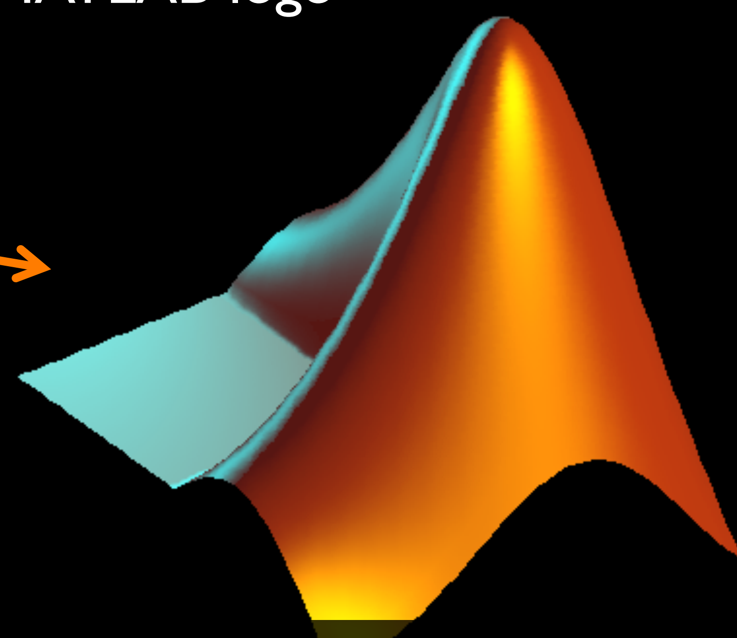
↓

Helmholtz equation $\quad \Delta_X F = \lambda F$



L-shaped membrane

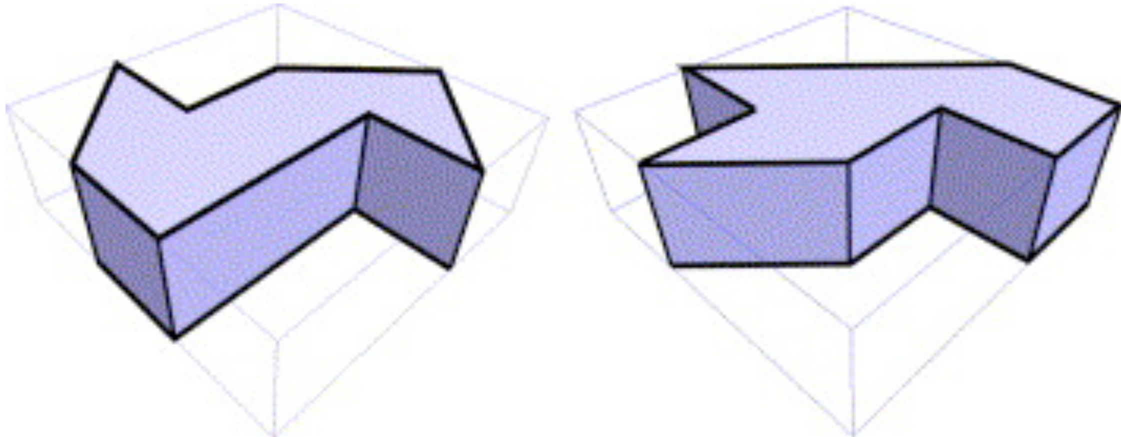
MATLAB logo



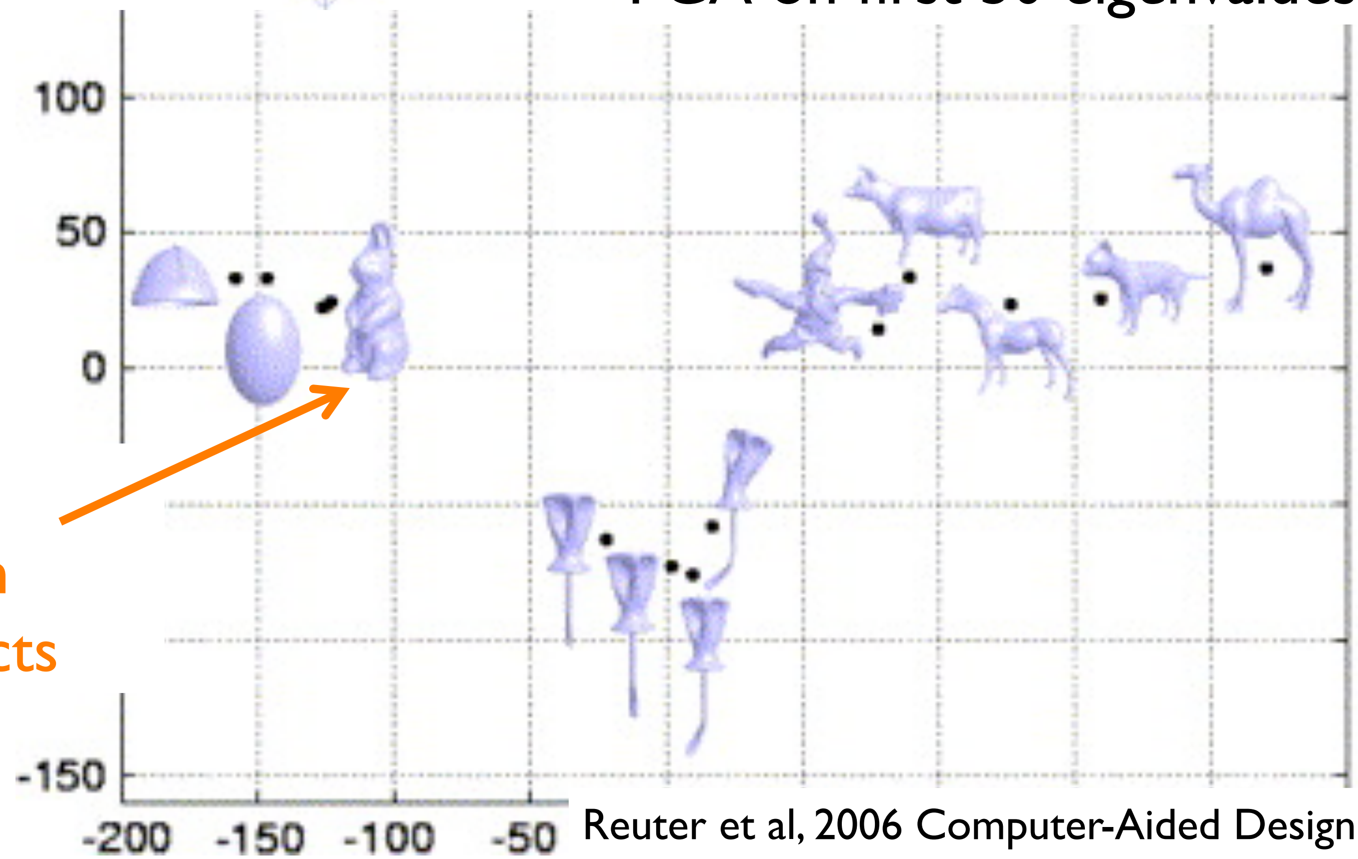
http://www.mathworks.com/company/newsletters/news_notes/clevescorner/win03_cleve.html

Shape spectrum

Isospectral shapes



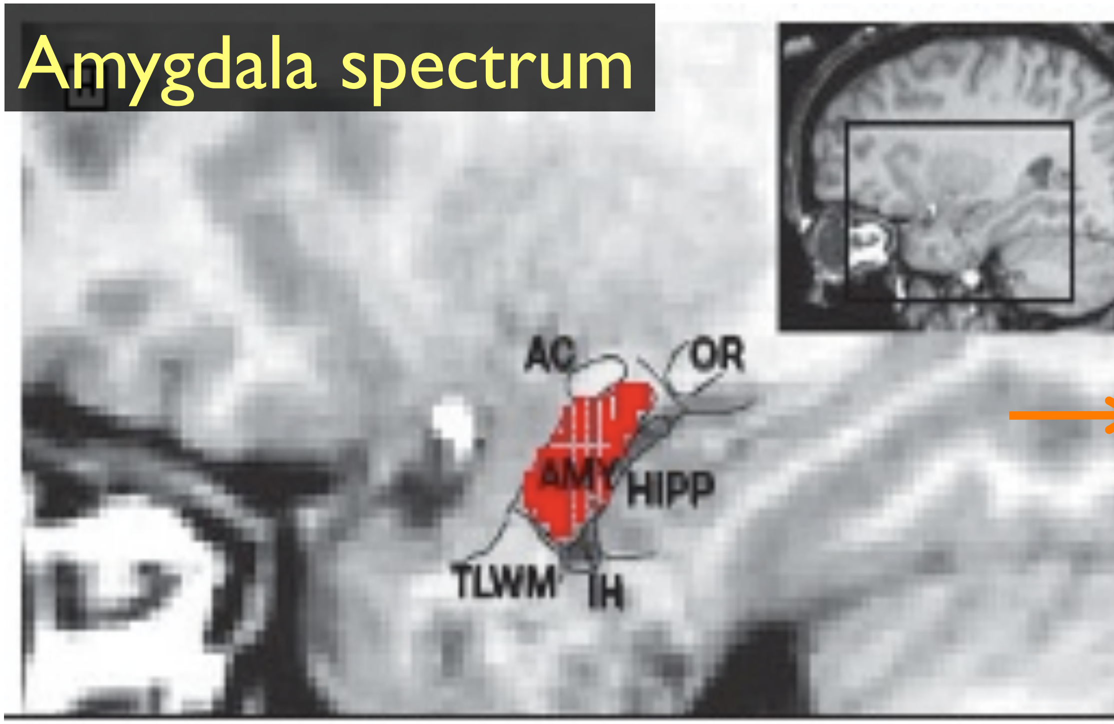
PCA on first 50 eigenvalues

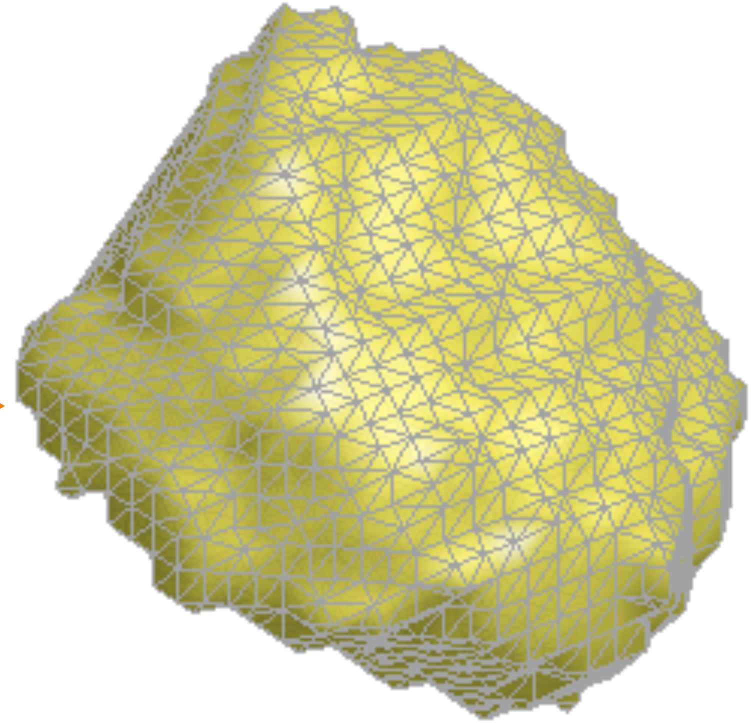


A bunny can't be distinguished from other stupid objects

Reuter et al, 2006 Computer-Aided Design

Amygdala spectrum





Generalized eigenvalue problem

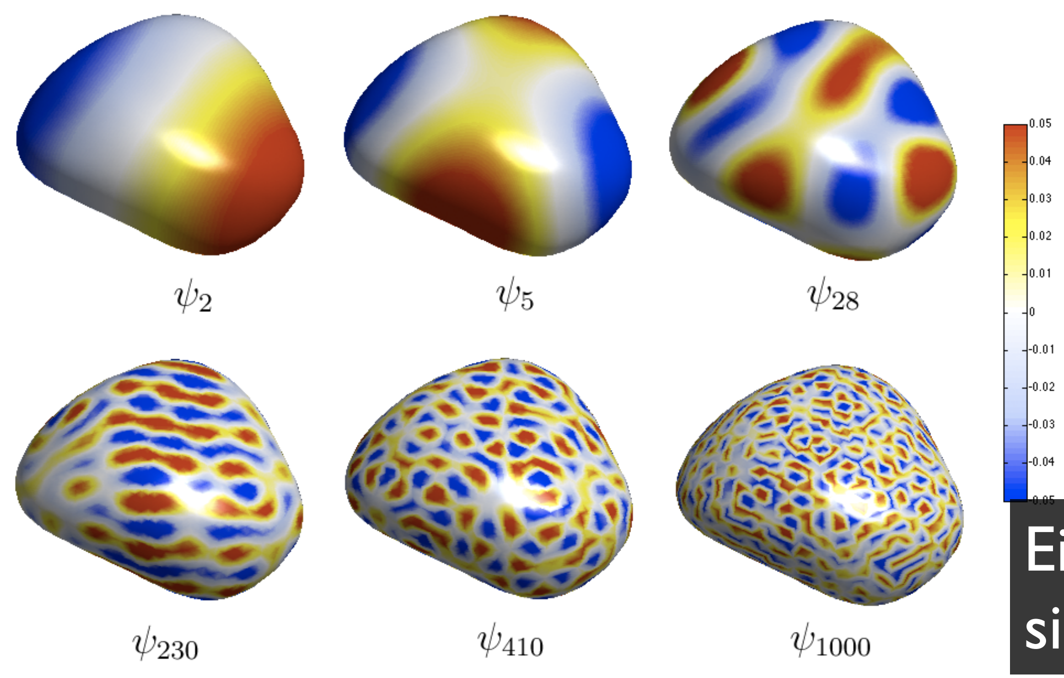
$$\Delta_X F = \lambda F$$

$$\Big\downarrow \text{discretization}$$

$$\lambda A \psi = C \psi$$

$$a_{ii} = \frac{1}{12} \sum_{p_j \in N(p_i)} T_{ij}^{+} + T_{ij}^{-}$$

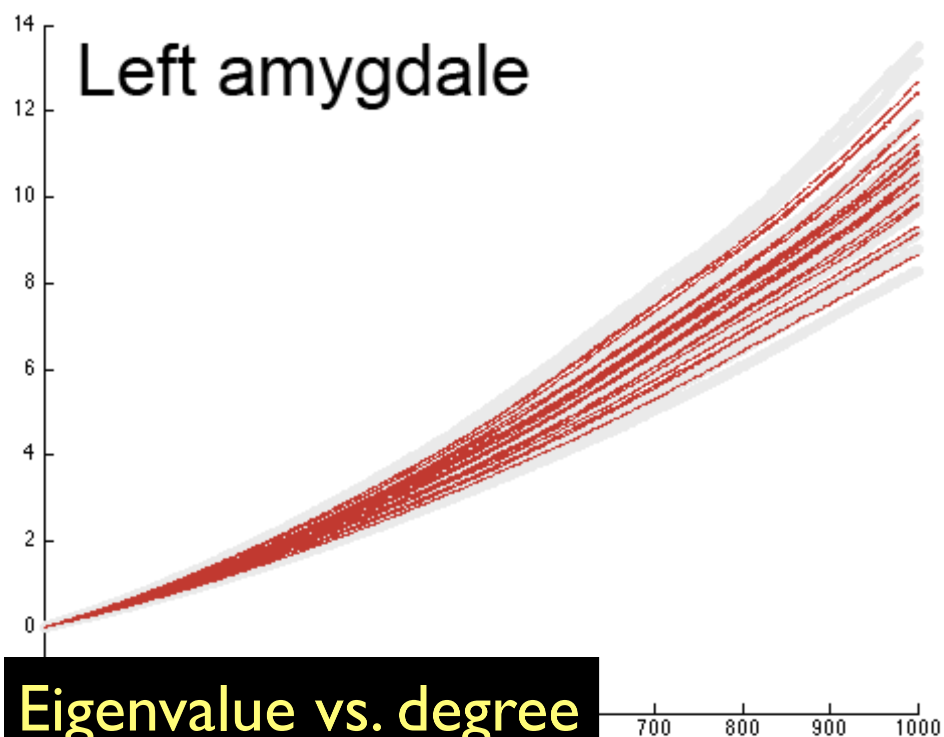
$$c_{ii} = \frac{1}{2} \sum_{p_j \in N(p_i)} (\cot \theta_{ij} + \cot \phi_{ij})$$

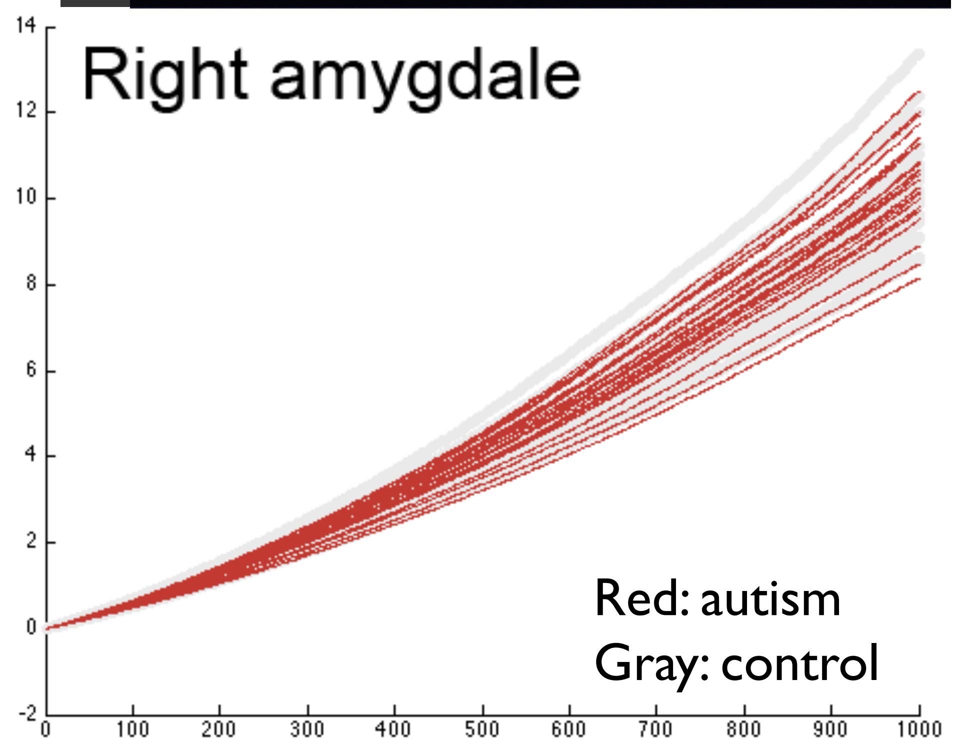


Weyl's formula

$$\lambda_k \rightarrow \frac{4\pi k}{\mu(\mathcal{M})}$$

Eigenvalues can't discriminate similarly shaped objects.





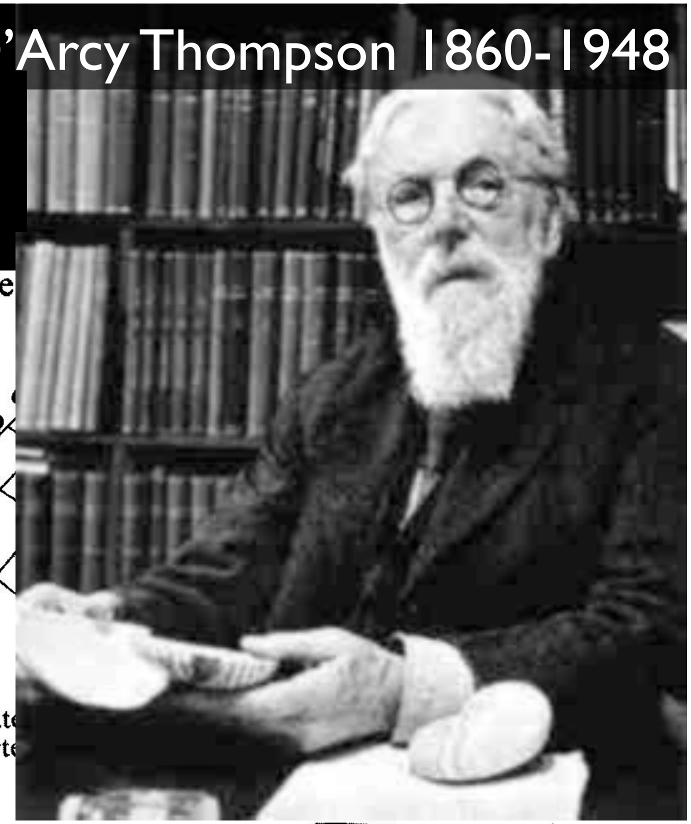
Eigenvalue vs. degree

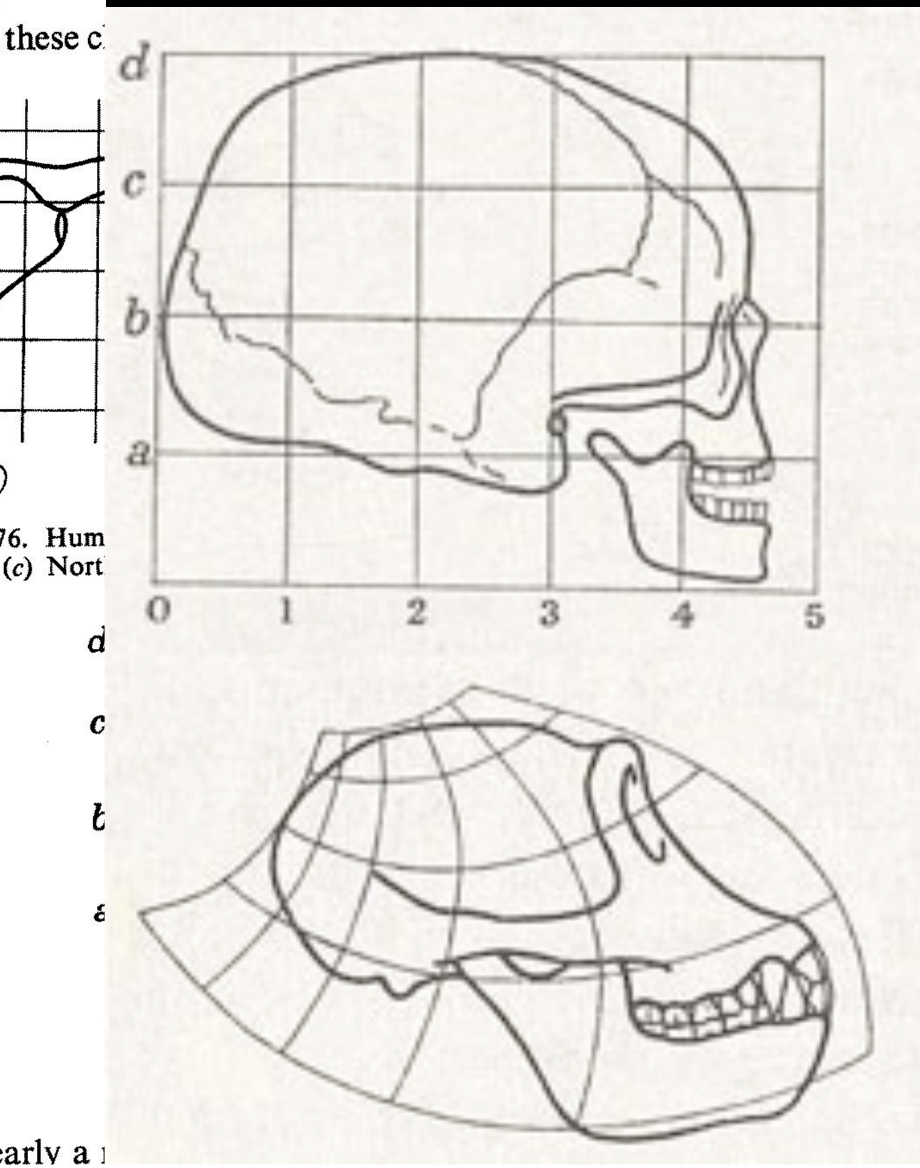
Red: autism
Gray: control

Extrinsic approach
Fourier Descriptors

Extrinsic approach: Fourier Discriptors

D’Arcy Thompson 1860-1948





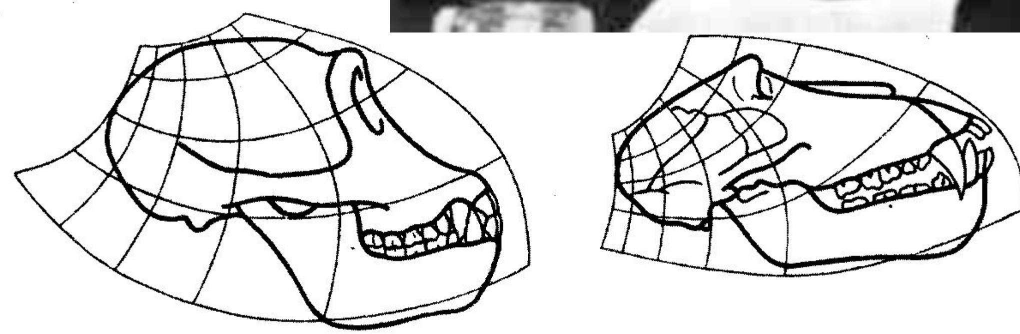
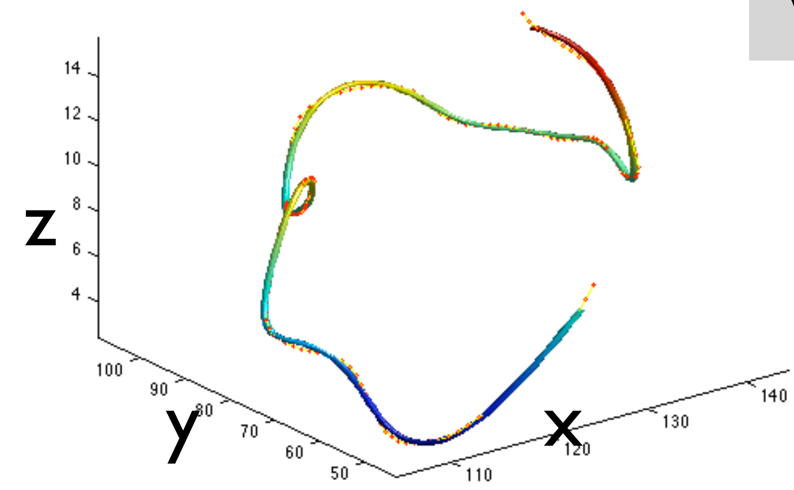


Fig. 179. Skull of chimpanzee.

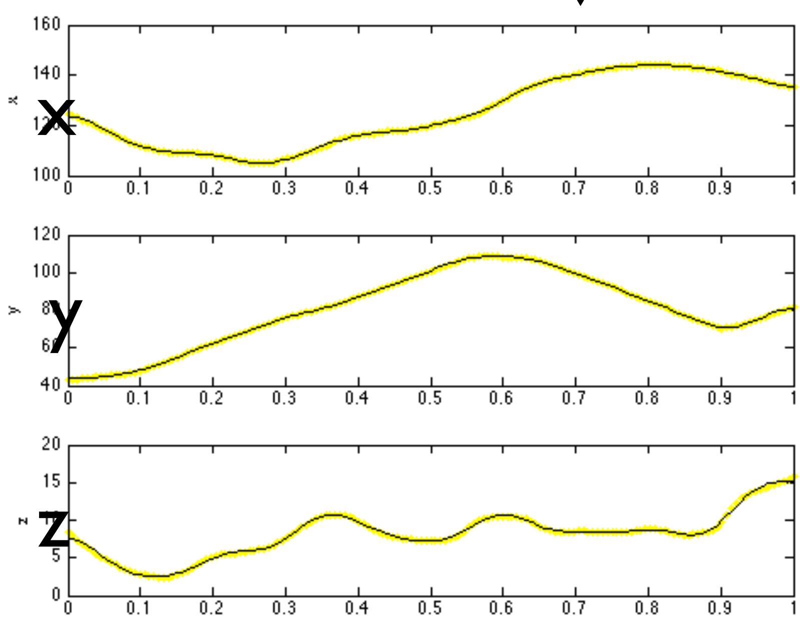
Fig. 180. Skull of baboon.

On Growth and Form
D’Arcy Thompson

White matter fiber tract model



parameterization



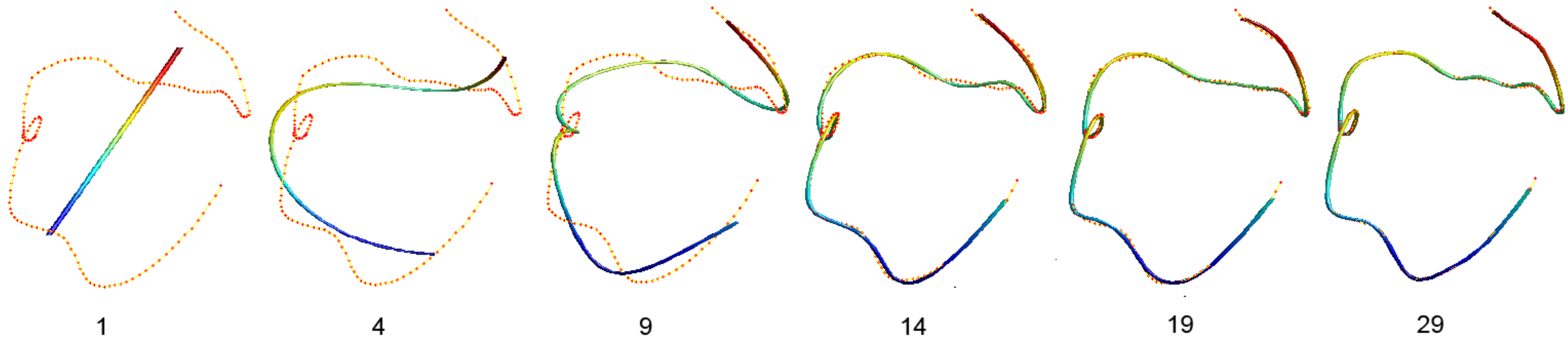
88.1799	56.6336	5.7367
-12.4775	-11.2552	-2.0791
2.4336	-15.4428	-0.4021
4.3956	2.2733	-0.9354
-0.0106	-0.0674	0.6999
2.1773	-2.4194	-0.1176
0.5808	0.8390	1.2942
0.0615	-0.1893	0.1188
-0.2629	0.7524	0.1089
0.7909	-0.7276	-0.1901
0.5458	0.6236	0.6939
0.4295	-0.4337	0.2185
0.2150	0.4157	0.0254
0.1584	-0.1973	0.0762
-0.1557	0.2466	-0.1086
0.0632	-0.0978	-0.0208
0.0389	-0.0143	-0.0284
-0.0014	-0.1193	0.1970
0.0004	0.0129	-0.0198
0.1342	0.0002	0.0260

Any tract can be compactly parameterized with only 60 coefficients.

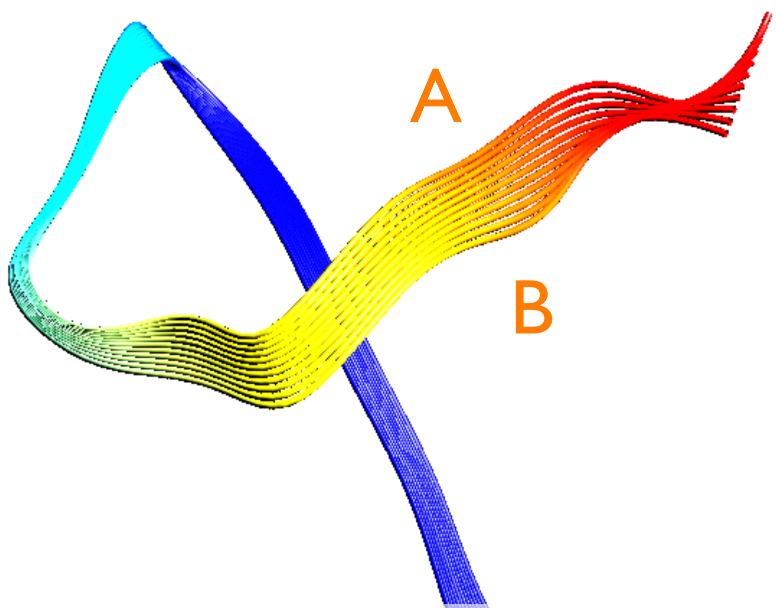
basis expansion

$$(x, y, z)' = \sum_{l=0}^{19} \beta_l \cos(l\pi t)$$

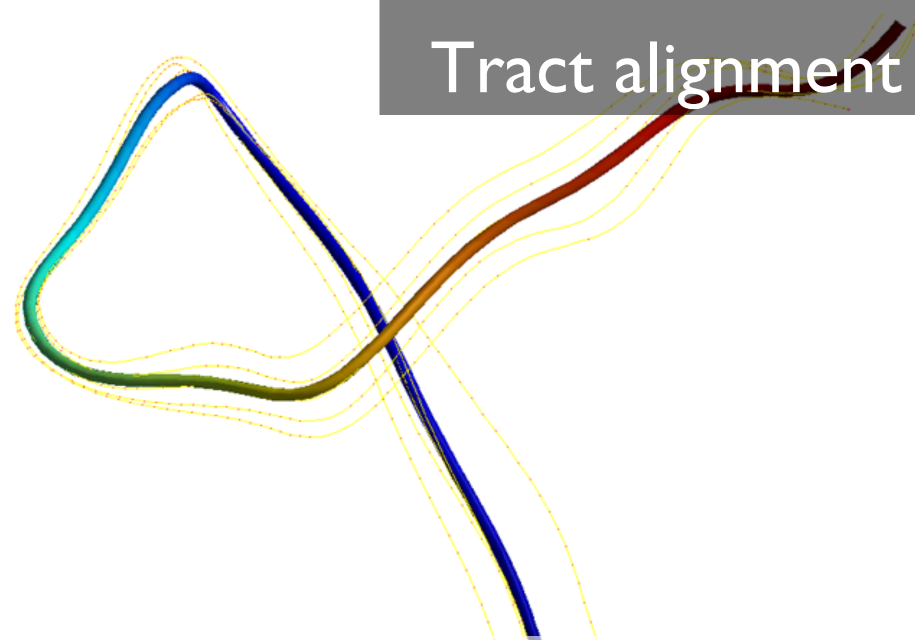
Cosine series representation



Tract alignment

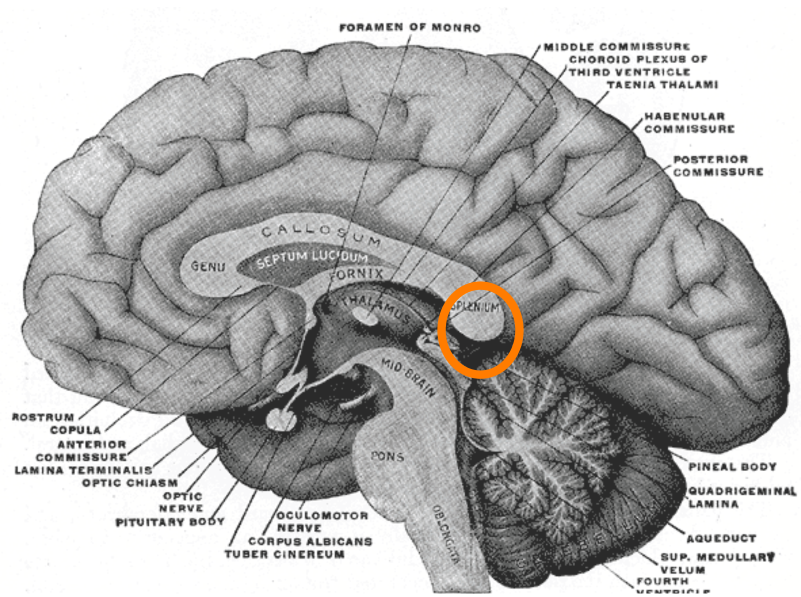


Tract A is registered to tract B

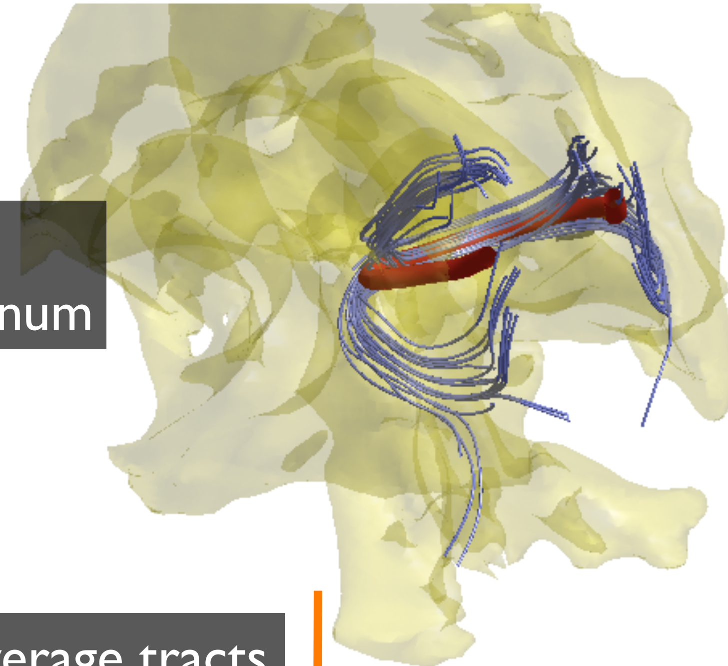


Average of 5 tracts

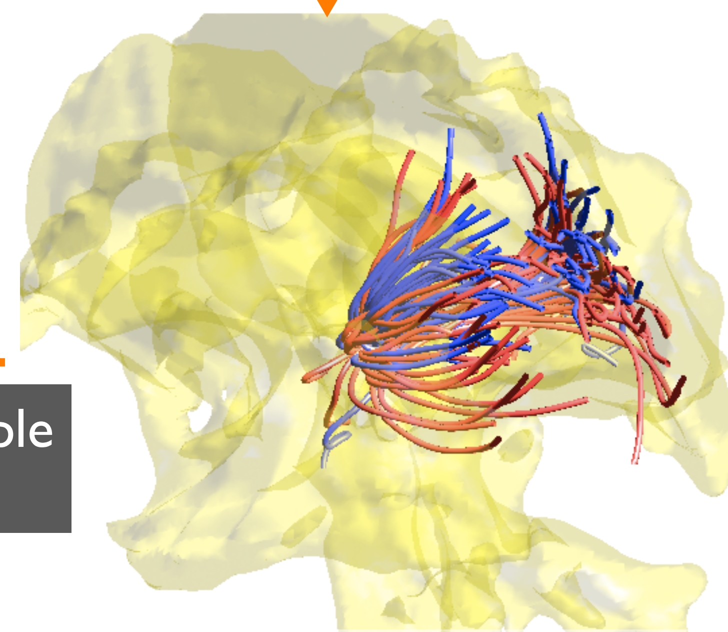
Fiber concentration analysis



tracts passing through spleninum

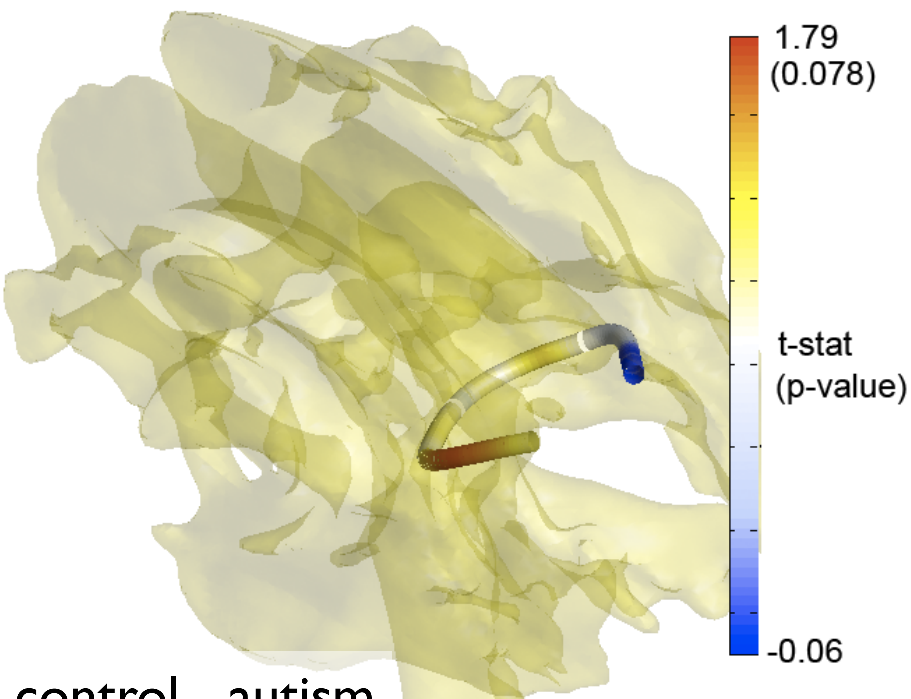


Average tracts



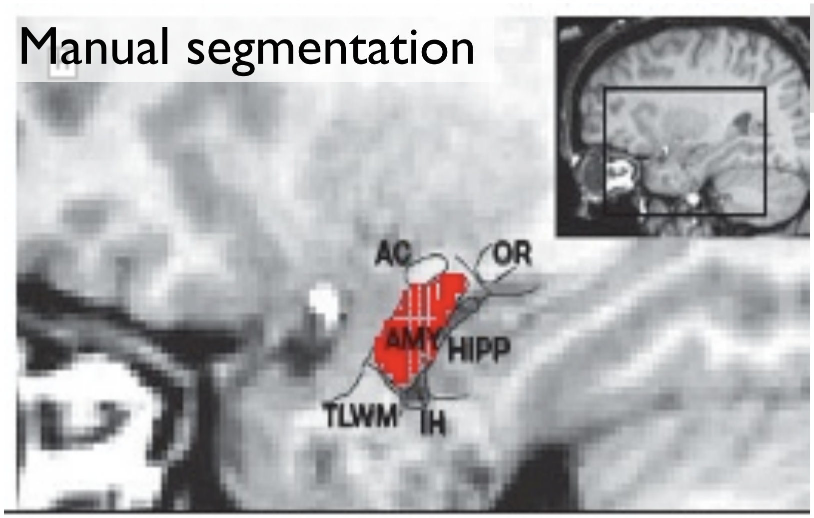
42 autistic & 32 control

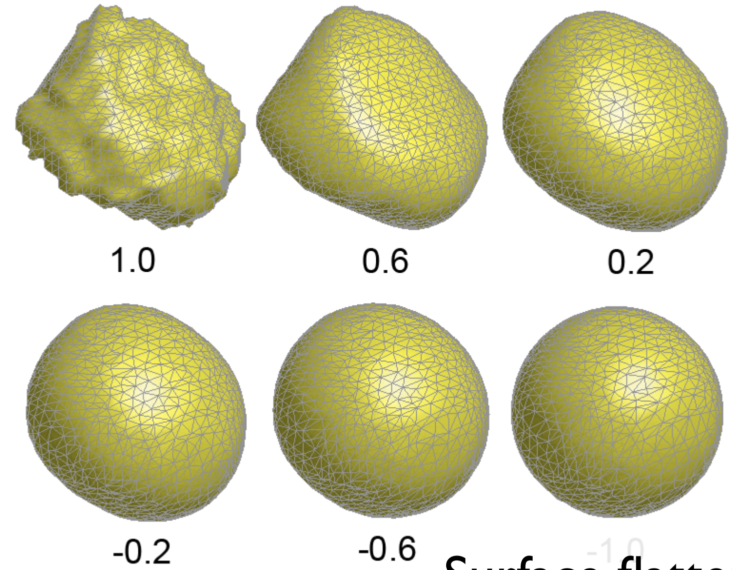
two sample t-test



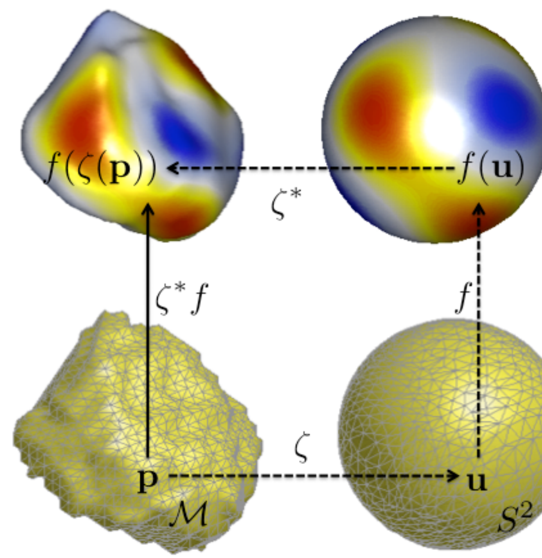
control - autism

Spherical coordinates on amygdala

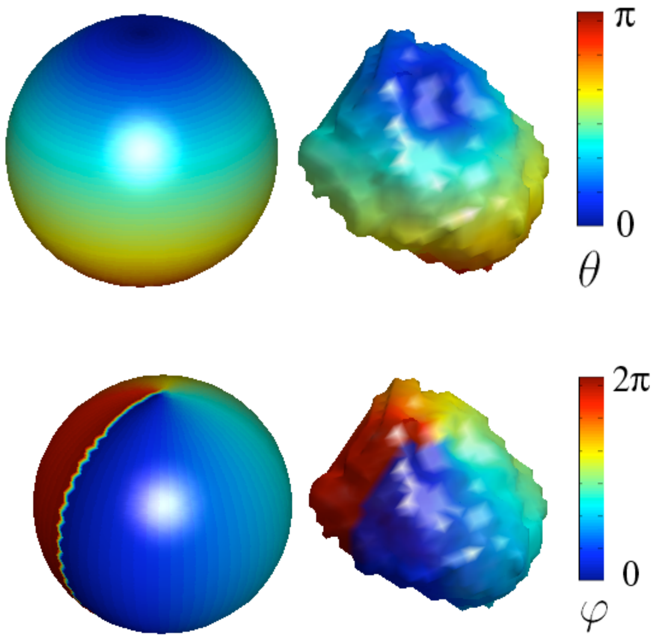




Surface flattening



spherical angles



Fourier Descriptors
Real hard example

566 IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 26, NO. 4, APRIL 2007

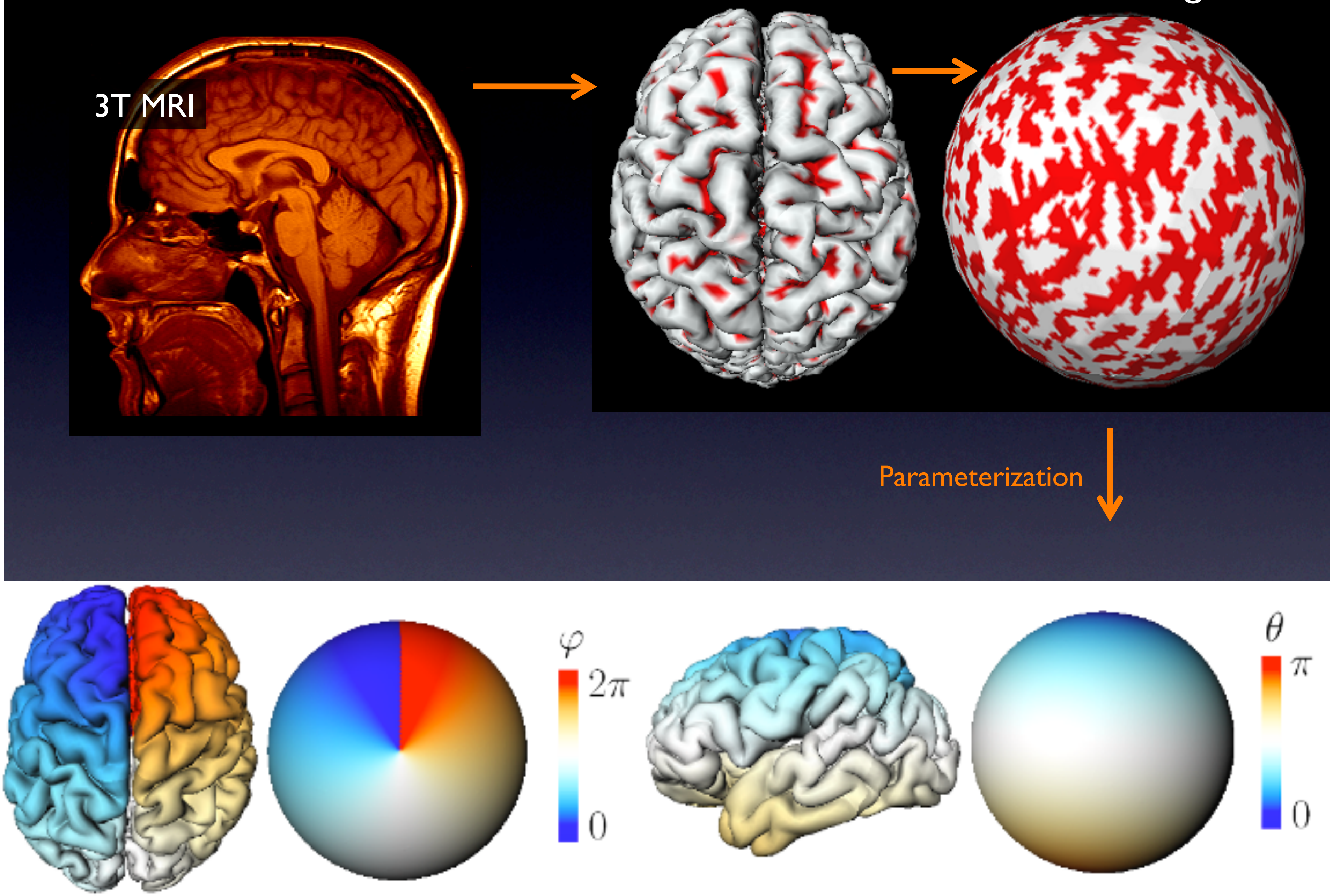
Weighted Fourier Series Representation and Its Application to Quantifying the Amount of Gray Matter

Moo K. Chung*, Kim M. Dalton, Li Shen, Alan C. Evans, and Richard J. Davidson

This problem was beyond the capability of
average PC (Pentium-3 with 1GB memory) in 2005.

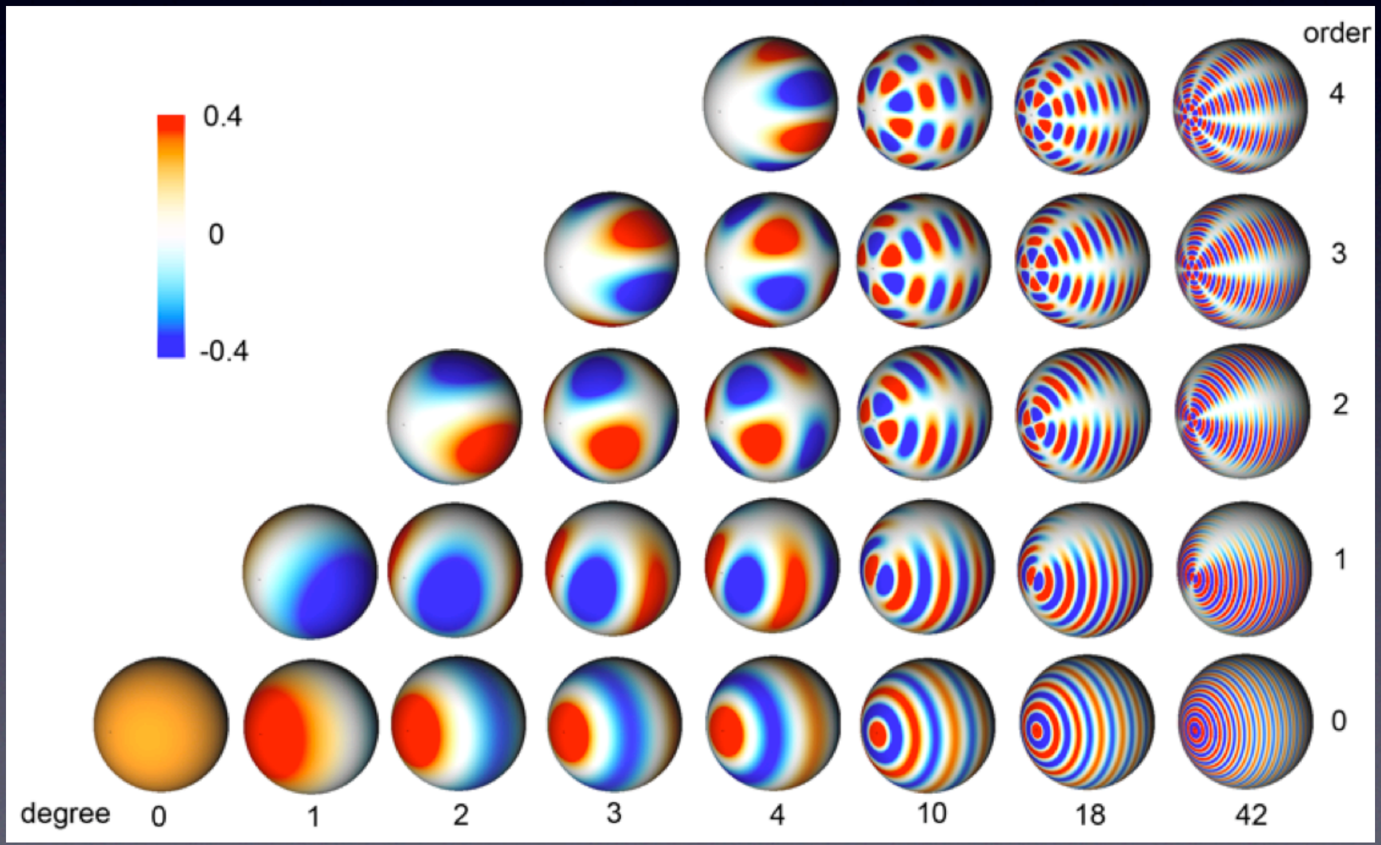
But can be solved with 9 year old laptop with 500MB memory.

Spherical Coordinates on Cortex



Spherical harmonic of degree *l* and order *m*

$$Y_{lm} = \begin{cases} c_{lm} P_l^{|m|}(\cos\theta)\sin(|m|\varphi), & -l \le m \le -1, \\ \frac{c_{lm}}{\sqrt{2}} P_l^0(\cos\theta), & m = 0, \\ c_{lm} P_l^{|m|}(\cos\theta)\cos(|m|\varphi), & 1 \le m \le l, \end{cases}$$



Coarse detail ⟷ Fine detail

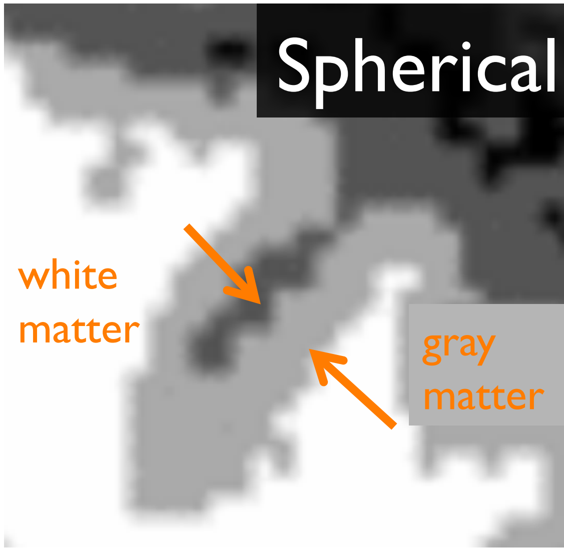
Spherical harmonic expansion of cortical thickness

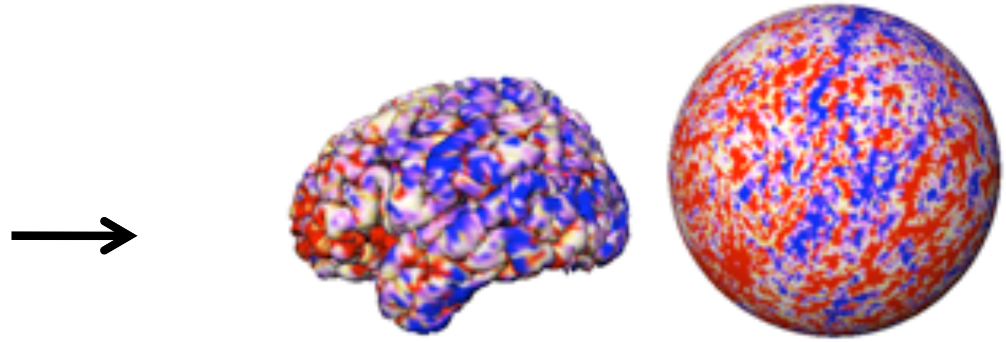
$$\sum_{l=0}^{k} \sum_{m=-l}^{l} f_{lm} Y_{lm}(\theta, \varphi) \to f$$

$$f_{lm} = \int_{S^2} f(\theta, \varphi) Y_{lm}(\theta, \varphi) \, d\theta d\varphi$$

Decomposition of signal on unit sphere

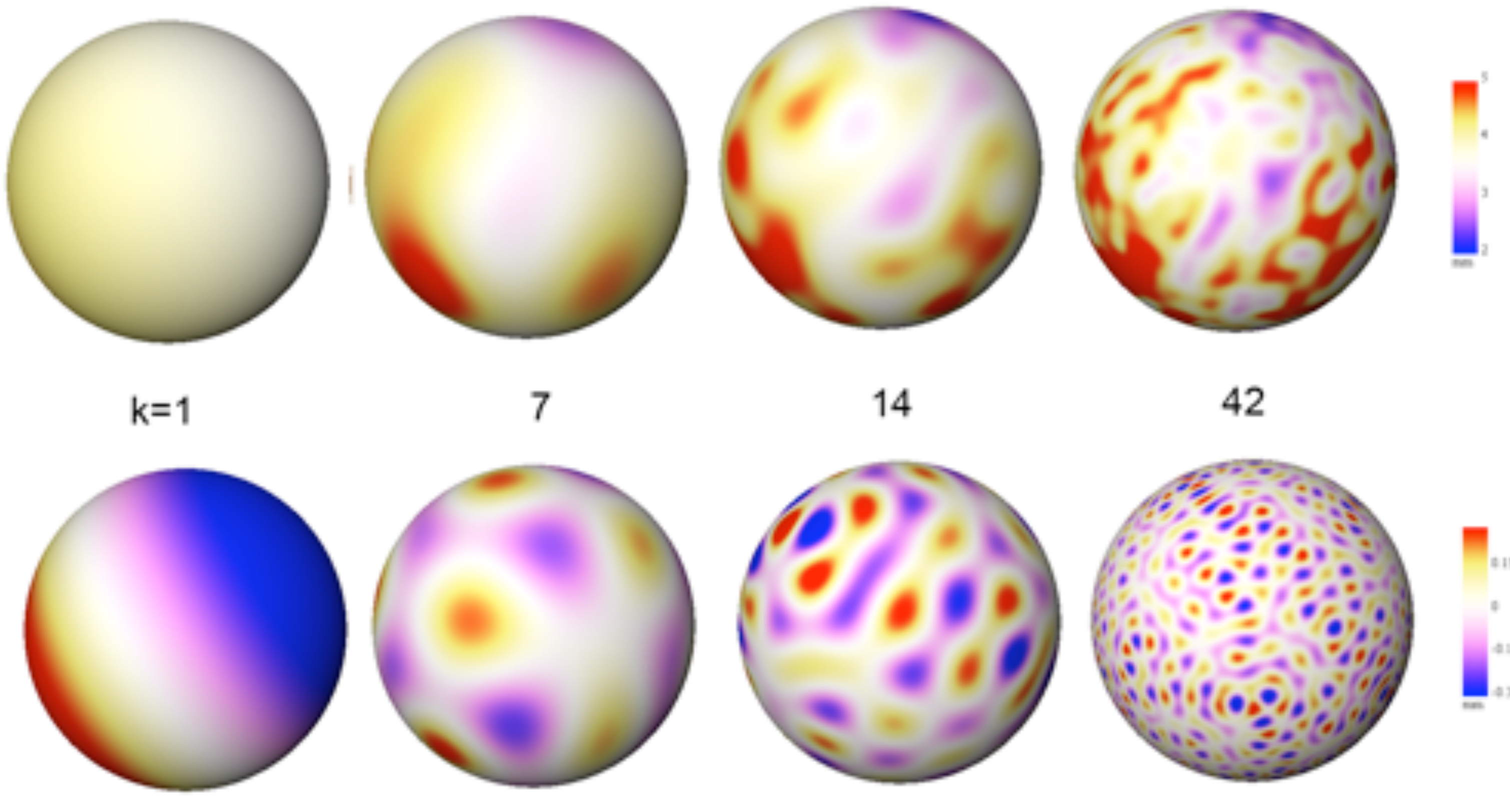
Spherical harmonic expansion of cortical thickness



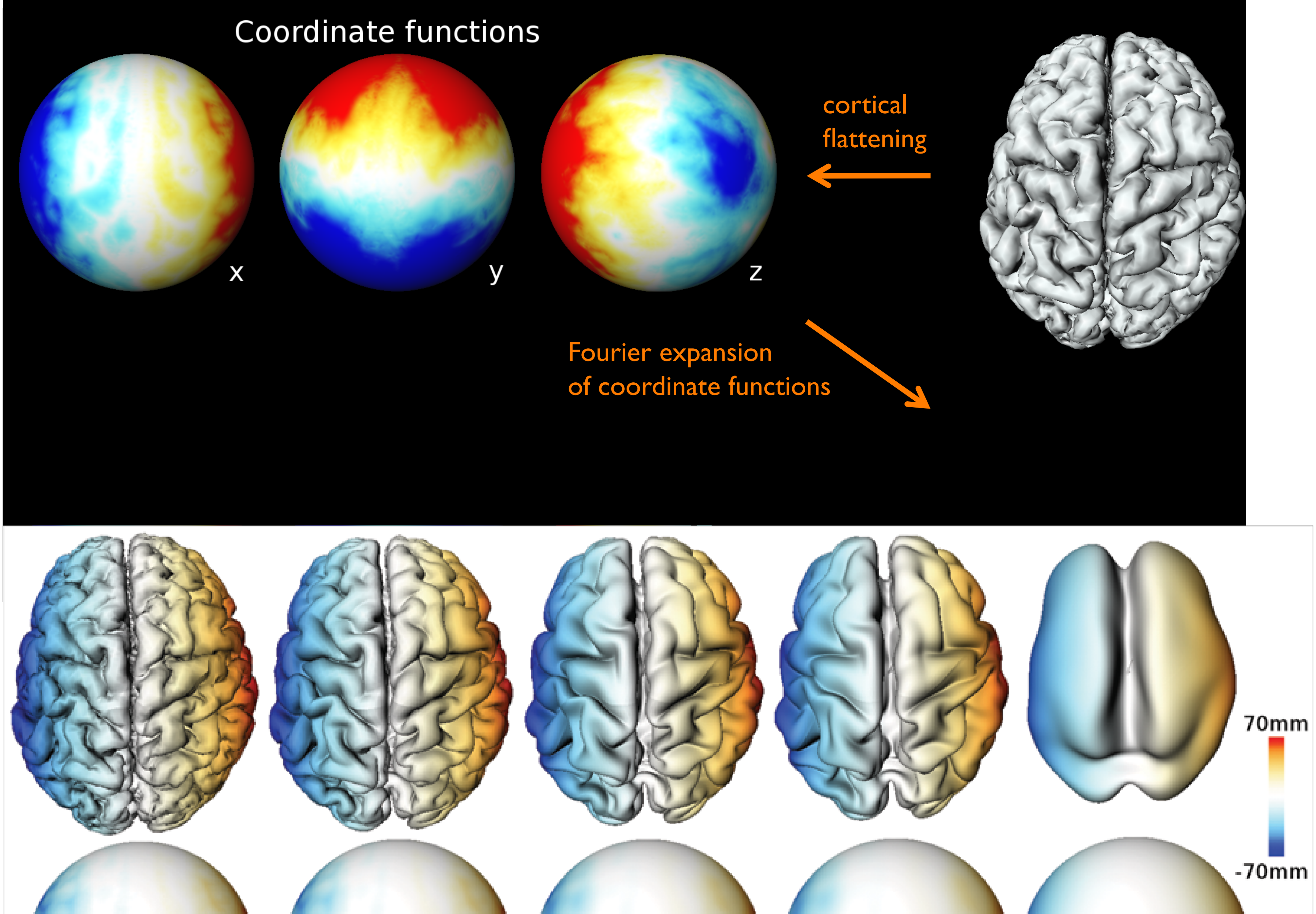


$\sum_{l=1}^{k}$ k-th degree Expansion

spherical harmonic basis



Fourier expansion of cortex



Computing Fourier coefficients

$$f_{lm} = \int_{S^2} f(\theta,\varphi)\, Y_{lm}(\theta,\varphi)\; d\theta\, d\varphi$$

Compute for all *l* and *m* and three coordinates = 20000 coefficients. MATLAB (LAPACK) breaks down after order 80. No imaging papers beyond order 40.

dd

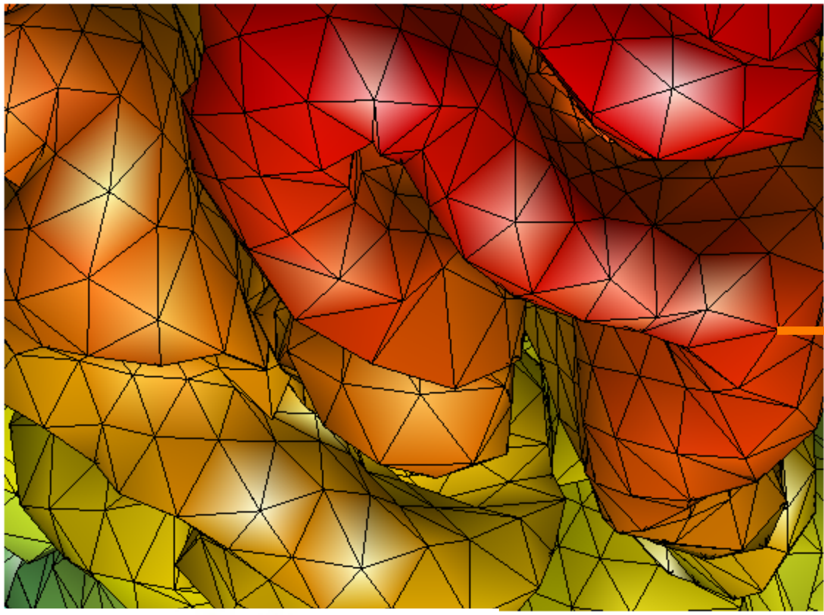
Available techniques:

Direct numerical & Monte-Carlo integration

Fast spherical harmonic transform

Least squares

Least squares estimation



at the i-th vertex p_i

$$f(p_i) = \sum_{l=0}^{k} \sum_{m=-l}^{l} \beta_{lm} Y_{lm}(p_i)$$

$$\mathbf{f} = \mathbf{Y}\beta$$

Matrix inversion up to size 1 million!

$$\beta = (\mathbf{Y}'\mathbf{Y})^{-1}\mathbf{Y}'\mathbf{f}$$

After two months of stupid struggle,
I was like this in 2006

Iterative residual fitting (IRF) algorithm

Step 1. measurements $f(p_1), \cdots, f(p_n)$

Step 2. Set initial degree=0 $k = 0$

Step 3. Solve $f(p_i) = \sum_{m=-k}^{k} \beta_{km} Y_{km}(p_i)$

Project data into a finite subspace

Iterate

Step 3.5. $f \leftarrow f - \hat{f}$

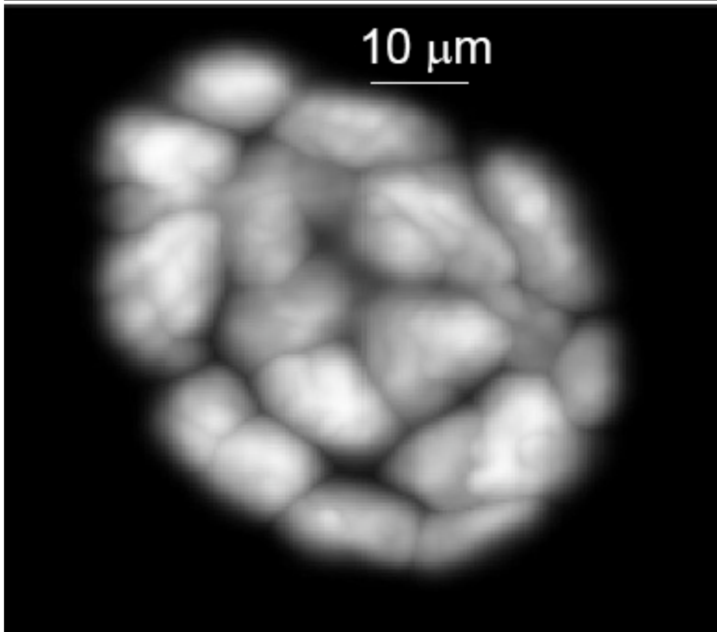
Once low frequency parts are estimated, we throw them away

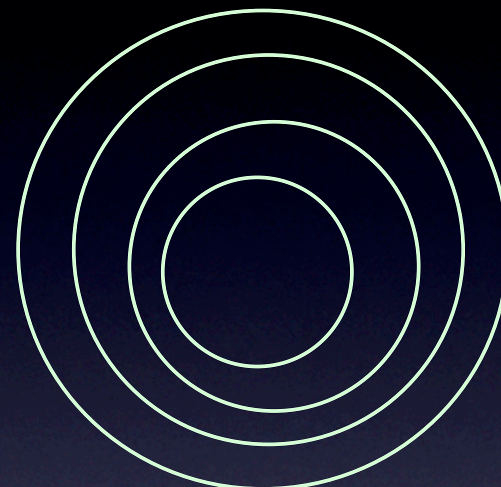
Step 4. Set degree $k \leftarrow k + 1$

MATLAB code available at http://www.stat.wisc.edu/~mchung/

Direct application of IRF

Reconstruction of 3D microscopic image data using spherical harmonics



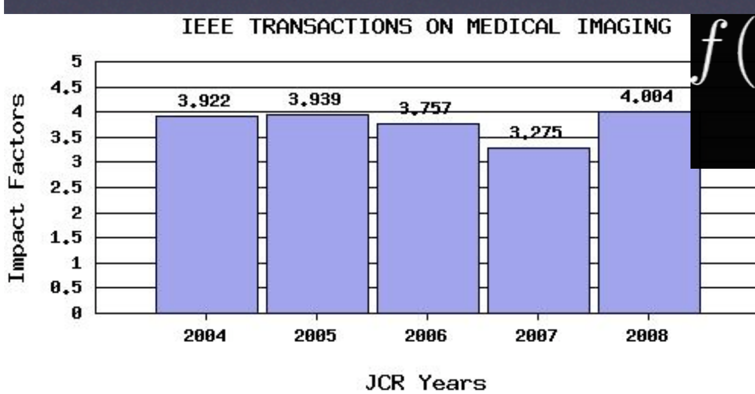


Humongous linear system involving spherical harmonics

$$f(\theta,\varphi,r)=\sum_i\sum_j\sum_k \beta_{ijk}Y_{ijk}(\theta,\varphi,r)$$

TMI 2007: 9 references

TMI 2008: 11 references

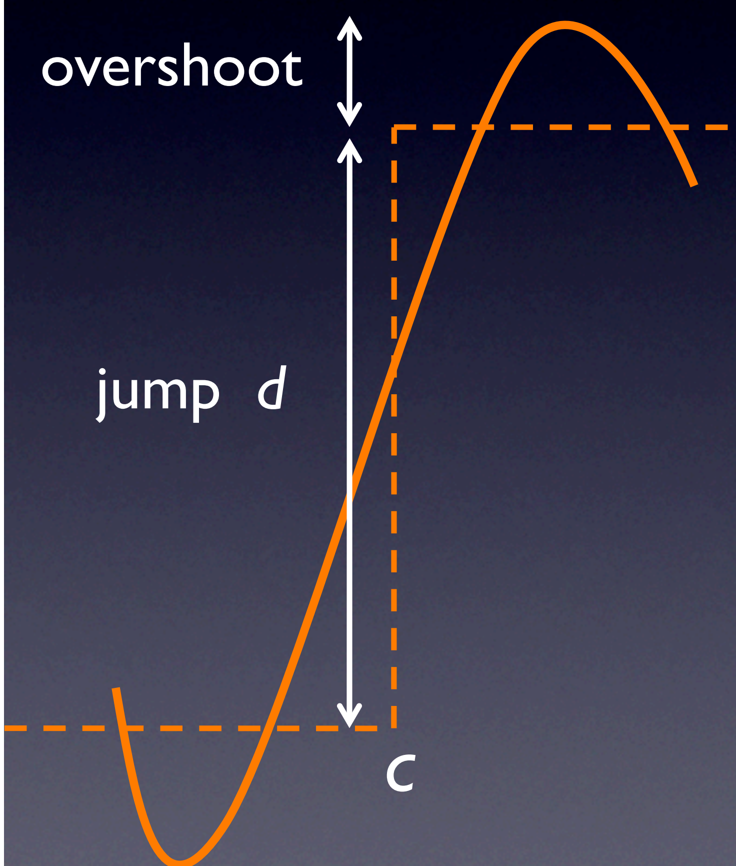


Khairy et al., 2008 MICCAI

Fourier approach is not perfect

Gibbs phenomenon

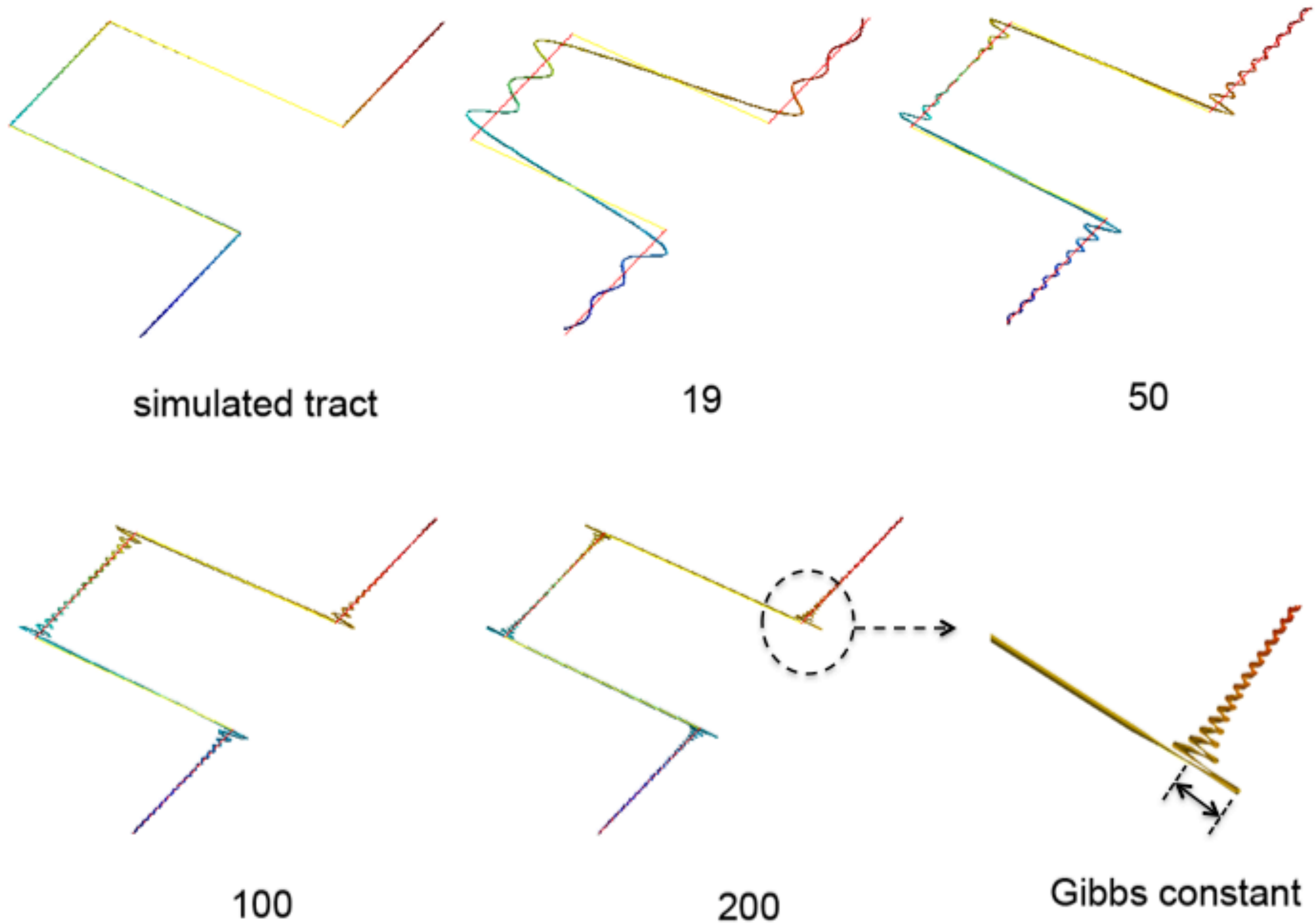
Spherical harmonic expansion is only good for smooth & continuous signal



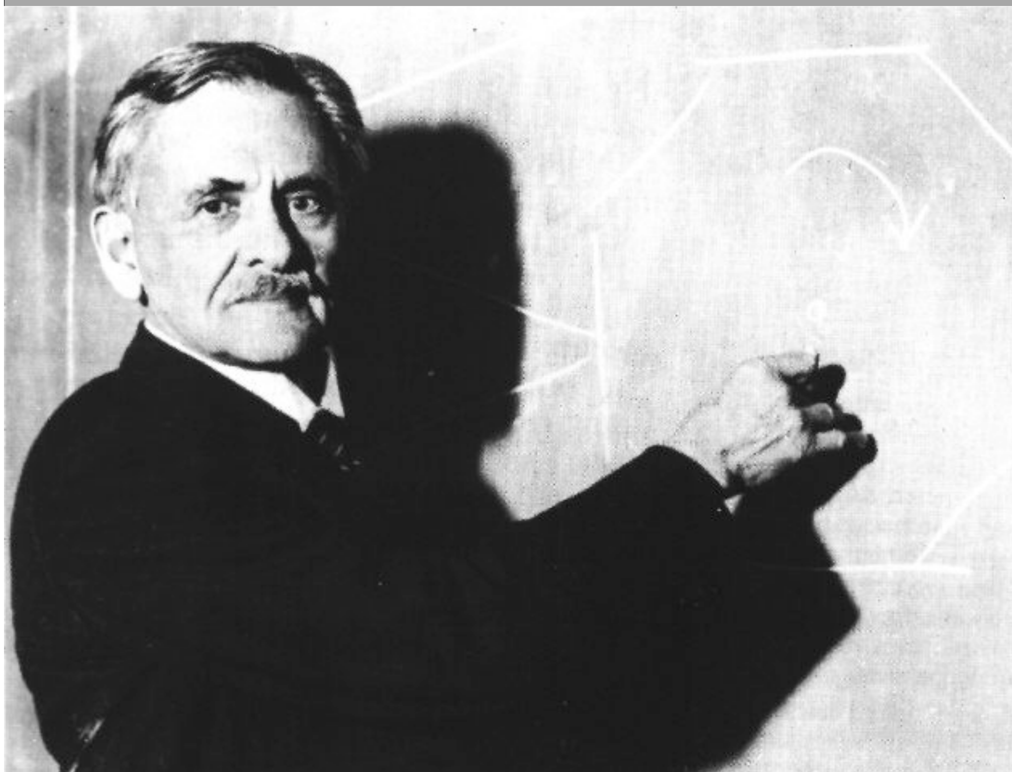
Gibbs Phenomenon

Mathematician Henry Willbraham published a paper on Gibbs phenomenon in 1848 but did not attract any attention.

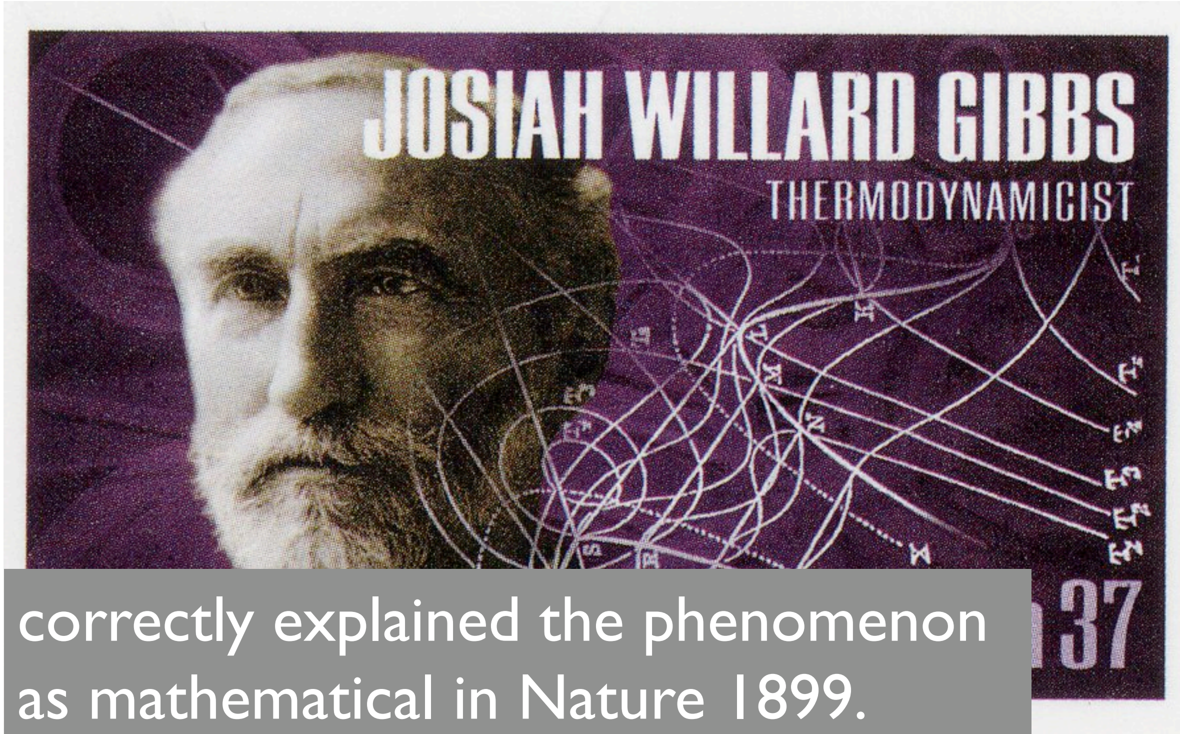
Gibbs phenomenon on a simulated white matter tract
Overshooting at jump discontinuity



Albert Abraham Michelson

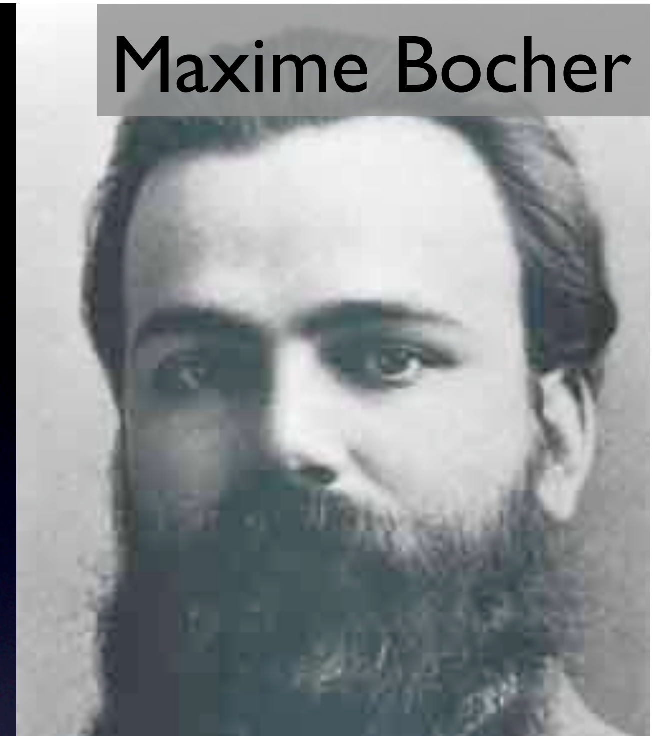


The Michelson-Stratton harmonic analyzer, one of the first mechanical analogue computers, recorded data from spectroscopic experiments. → Observed the phenomenon but assumed it to be mechanical error

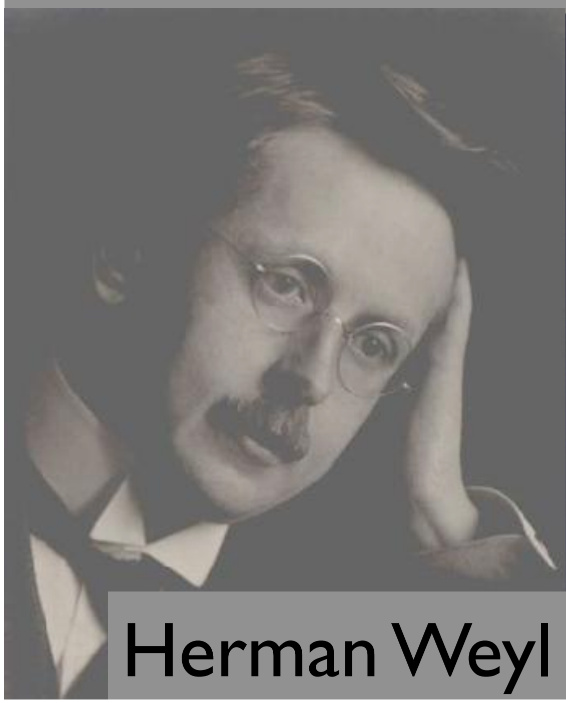


correctly explained the phenomenon as mathematical in Nature 1899.

Maxime Bocher



Gave detailed mathematical analysis and named it Gibbs phenomenon in 1906.

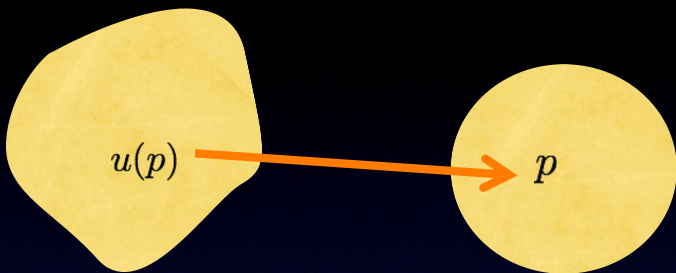


Herman Weyl

Investigated the Gibbs phenomenon associated with spherical harmonics in 1968.

How do we reduce the Gibbs phenomenon?

Weighted Fourier Analysis



manifold $\mathcal{M}$ parameter space $\mathcal{N}$

tracts, amygdala, hippocampus, cortical surface

$$\mathcal{L}\psi_j = \lambda_j \psi_j$$

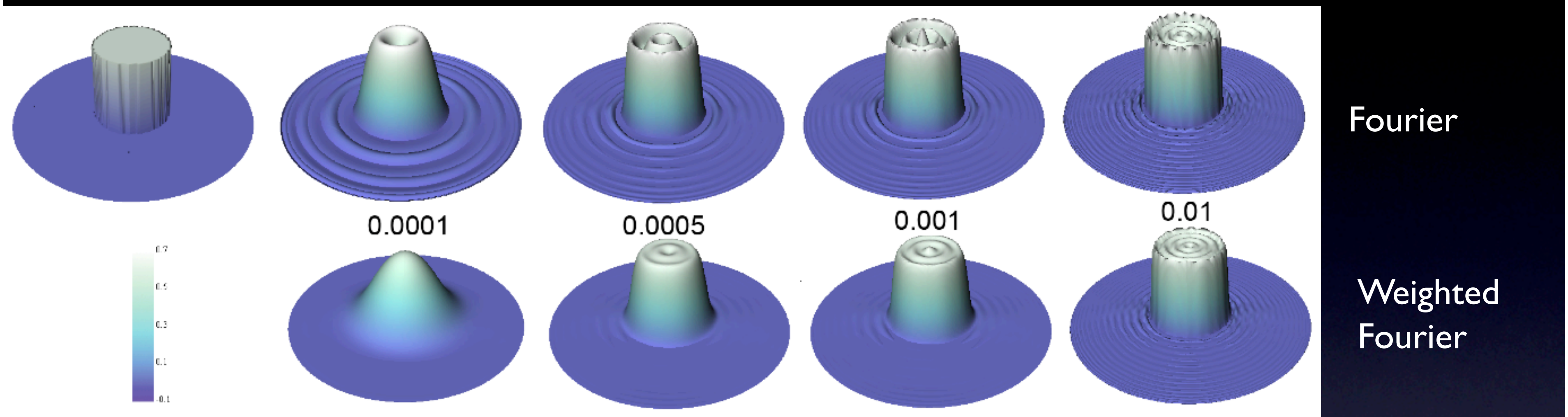
Input signal

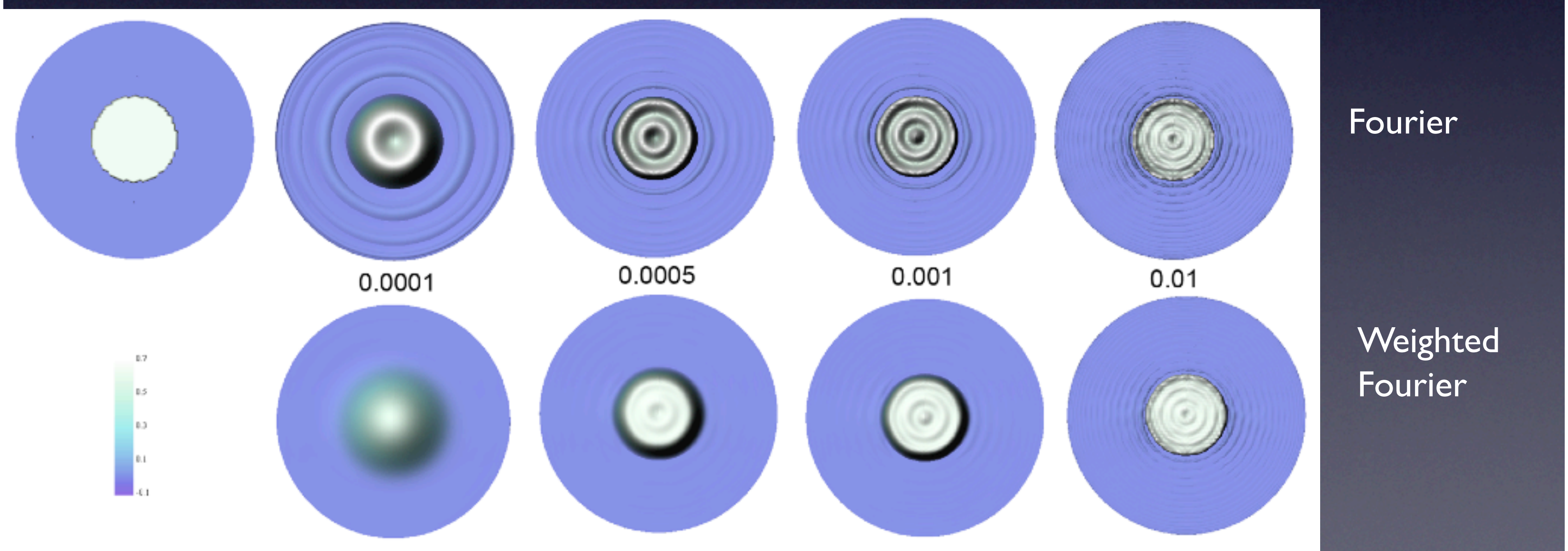
PDE: $\partial_t g + \mathcal{L}g = 0, g(p, t=0) = f(p)$

Weighted Fourier series
$$g(p,t) = \sum_{j=0}^{\infty} e^{-\lambda_j t} \langle f, \psi_j \rangle \psi_j(p)$$

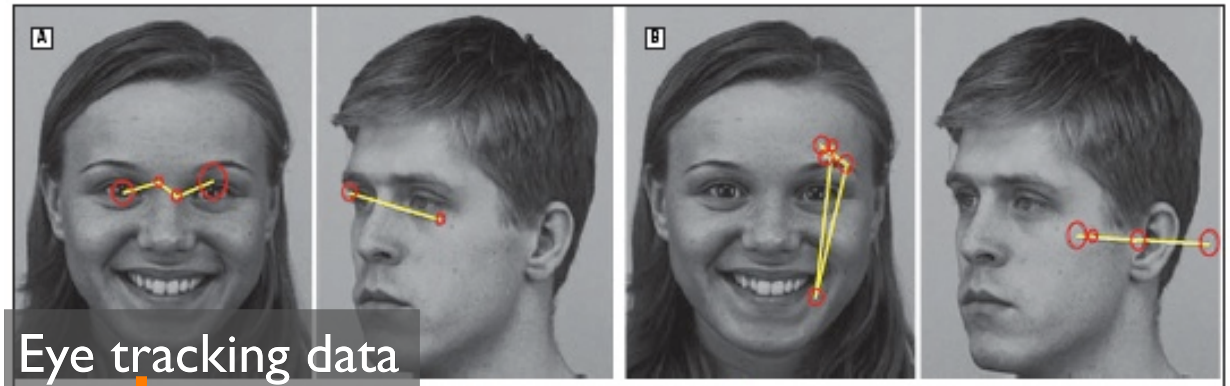
Kernel smoothing
$$= \int_{\mathcal{N}} K_t(p,q) f(q)\, d\mu(q)$$

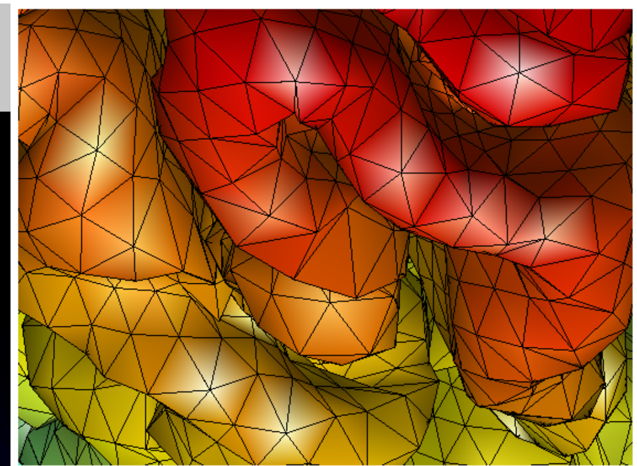
Gibbs phenomenon on hat shaped surface





Brain & behavior correlation





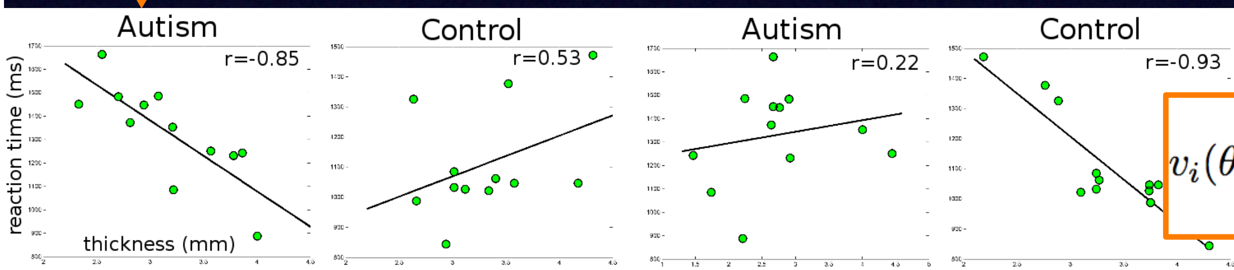
Partial correlation of thickness & gaze duration

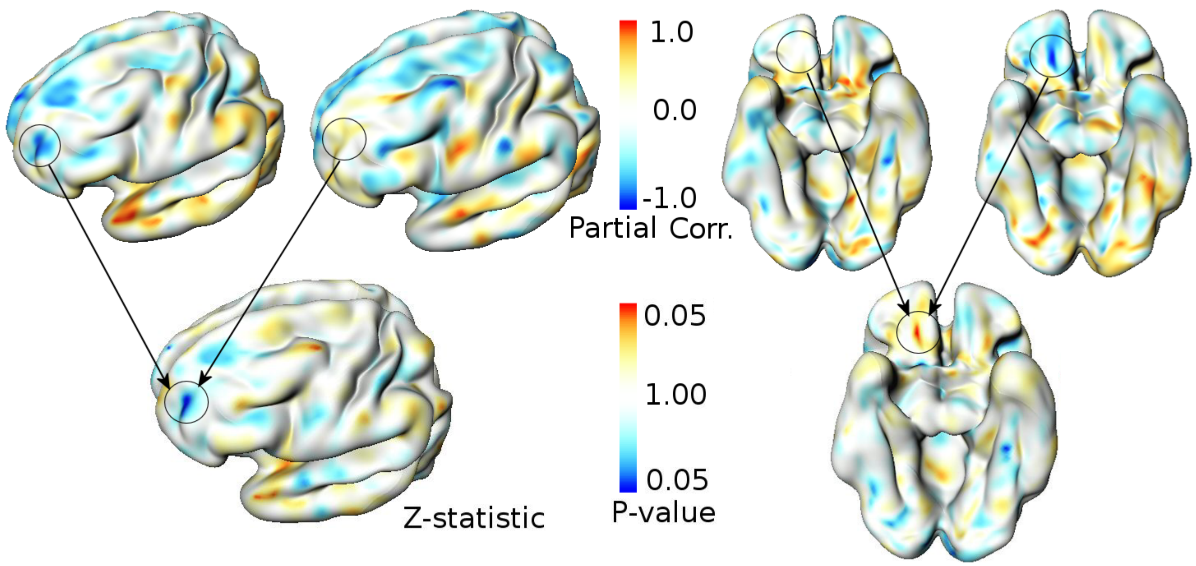
Weighted Fourier representation

$$v_i(\theta,\varphi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} e^{-l(l+1)\sigma} f_{lm}^{i} Y_{lm}(\theta,\varphi)$$

88.1799	56.6336	5.7367
-12.4775	-11.2552	-2.0791
2.4336	-15.4428	-0.4021
4.3956	2.2733	-0.9354
-0.0106	-0.0674	0.6999
2.1773	-2.4194	-0.1176
0.5808	0.8390	1.2942
0.0615	-0.1893	0.1188
-0.2629	0.7524	0.1089
0.7909	-0.7276	-0.1901
0.5458	0.6236	0.6939

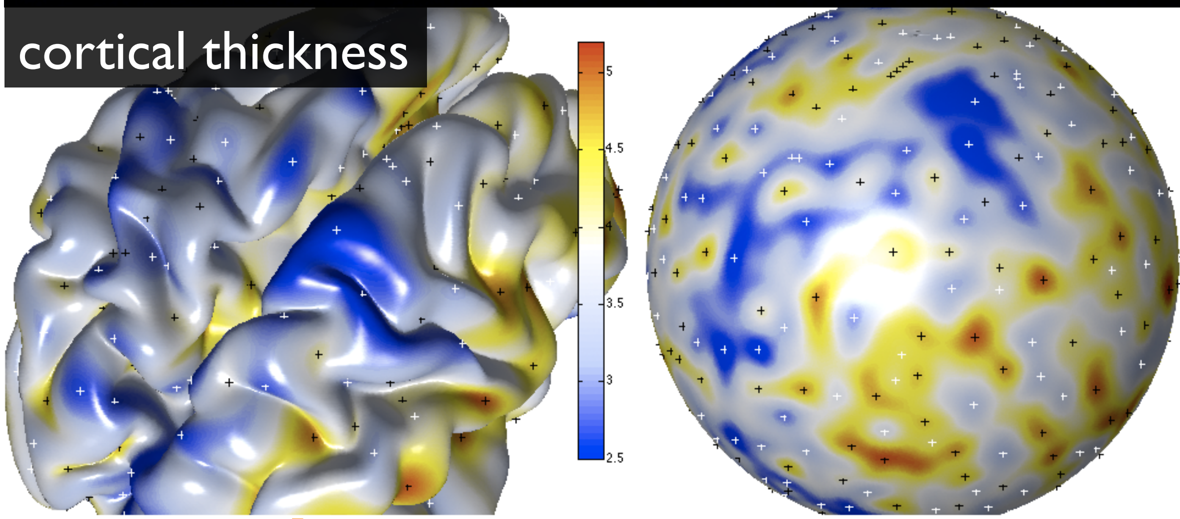
....

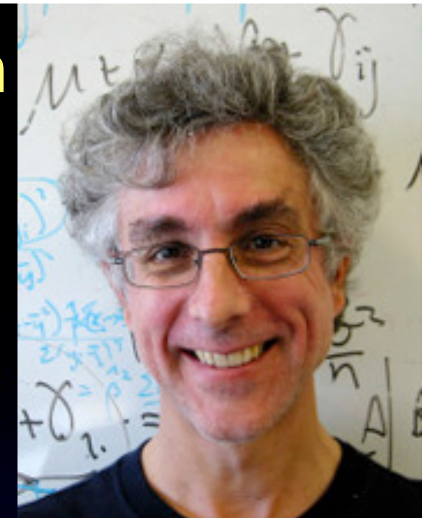




Persistence homology based signal detection

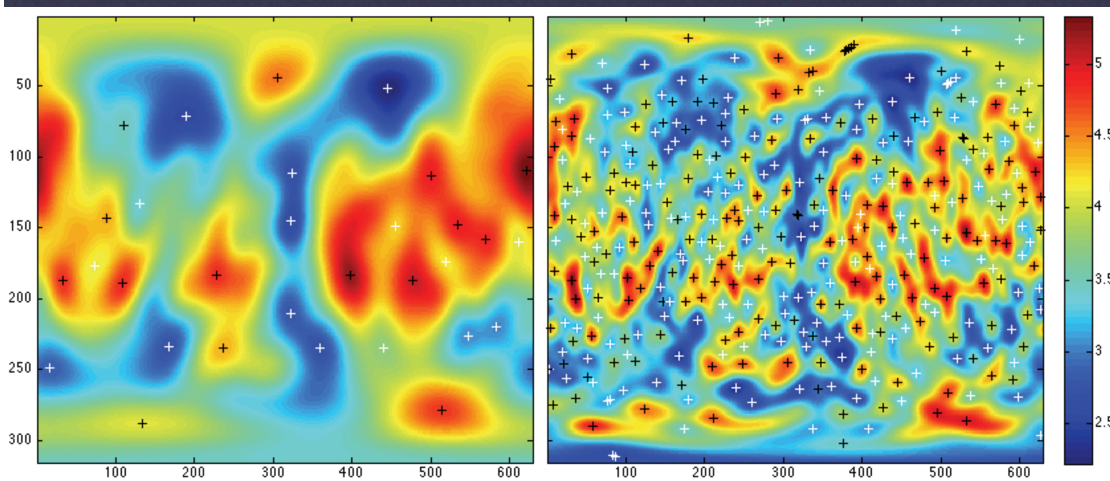
Euler characteristic, Betti numbers, Morse functions,
Worsley's random field theory.

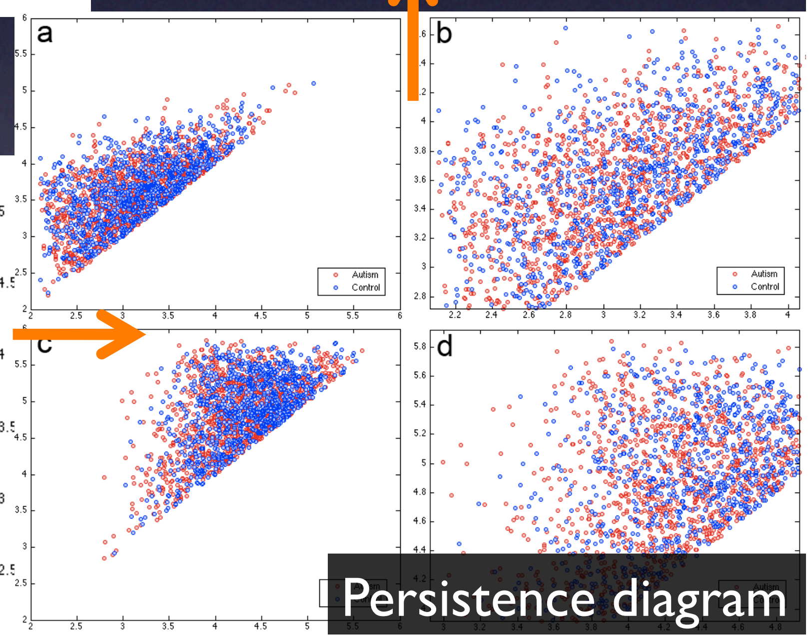




Topological classification 96%
Previous method 90%

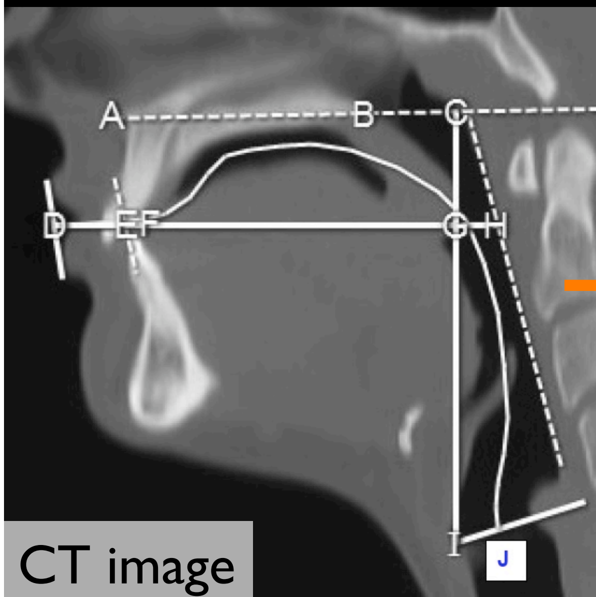
cortical flattening



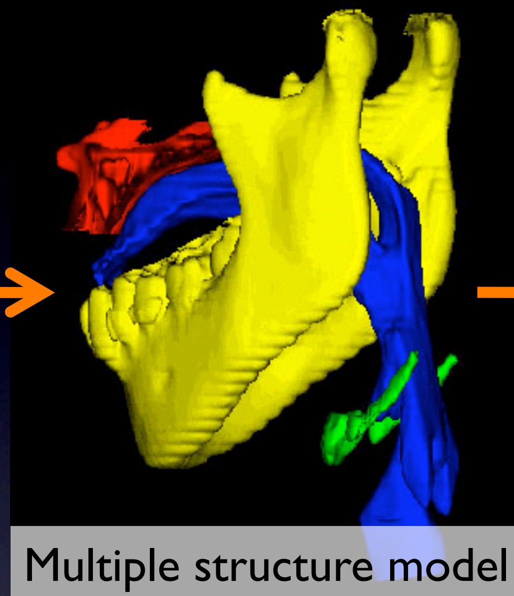


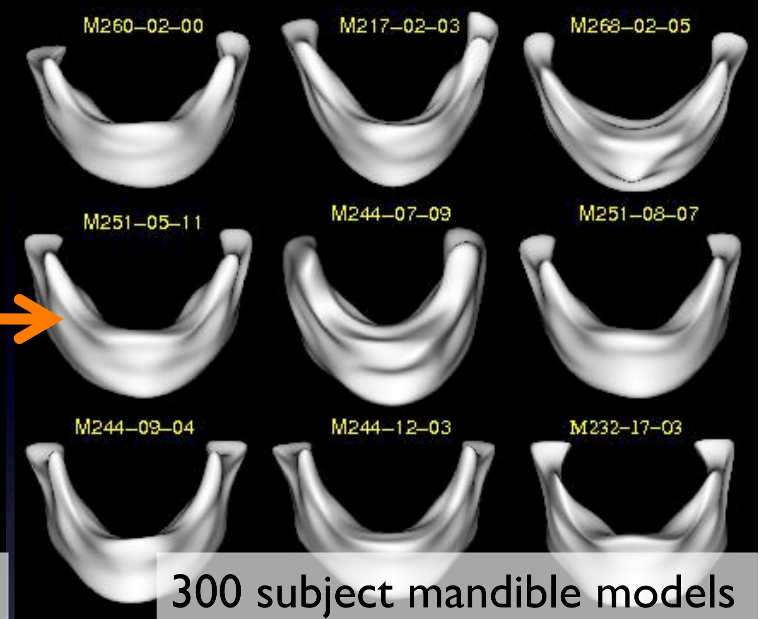
Persistence diagram

Longitudinal growth modeling on mandible CT images



CT image

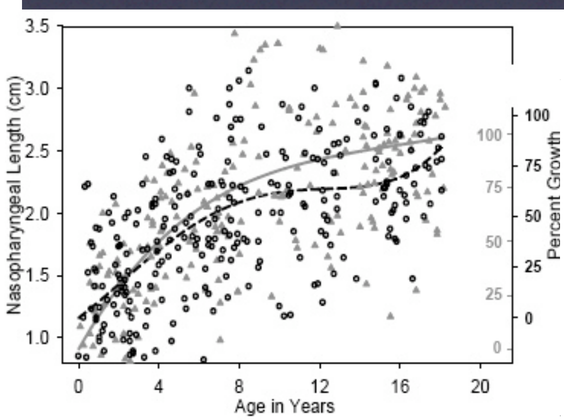

Multiple structure model

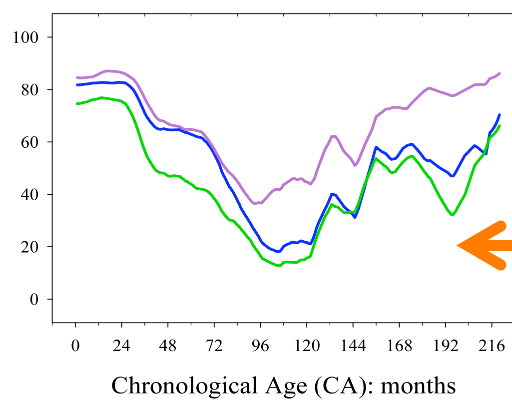


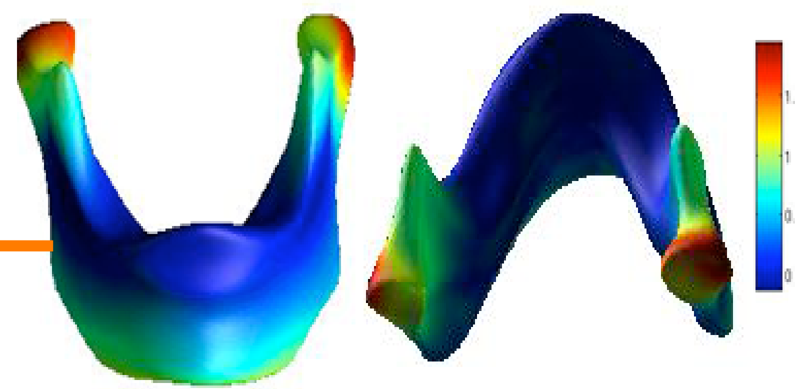
300 subject mandible models

Longitudinal modeling

Growth rate modeling with weighted Fourier







WCU-Project till the end of 2011

with Drs. Dong Soo Lee and Jae-Sung Lee (SNU), and
Drs. Andrew Alexander and Richard Davidson (Madison)
plus many *hardworking* postdocs in both places

Brain connectivity analysis
Wiring brain

↓ teaching

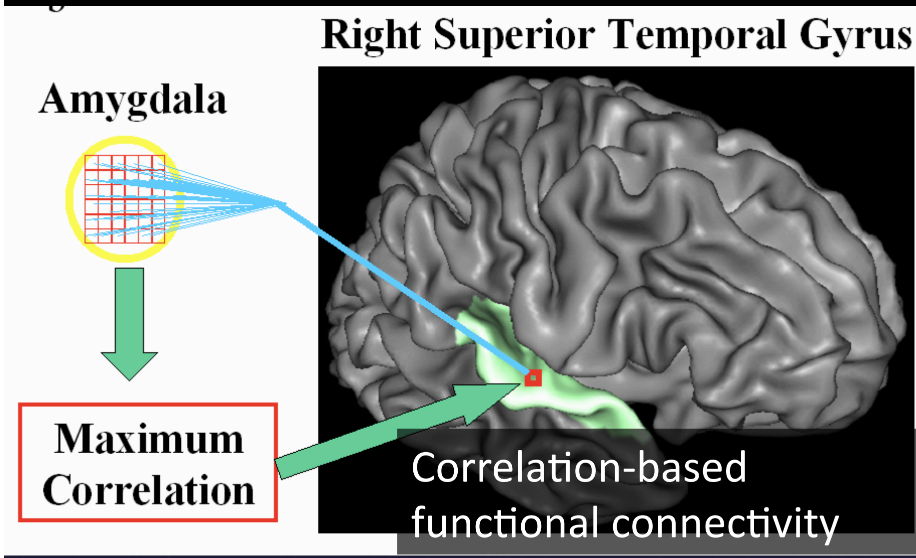
Neuroimage processing (2009 fall)
Computational Methods in neuroImage Analysis (2010 fall)
Statistical methods in neuroimage analysis (2011 fall)

↓ book publication

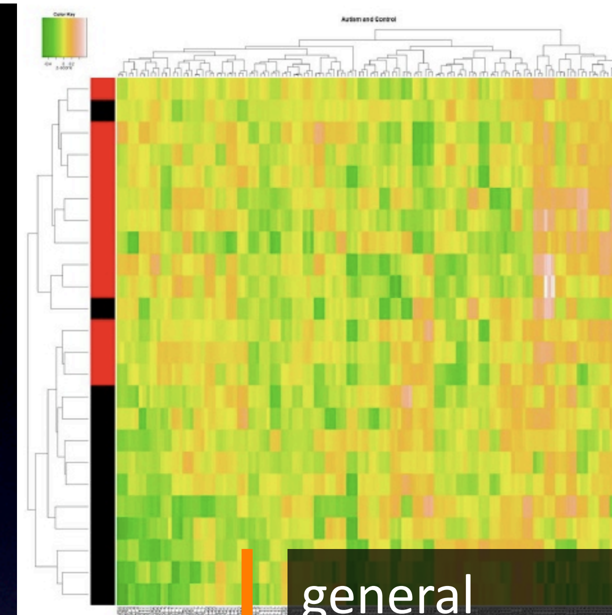
Computational Neuroanatomy (2011)
Statistical and Computational Methods in Brain Image Analysis (2012)

functional (fMRI) connectivity: effective style

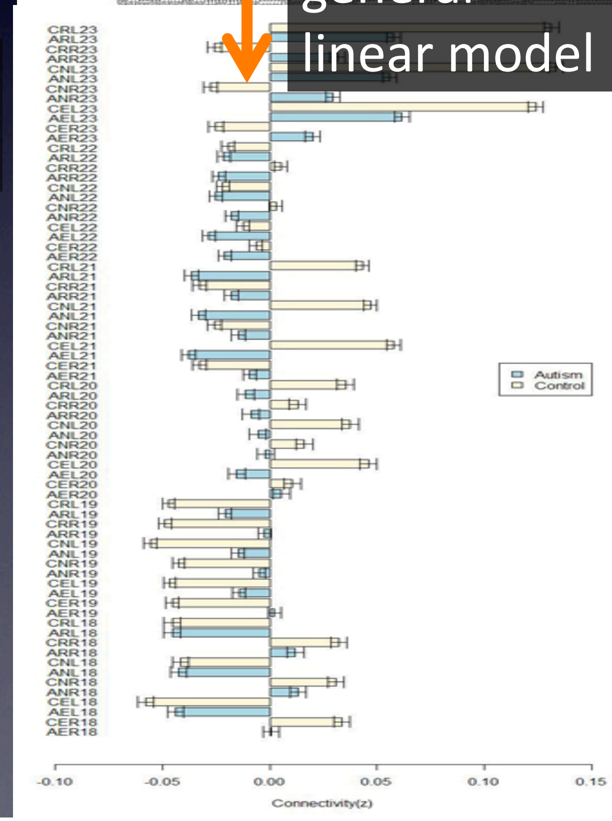
Daniel J. Kelley PhD thesis



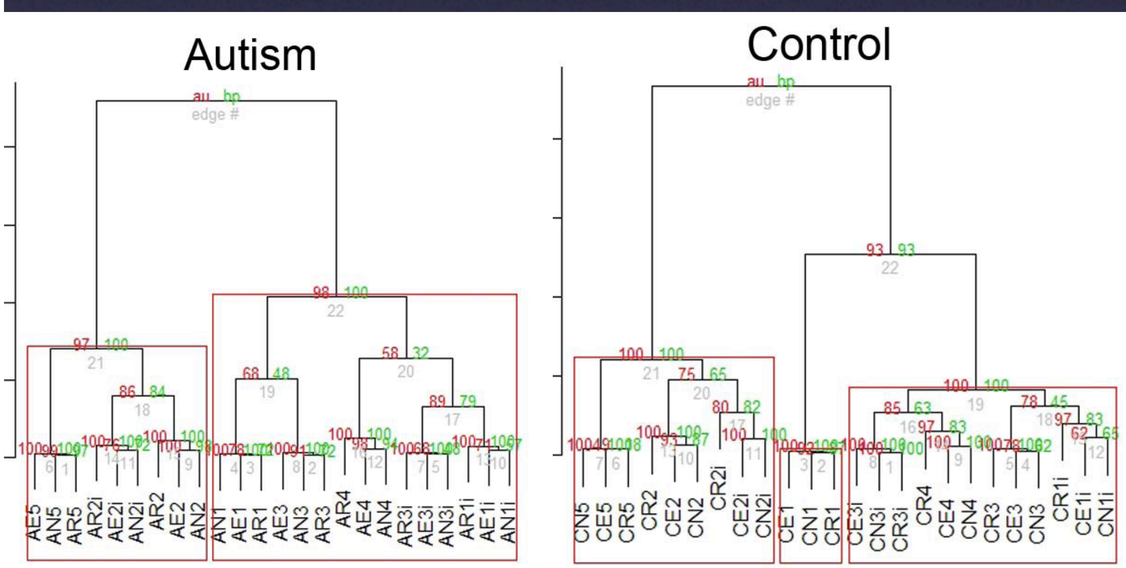
distance on correlation



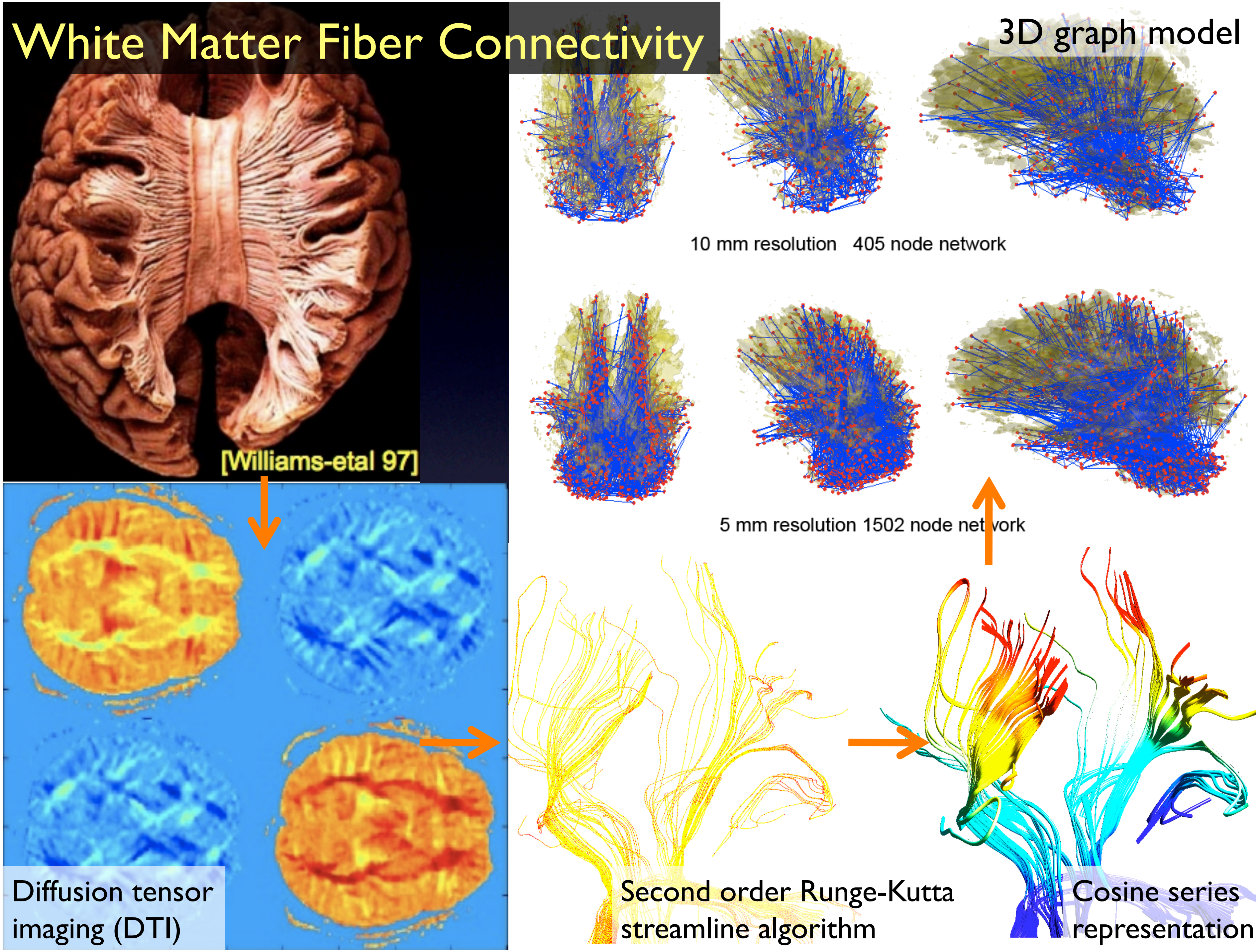
general linear model

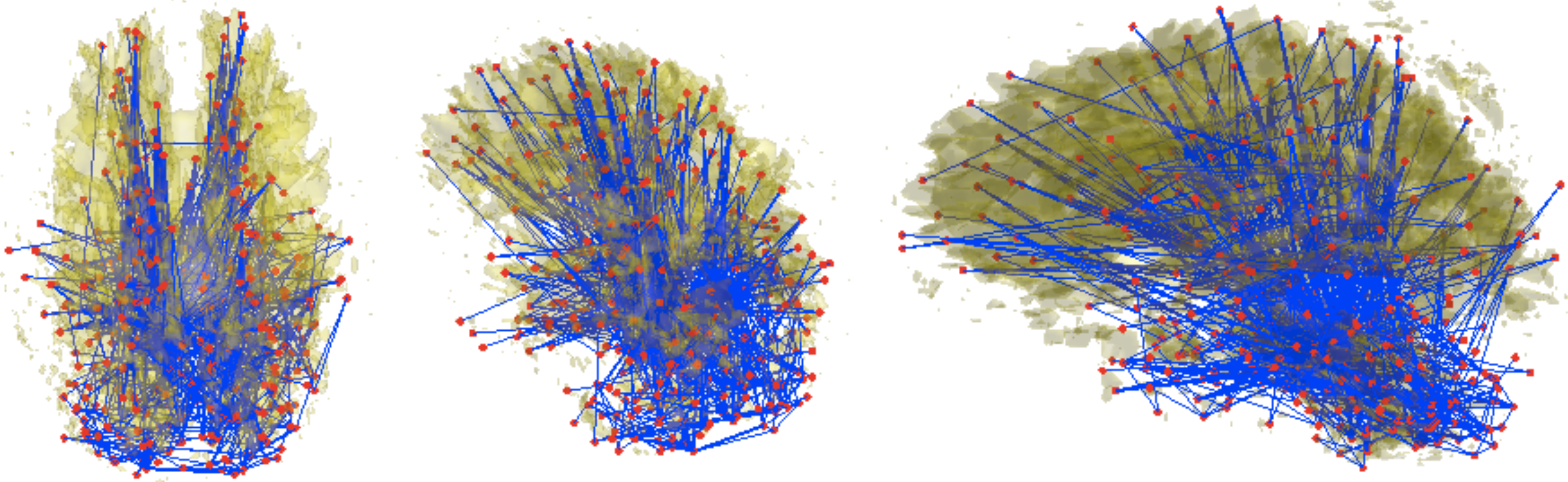


hieratical clustering

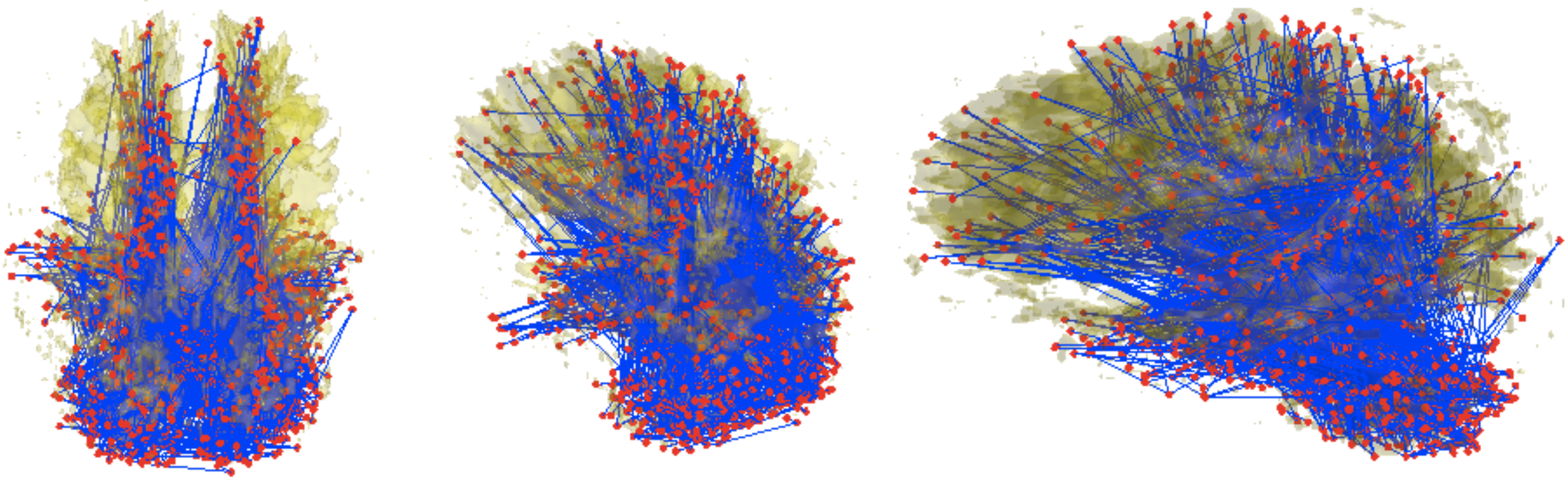


White Matter Fiber Connectivity





10 mm resolution   405 node network



5 mm resolution 1502 node network

MATLAB demonstration

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Daniel Kelley, Brendon Nacewicz

University of Wisconsin-Madison

Thank you.
send email for whatever questions,
collaboration request to
mkchung@wisc.edu