

# Statistical Power Maps for Sparse Representation of Subcortical Brain Structures

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## Motivation

The Laplace-Beltrami (LB) eigenfunctions are used in representing the surface displacement as a Fourier series expansion on amygdala and hippocampus surfaces. However, some Fourier coefficients may not necessarily contribute significantly in the representation. We propose to filter out these insignificant terms via a sparse regression.

## MRI Data

We have T1-weighted MRI of 69 normal adults (age range 38-79; 23 men and 46 women). Amygdalae and hippocampi were manually segmented and nonlinear image registration was performed using ANTS. The length of displacement of warping the template to an individual structure is used as a main anatomical feature of interest.

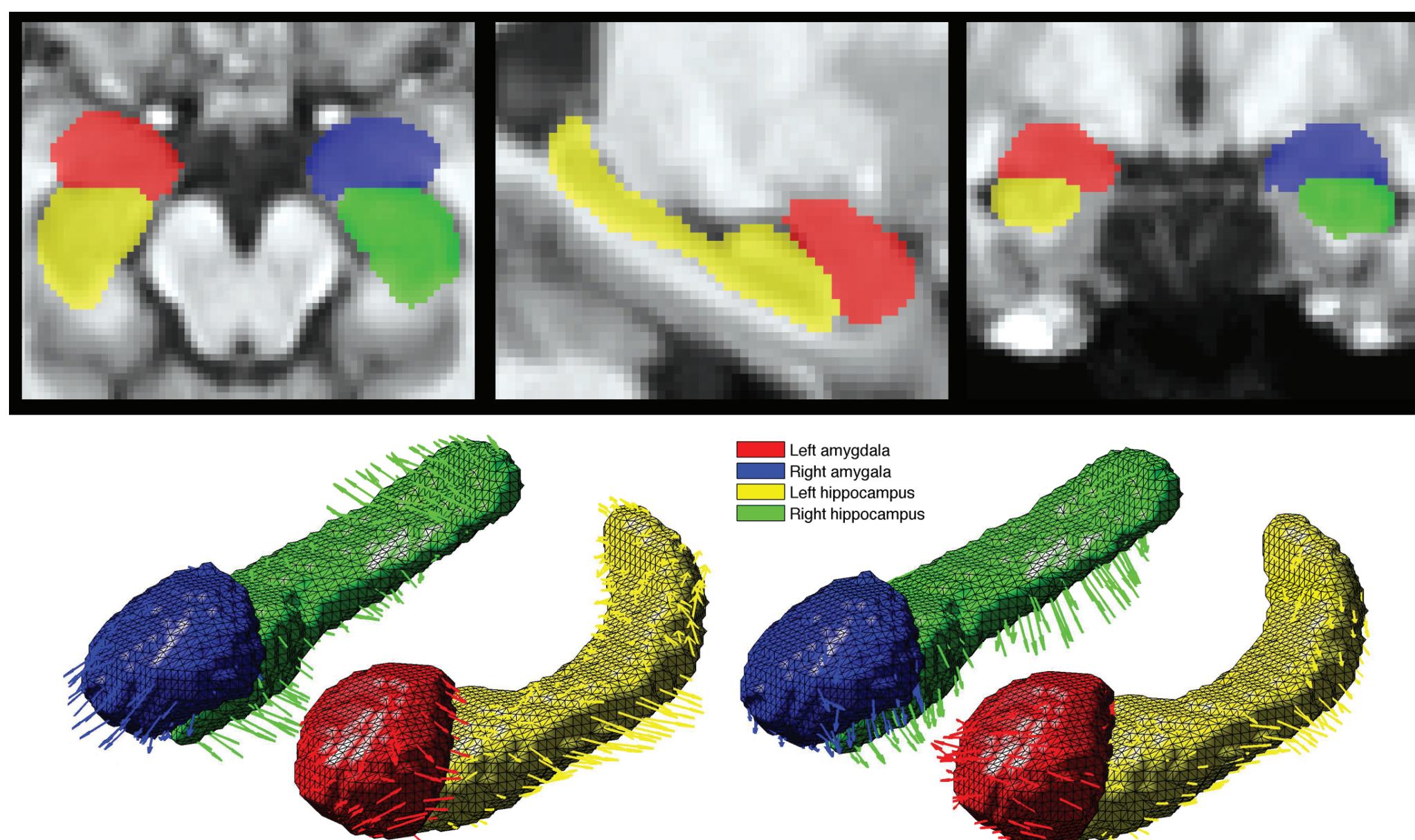


Figure 1: Manual segmentation of hippocampus and amygdala. Template surfaces are constructed by averaging the binary segmentation in the normalized space. The arrows are the displacement of warping from the template to individual surfaces.

## Laplace-Beltrami Eigenfunctions

The eigenfunctions  $\psi_j$  of the LB-operator  $\Delta$  on a surface are given by

$$\Delta\psi_j = -\lambda_j\psi_j.$$

The LB-eigenfunctions are computed on the template surface using the FEM discretization (Figure 2). The eigenfunctions are used to represent the surface displacement length as

$$\text{Length} = \sum_{j=0}^k \beta_j \psi_j = \psi \beta,$$

where  $\psi = (\psi_0, \dots, \psi_k)$  and  $\beta = (\beta_0, \dots, \beta_k)'$ .  $k = 1000$  is used.

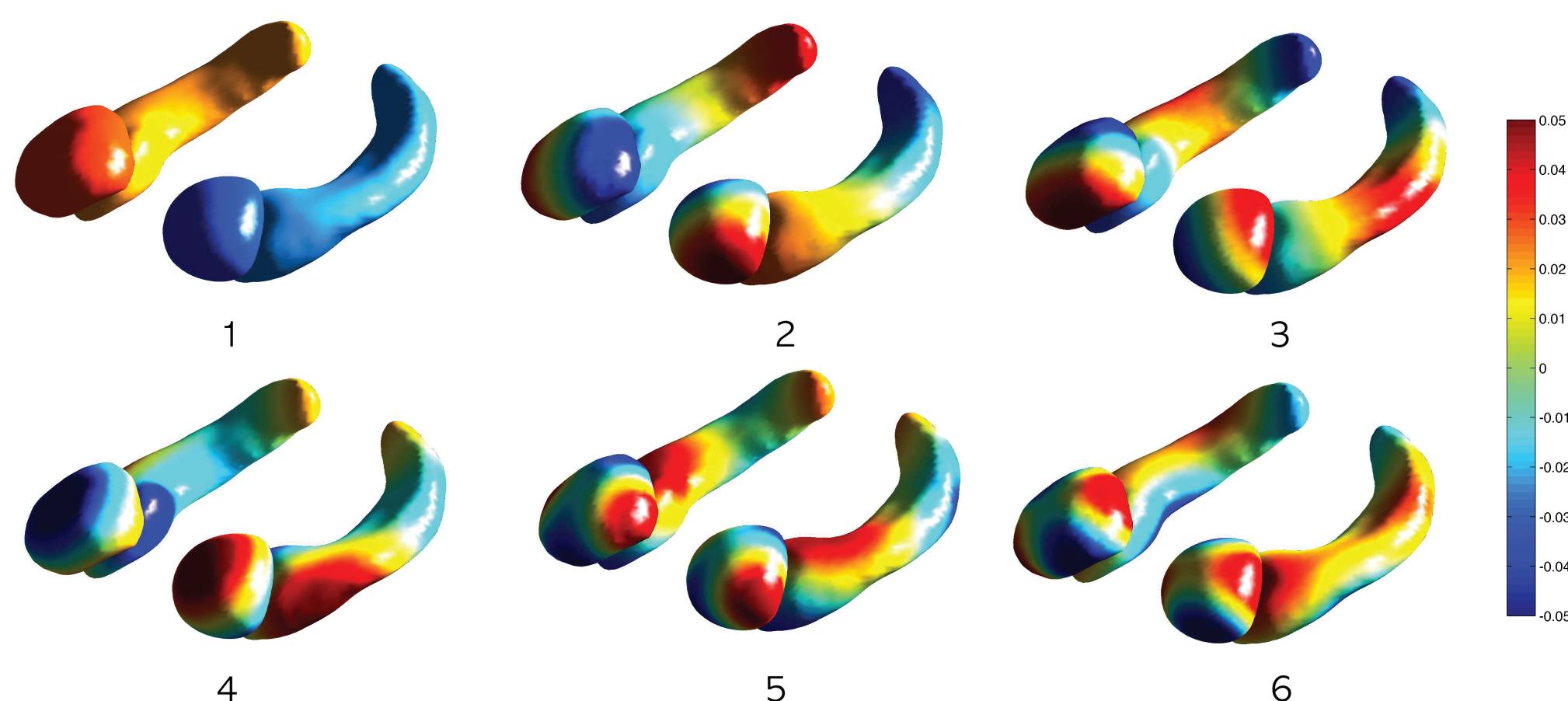


Figure 2: First six LB-eigenfunctions on amygdala and hippocampus surfaces.

## Sparse Representation

The least squares estimation method is previously used in estimating the coefficients while high frequency terms were truncated to reduce noise. However, some lower frequency terms may not necessarily contribute significantly. So we sparsely filter out insignificant terms by minimizing  $\ell_1$ -norm penalty. The coefficients  $\beta$  are then estimated as

$$\hat{\beta} = \arg \min_{\beta} \|\text{Length} - \psi\beta\|_2^2 + \lambda \|\beta\|_1.$$

$\lambda = 1$  is used. Only 50 largest coefficients among 1000 estimated coefficients are selected as significant. The age effect on the displacement length is then modeled as a linear model:

$$\text{Length} = 1 + a \cdot \text{Brain} + b \cdot \text{Age} + c \cdot \text{Gender} + \epsilon,$$

where Brain is the total brain volume (Figure 3).

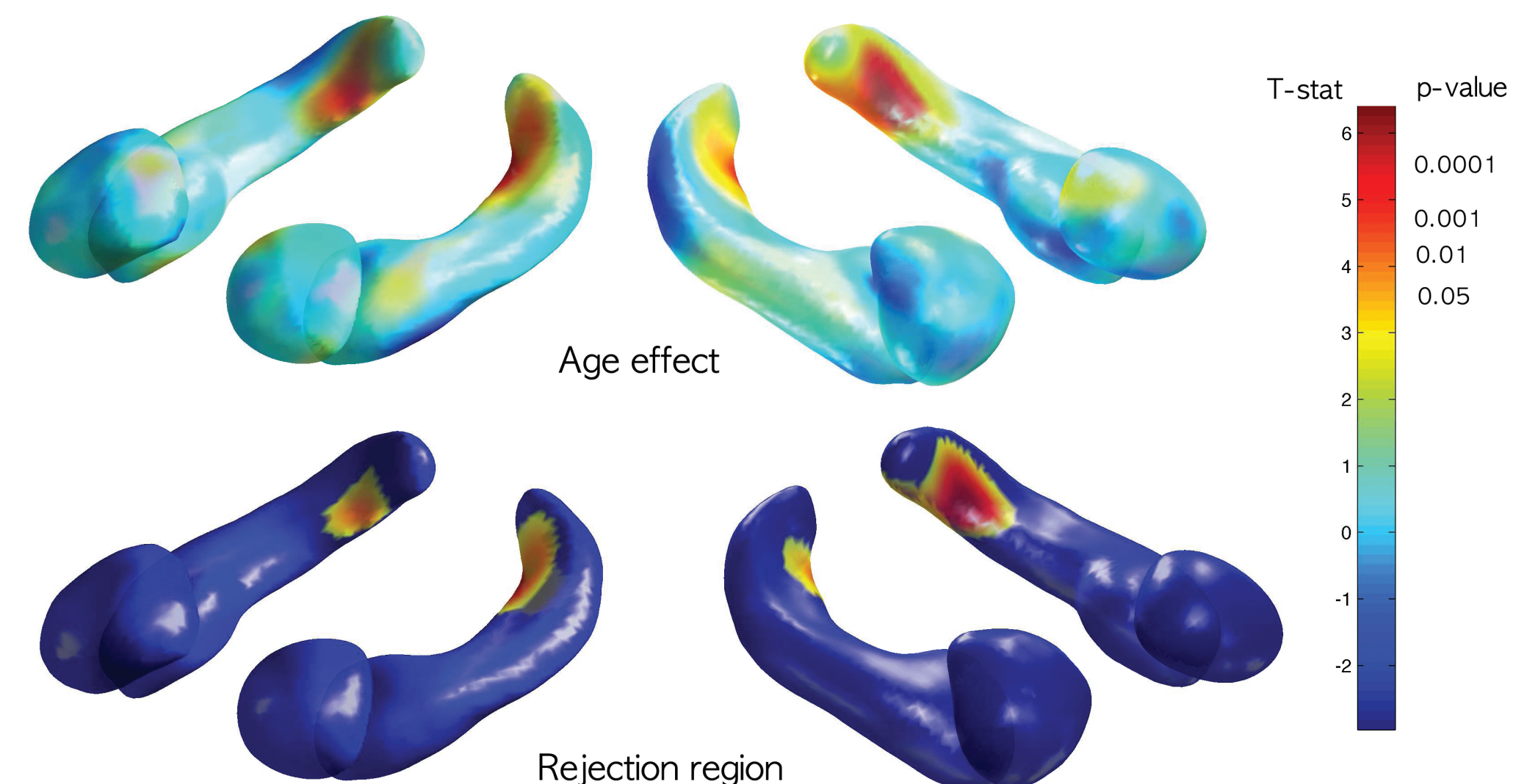


Figure 3: Age effect on hippocampus shape. The T-stat. and the corrected p-value are also shown. There is no age effect on amygdala. Rejection regions  $\mathcal{M}_1$  corresponding to 0.05 level are also shown.

## Statistical Power under Multiple Comparisons

For a T-random field  $T(x)$ , the threshold  $h$  corresponding to the type-I error at 0.05 is given by

$$P\left(\max_{x \in \mathcal{M}} T(x) > h\right) = 0.05,$$

where  $\mathcal{M}$  is the surface area. Then the statistical power is given by

$$\text{Power} = 1 - P\left(\max_{x \in \mathcal{M}_1} T(x) > h\right),$$

where the maximum is restricted to the rejection region  $\mathcal{M}_1$  (Figure 1). To estimate the power, we opted for numerical simulations with 5000 permutations.

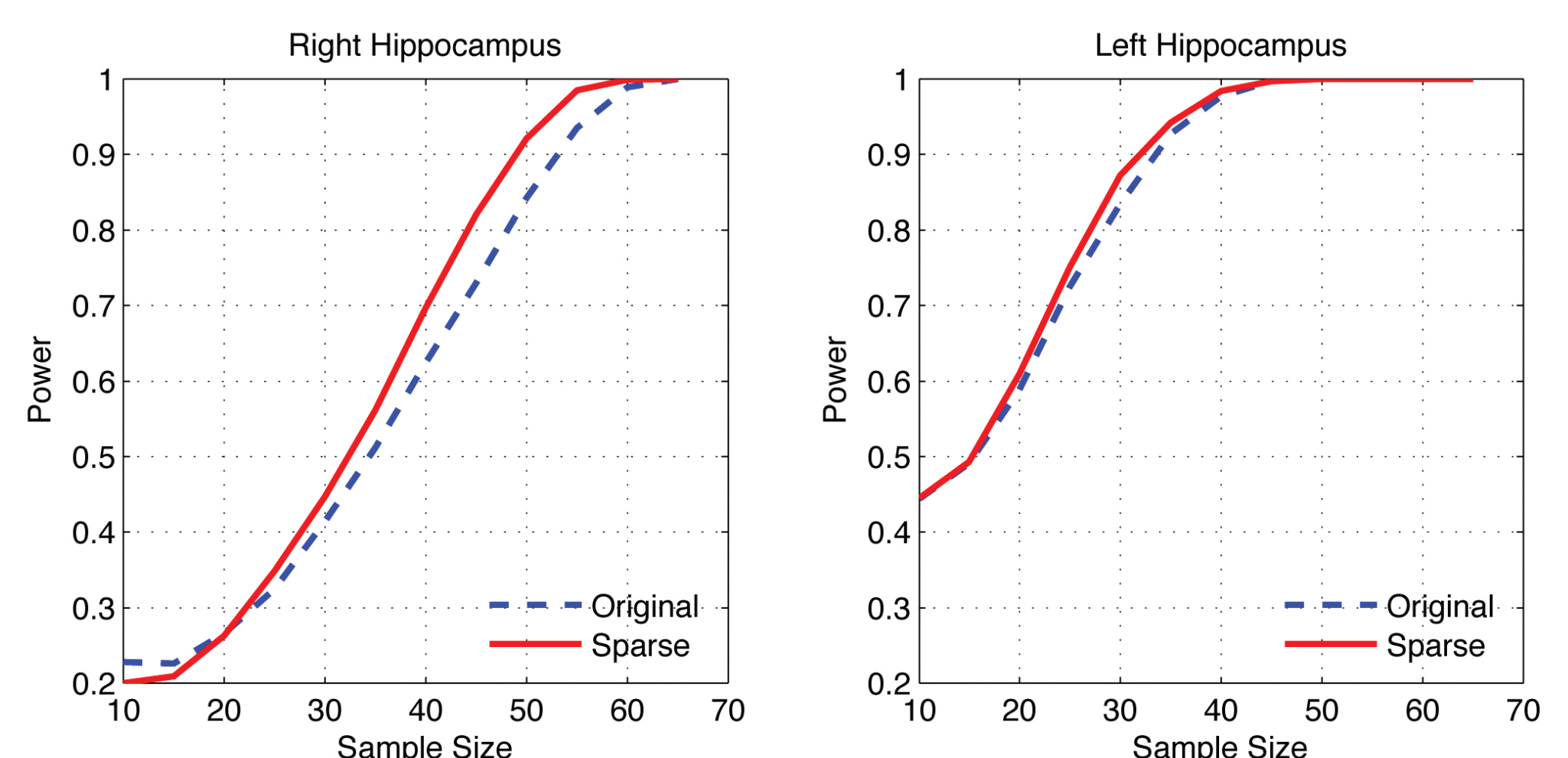


Figure 4: Statistical power over sample size computed under multiple comparisons.

## Conclusion

As seen in the overall power curves (Figure 4),  $\ell_1$ -norm minimization gives higher power for a given sample size. The proposed sparse regression requires smaller sample size to achieve a given power level demonstrating the advantage of the proposed method.

The proposed sparse shape model is shown to smooth surface data with only 5% of Laplace-Beltrami eigenfunctions and improves the statistical power and reduces the sample size requirement.