A rotation-invariant spherical harmonic decomposition method for mapping intravoxel multiple fiber structures

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A new rotation-invariant spherical harmonic decomposition (SHD) method is proposed in this paper for analyzing high angular resolution diffusion (HARD) imaging. Regular SHD methods have been used to characterize the features of the apparent diffusion coefficient (ADC) profile measured by the HARD technique. However, these regular SHD methods are rotation-variant, i.e., the magnitude and/or the phase of the harmonic components changes with the rotation of the ADC profile. We propose a new rotation-invariant SHD (RI-SHD) method based on the rotation-invariant property of a diffusion tensor model. The basic idea of the proposed method is to reorient the measured ADC profile into a local coordinate system determined by the three eigenvectors of the diffusion tensor in each imaging voxel, and then apply a SHD to the ADC profile. Both simulations and in vivo experiments were carried out to validate the method. Comparisons were made between the component maps from a regular SHD method, diffusion circular spectrum mapping (DCSM) method and the proposed RI-SHD method. The results indicate that the regular SHD maps vary significantly with the rotation of the diffusion-encoding scheme, whereas the maps of the DCSM and the proposed RI-SHD method remain unchanged. In particular, the (0,0)-th, (2,2)-th and (4,4)-th component maps from the RI-SHD method exhibited good consistency with the 0th, 2nd and 4th order maps of the DCSM method, respectively. Compared with the regular SHD methods used in HARD imaging, the proposed RI-SHD method is superior in characterizing the diffusion patterns of multiple fiber structures between different brain regions or across subjects.

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Introduction

Diffusion tensor imaging (DTI) has been established as a powerful tool to non-invasively investigate white matter structures in vivo (Basser et al., 1994; Le Bihan et al., 2001). An important advantage of DTI over traditional diffusion-weighted imaging (DWI) is that the diffusion tensor offers a rotation-invariant model that justifies the quantitative comparison of diffusion structures between different parts of the brain or across different subjects (Basser and Pierpaoli, 1996). Tractography techniques have also been developed to delineate neural pathways based on the assumption that the major eigenvector of the diffusion tensor should be oriented parallel with local white matter fibers (Basser et al., 2000; Mori et al., 2002; Poupon et al., 2000). However, the validity of the tractography reconstructed from DTI is confounded by the fact that the tensor model is only a 2nd order approximation of a possible complex diffusion pattern (Basser, 2002), and that the primary eigenvector of the diffusion tensor may be seriously biased from the actual fiber direction if multiple fibers share a single voxel (Alexander et al., 2001; Basser et al., 2000). To resolve the problem caused by intravoxel multiple fibers, more elaborate acquisition and analysis strategies beyond the tensor model are generally needed.

One strategy of characterizing intravoxel multiple fibers is to calculate the probability distribution function (PDF) of the diffusion process in each voxel based on the Fourier transform relationship between the PDF of diffusion displacement and the diffusion-weighted signal attenuation in q-space (Assaf and Cohen, 2000). Wedeen et al. (2000) proposed the idea of diffusion spectrum imaging (DSI) that probes complex white matter structures by calculating the 3-D diffusion displacement PDF from a large number of data acquisitions in q-space. Theoretically, the q-space Fourier relationship is strictly held when the diffusion-weighting gradients have infinitely narrow width and infinitely high amplitude (Callaghan, 1990). However, there is growing evidence that despite the pulse width violation, it is still a reasonable description of local diffusion and microstructural organization in brain tissues (Assaf et al., 2004). Lin et al. (2003) performed experiments on phantoms and animal models to assess the accuracy of DSI in practical MRI settings. As a modified version of DSI, a “q-ball imaging” (QBI) technique was proposed to acquire q-space data only on a spherical surface (Tuch et al., 2003). In general, the relatively high diffusion gradient requirements and the large acquisition numbers are still
the major barriers of q-space imaging techniques for clinical implementation.

Another strategy is to directly characterize the measured high angular resolution diffusion (HARD) profile in each voxel, although how to effectively quantify the HARD information remains an open question (Frank, 2001). Alexander et al. (2002) and Frank (2002) proposed the idea of using spherical harmonic decomposition (SHD) to characterize the 3-D apparent diffusion coefficient (ADC) profile measured by HARD imaging. Recently, SHD method was also employed to directly estimate the orientation density function (ODF) of a diffusion pattern (Tournier et al., 2004). In general, the lower order (0th or 2nd) spherical harmonics (SH) obtained by SHD represent the isotropic diffusion or single fiber diffusion patterns, whereas the higher orders (4th or higher) represent non-Gaussian patterns associated with intravoxel multiple fiber components. However, compared with DTI, a major disadvantage of the SHD method is that the calculated SHs are actually rotation-variant, i.e., the magnitude and the phase value of the decomposed SH (1st order or higher) change with the rotation of the diffusion profile with respect to the coordinate system. This drawback was indicated by a simulation presented by Frank (2002) but has not yet been explicitly addressed. A “grouped” index from the different order harmonics, e.g., the sum of the squared harmonic magnitudes, as suggested by Goldberg-Zimring et al. (2004), is less sensitive to the imaging objects’ rotation. However, the information of individual SHD maps is mixed up within the grouped index. As a result, the SHD maps calculated from the existing SHD methods cannot be used individually to ensure rotation-invariant comparisons between different brain regions or across subjects.

Zhan et al. (2003) proposed an alternative method named “diffusion circular spectrum mapping” (DCSM) for characterizing the HARD profiles of multiple fiber components. DCSM only examines the ADC distribution along the circle spanned by the major and medium eigenvectors and applies a 1-D Fourier transform onto this circular ADC distribution. It has been demonstrated that the 0th, 2nd and 4th order circular harmonics are associated with isotropic, single fiber and orthogonal fiber crossing diffusion patterns, respectively. Recently, the DCSM method was further extended to identify not only the existence but also the orientation of intravoxel fiber crossings by using the phase information of the circular harmonics (Zhan et al., 2004). Unlike the regular SHD method mentioned above, the DCSM components can be used as rotation-invariant maps as DTI-based indices such as fractional anisotropy (FA) and mean diffusivity (MD).

In this paper, a new rotation-invariant SHD (RI-SHD) technique is presented based on the rotation-invariant property of DTI. The basic idea of this method is to reorient the measured diffusion profile according to the eigen-system of the diffusion tensor in each voxel, and then apply a SHD algorithm to the reoriented ADC profile. Computer simulations and in vivo experiments were performed to validate this method by using an equivalent rotation created by different diffusion encoding schemes. Results are compared between the regular SHD method and the proposed method, indicating that while the regular SHD maps vary significantly with rotation, the corresponding maps of proposed method remain unchanged. In particular, the (0,0)-th, (2,2)-th and (4,4)-th harmonic component maps obtained from the RI-SHD method exhibited good consistency with the 0th, 2nd and 4th order maps from the DCSM method, respectively. Compared with regular SHD methods, the proposed RI-SHD method is superior in characterizing intravoxel multiple fibers in different brain regions or across subjects.

Theory

For an arbitrary imaging voxel, let \( D_{app}(\theta, \phi) (0 \leq \theta \leq \pi, 0 < \phi < 2\pi) \) be the ADC profile measured by a HARD technique, where \((\theta, \phi)\) denotes the polar and azimuths angles respectively of the spherical coordinate system of the measurement. The spherical harmonic decomposition (SHD) is regularly performed such that

\[
a_{lm} = \int_0^{2\pi} \int_0^\pi D_{app}(\theta, \phi) Y_{lm}^*(\theta, \phi) \sin(\theta) d\theta d\phi
\]

where \( a_{lm} \) is generally a complex coefficient representing the \(m\)-th degree of the \(l\)-th \((-l \leq m \leq l)\) order spherical harmonic (i.e., abbreviated as \((l,m)\)-th SHD component), whereas \( Y_{lm}^*(\theta, \phi) \) is the corresponding SH kernel function given by

\[
Y_{lm}(\theta, \phi) = \left( \frac{(2l+1)(l-m)!}{4\pi(l+m)!} \right)^{1/2} P_l^m(\cos\theta) \exp(-jm\phi)
\]

where \( P_l^m(\cdot) \) denotes the \(m\)-th degree of the \(l\)-th order associated Legendre function. In the context of the SHD defined above, the ADC profile can be written as an expansion of the Laplace series up to an infinite order, i.e.,

\[
D_{app}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}^*(\theta, \phi)
\]

According to the Parseval’s theorem (Champeney, 1989) of the spherical Fourier transform, we have

\[
\int_0^{2\pi} \int_0^\pi |D_{app}(\theta, \phi)|^2 \sin(\theta) d\theta d\phi = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} |a_{lm}|^2
\]

Eq. (4) implies that the sum of squared magnitude of all SHD components is strictly rotation-invariant, although the specificity of individual SHD components is lost in the sum.

To give an intuitive impression about the kernel functions used in SHD, the lower order \((0 \leq l \leq 6)\) components of the SH kernel functions are illustrated in Fig. 1. In each subfigure, the profile of the SH kernel function \( Y_l^m(\theta, \phi) \) is plotted with a normalized size, and the SH orders \((0 \leq l \leq 6)\) are arranged vertically from bottom to top, while the SH degrees \((0 \leq m \leq l)\) are horizontally aligned from left to right in the row of order \(l\). Note that those SH kernels with negative degrees \(-l \leq m < 0\) are omitted in the figure, because \( Y_l^{-m}(\theta, \phi) \) generally has the symmetrical shape as \( Y_l^{m}(\theta, \phi) \) except for a different size and orientation as indicated in Eq. (2). Fig. 1 shows that the SH profiles are more complex in shape with the increase of the SH orders. For example, the 0th order SH kernel has a spherical profile, while the SH kernels with \(l = 2\) order have cylindrical profiles. Therefore, the SH kernels with \(l = 0\) and 2 orders can be used to measure the diffusion contributions from isotropic (no fiber) and cylindrical (single fiber) patterns, respectively. SH kernels with odd orders \((l = 1, 3\) and 5) have asymmetric shapes about the center,
and thus may only reflect imaging noises and/or artifacts rather than real diffusion patterns (Frank, 2002). The SH kernel profiles with an order $l = 4$ or 6 have shapes with multiple maxima, and thus may be used to characterize the complex diffusion patterns of multiple fibers. In particular, the $(4,4)$-th SH kernel component (i.e., $l = 4$ and $m = 4$) has a four-leaf shape similar to the ADC profile of an ideal orthogonal fiber crossing, and therefore the $(4,4)$-th SHD component could be used to provide an intravoxel fiber crossing map. Due to the anisotropic profiles of the SH kernels (except the 0th order), the rotation of any anisotropic ADC profile with respect to the spherical coordinate system may lead to the change of corresponding SHD coefficients, resulting in the rotation-variant property of the regular SHD method.

The proposed RI-SHD method is based on the rotation-invariant property of the diffusion tensor model. Let $D$ be the diffusion tensor estimated from the spherical ADC distribution $D_{\text{app}}(\theta, \phi)$. In our method, a rotation manipulation is performed in each voxel on the $D_{\text{app}}(\theta, \phi)$ from the laboratorial coordinate system $(\theta, \phi)$ to a “local” coordinate system $(\theta', \phi')$ that is determined by $D$, such that

$$D'_{\text{app}}(\theta'\phi') = [V_1 V_2 V_3] D_{\text{app}}(\theta, \phi)$$  \hspace{1cm} (5)

where $V_1$, $V_2$, and $V_3$ are the major, medium and minor eigenvectors of the tensor $D$ associated with the corresponding eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$, respectively. In Eq. (5), the ADC profiles $D_{\text{app}}(\theta, \phi)$ and $D'_{\text{app}}(\theta', \phi')$ should be written in a form of $3 \times N$ matrix, in which each column corresponds to the vector of a diffusion weighting on the ADC profile and $N$ is the number of all measurements. The rotation matrix $[V_1 V_2 V_3]$ can be alternatively written as a function of the rotation angles, i.e.,

$$[V_1 V_2 V_3] = R_z(\alpha) R_y(\beta) R_z(\gamma)$$  \hspace{1cm} (6)

where $\alpha$, $\beta$ and $\gamma$ are the rotation angles about $x$-, $y$- and $z$- coordinate axis, respectively, and $R_z(\alpha)$, $R_y(\beta)$, and $R_z(\gamma)$ are the matrix factors of the rotation angle $\alpha$, $\beta$ and $\gamma$, respectively, such that
Thus, reorientation manipulation can be regarded as to rotate the profile $D_{\text{app}}(\theta, \phi)$ of angle $(\alpha, \beta, \gamma)$ such that the major, medium, and minor eigenvectors of the orientated diffusion tensor are parallel with the $x$-, $y$- and $z$-coordinate axis, respectively. The proposed RI-SHD is obtained by applying a SHD to the reoriented ADC profile $D_{\text{app}}'((\theta', \phi'))$, i.e.

$$a_{lm} = \int_{0}^{\pi} \int_{0}^{2\pi} D_{\text{app}}''((\theta', \phi')) Y_l^m(\theta', \phi') \sin(\theta') d\theta d\phi'$$

where $a_{lm}$ is the calculated RI-SHD coefficient with the $m$-th degree of the $l$-th order. Due to the fact that $D_{\text{app}}''((\theta', \phi'))$ has been reoriented according to the eigenvectors of the local diffusion tensor $D$, the magnitude $|a_{lm}|$ would be insensitive to the orientation of the diffusion tensor, i.e., they have similar rotation-invariant property as those DTI-based indices calculated directly from the tensor eigen-system.

**Methods**

**Simulation study**

For the convenience of comparison, we used a digital phantom with the same structures as in our previous studies (Zhan et al., 2003, 2004) to validate the rotation-invariant SHD method described above. As shown in Fig. 2, objects I and II were used to simulate tissues with isotropic and planar diffusion patterns, with tensor eigenvalue ratios of 1:0.95:0.9 and 1:0.95:0.1, respectively. Objects III, IV, V and VI simulated linear diffusion patterns, with an eigenvalue ratio of 1:0.1:0.1, and the major eigenvector placed parallel to the object’s axis. For object VI, the axis was perpendicular to the imaging slice. Fiber intersection and “kissing” were simulated in areas (A), (B), (C) and (D). In each fiber-crossing voxel, signal contributions from individual fiber compartments were assumed to be equal. The maximal ADC was corresponding to $\frac{1}{C_2} \frac{10}{C_0} 3$ (mm$^2$/s). The in-plane matrix size was 128 x 128. Diffusion-weighted MRI was simulated with 256 diffusion-encoding directions approximately equally spaced on a spherical surface with a $b$ value of 2500 (s/mm$^2$). An image without diffusion...
weighting ($h = 0$) was also generated. The simulations were performed in a noisy environment with a signal-to-noise ratio (SNR) of 200.

To assess the effects of object rotation, two rotation-related diffusion encoding schemes were used to perform simulations on the above digital phantom, simulating the diffusion-weighted MRI experiments before and after phantom rotation, respectively. Let two $3 \times 256$ matrices $E_1$ and $E_2$ denote the two diffusion encoding schemes with each column representing the vector of one encoding direction (Fig. 3). The rotation relationship between the two schemes can be written as

$$E_2 = R_z(\gamma)R_y(\beta)R_x(\alpha)E_1$$

That is, the second diffusion encoding scheme $E_2$ can be obtained by rotating the first scheme $E_1$ with angles $\alpha$, $\beta$ and $\gamma$ about the $x$-, $y$- and $z$-axis respectively. It is noted that, for the diffusion-weighted MRI experiments on a static phantom, the rotation of a diffusion encoding scheme at angles $(\alpha, \beta, \gamma)$ can be alternatively interpreted as an equivalent rotation of the phantom at angles $(-\alpha, -\beta, -\gamma)$ while the diffusion encoding scheme remains unchanged. The obvious advantage of this strategy for generating an equivalent phantom rotation is to avoid the difficulties of repositioning the imaging slice(s).

In the present study, the rotation angles were set as $(\alpha = 0, \beta = \pi/2, \gamma = \pi/4)$. Therefore, the fiber crossings in areas (A), (B) and (C) are reoriented to a plane approximately perpendicular to the $x$–$y$ plane, while the fiber crossings in area (D) are reoriented to a plane approximately parallel to the $x$–$y$ plane.

**In vivo experiments**

MRI experiments were performed on five normal human subjects (2 males and 3 females, all right handed, aged 22–39) on a 3T Siemens Allegra scanner. Informed consents were obtained in accordance with the guidelines of the Institutional Review Board at the National Institute on Drug Abuse. For each subject, diffusion-weighted images were acquired using a modified spin-echo EPI pulse sequence with the same $b$ value and diffusion-weighting ($h = 0$) as those used in the digital phantom simulations. The sampling bandwidth was 752 (Hz/pixel) and the in-plane image matrix was 128 $\times$ 128. Ear plugs were used to reduce noise, and foam packs were applied to restrict head motion. Other basic imaging parameters were: $TR = 1.3$ s, $TE = 136$ ms, $FOV = 24 \times 24$ cm$^2$. Two coronal imaging slices (4 mm thickness and 1 mm gap) were acquired approximately parallel to the extension of the brain stem covering the pons region. The acquisition was repeated 4 times to ensure an enhanced signal-to-noise ratio (SNR) of about 70 and 15 for the reference ($h = 0$) and the diffusion-weighted images, respectively. The SNR was estimated by the ratio of the mean intensity of a foreground region (e.g., the temporal lobe gray matter) and the standard deviation of the image background region.

**Data processing**

For each diffusion-weighted dataset acquired with a diffusion encoding scheme, the following data processing procedures were performed: (i) a procedure to correct the geometrical distortion due to the susceptibility-induced field inhomogeneities (Jezzard and Balaban, 1995) on the in vivo EPI images; (ii) the reference and diffusion-weighted images were used to calculate the diffusion anisotropy (FA) map of the digital phantom acquired with encoding scheme $E_1$. No difference is detected between the maps before and after the rotation, indicating that the DTI and DCSM methods are rotation-invariant.

In the 4th order DCSM maps, all the fiber crossing areas (A–D) are identified.

![Fig. 5. The 0th, 2nd and 4th order DCSM maps and the fractional anisotropy (FA) map of the digital phantom acquired with encoding scheme $E_1$. No difference is detected between the maps before and after the rotation, indicating that the DTI and DCSM methods are rotation-invariant.](image-url)
tensor eigen-system for each voxel (using a singular value decomposition (SVD) algorithm), and DTI-based indices mean diffusivity (MD) and fractional anisotropy (FA), etc.; (iii) the diffusion circular spectrum mapping (DCSM) method (Zhan et al., 2003) was performed to calculate the 0th, 2nd and 4th order circular harmonic maps by using the same procedure as described in our previous papers, except for the different number of diffusion encoding directions; (iv) the regular SHD method was used to calculate the lower order \((l \leq 6)\) SHD maps according to Eqs. (1) and (2); and (v) the proposed RI-SHD method was used to calculate the RI-SHD maps with the lower orders \((l \leq 6)\) according to Eqs. (5) and (8).

It should be noted that data were processed according to diffusion encoding scheme \(E_1\) in procedures (ii) through (v) for all datasets acquired by either \(E_1\) or \(E_2\). As explained in the phantom simulation method, the diffusion-weighted images acquired with scheme \(E_1\) can be regarded as an equivalent dataset acquired with scheme \(E_2\) while the objects were correspondingly rotated with angles \((x = 0, \beta = -\pi/2, \gamma = -\pi/4)\).

Results

Phantom simulations

The magnitude SHD component maps of the digital phantom calculated from the regular SHD method are plotted in Figs. 4a and b for the diffusion dataset generated by the diffusion encoding scheme \(E_1\) and \(E_2\), respectively. In each subfigure, the lower order SHD component maps are aligned in a similar triangle array as that of Fig. 2, and the grayscale of each SHD component is individually normalized. As a comparison, DTI-based FA map and the 0th, 2nd and 4th order DCSM maps are illustrated in Fig. 5 for the original \((E_1)\) diffusion dataset, and the results for the rotated \((E_2)\) diffusion dataset are essentially identical (data not shown). After the equivalent rotation of the digital phantom, however, the SHD component maps calculated from the regular SHD method change significantly with the rotation, except for the 0th order SHD map that indicates the isotropic diffusion pattern (e.g., in the object I of the phantom). In Fig. 4, the SHD components of the odd orders \((l = 1, 3 \text{ and } 5)\) are much more noisy than the even orders \((l = 0, 2, 4 \text{ and } 6)\), indicating that odd-order SHD components reflect noise and/or image artifacts due to their asymmetric structures. It is interesting to examine the maps of the \((4,4)\)-th SHD component (i.e., \(l = 4, m = 4\)) before and after the rotation and compare them with the 4th order DCSM maps shown in Fig. 5. In Fig. 4a, the \((4,4)\)-th SHD map highlights fiber crossings in areas (A), (B) and (C) where the fiber crossings lie parallel with the imaging slice, but it fails to identify the crossing area (D) where fiber crossings are embedded in the plane perpendicular to the imaging slice. In Fig. 4b, however, the \((4,4)\)-th SHD map highlights the area (D) but does not highlight the areas (A), (B) and (C) after the rotation.

In contrast, the magnitude maps calculated from the proposed RI-SHD method are illustrated in Figs. 6a and b for the diffusion dataset before and after the rotation, respectively. Clearly, the RI-SHD components exhibit the rotation-invariant property. Rotation does not result in any recognizable change in the displayed component maps. In particular, the \((4,4)\)-th component map in Fig. 6 successfully identifies all fiber crossings in areas (A) (B) (C) and (D). The \((0,0)\)-th, \((2,2)\)-th and \((4,4)\)-th RI-SHD component maps in Fig. 6 are in excellent agreement with the 0th, 2nd and 4th order DCSM maps shown in Fig. 5, respectively. Compared with the corresponding 4th order DCSM map (SNR \(\approx 80\)), the \((4,4)\)-th RI-SHD component map is less noisy (SNR \(\approx 120\)), suggesting its better noise-suppression ability for identifying fiber crossings. Fig. 6 also illustrates that the even-order (i.e., \(l = 2, 4, \ldots\)) SHD component maps with an odd-degree number (i.e., \(m = 1, 3, \ldots\)) are relatively noisier (SNR \(\approx 60\)) than the same order maps with an even degree number (i.e., \(m = 0, 2, 4, \ldots\)). The SNR of SHD maps is defined in the same way as that the MRI images (see Methods), except for one difference that the standard deviation of noise in the background region should be calculated before the background being clear out for display.

In vivo experiments

A typical coronal slice from one subject is used to illustrate results of the in vivo experiments, as the data well consistent across both subjects and brain slices. For the regular SHD method, the magnitude maps of the lower order SHD components are illustrated in Figs. 7a and b for the dataset acquired by diffusion encoding scheme \(E_1\) and \(E_2\), respectively. Similar to the simulation results, the SHD component maps change significantly with the equivalent rotation of the diffusion-encoding scheme, indicating again that the regular SHD method is rotation-variant. Similar to Fig. 5, the FA map and the 0th, 2nd and 4th order DCSM maps of the same slice are shown in Fig. 8 for the original \((E_1)\) diffusion dataset, and the results for the rotated \((E_2)\) diffusion dataset are essentially identical (data not shown). Comparing the DCSM maps in Fig. 8 with the \((0,0)\)-th, \((2,2)\)-th and \((4,4)\)-th SHD component maps shown in Fig. 7 illustrate the rotation-invariant property the DCSM method for the in vivo data. For example, the \((4,4)\)-th SHD component map in Fig. 7a fails to identify the fiber crossing in the pons area, whereas the corresponding map in Fig. 7b does not highlight the fiber crossings in the corpus callosum and the cingulum bundle.

For the proposed RI-SHD method, the calculated maps illustrated in Figs. 9a and b are calculated from the diffusion dataset acquired before and after the equivalent rotation. The rotation-invariant property of the proposed method is clearly demonstrated. Compared with the DCSM maps shown in Fig. 8, very little differences are found between the \((0,0)\)-th, \((2,2)\)-th and \((4,4)\)-th SHD component maps and the 0th, 2nd and 4th order DCSM maps, respectively. Again, the \((4,4)\)-th SHD
Fig. 7. A typical coronal imaging slice from one subject is used to illustrate the results of the in vivo MRI experiments. The regular SHD components maps of the slice are illustrated in subfigures (a and b) for the diffusion-weighted datasets acquired with encoding scheme $E_1$ and $E_2$, respectively. The SHD component maps are individually normalized in grayscale, and displayed in the same arrangement as used in Fig. 1. The differences between the corresponding maps shown in (a and b) indicate that the regular SHD method is rotation-variant. (a) The regular SHD component maps of the coronal slice for the diffusion-weighted dataset acquired with encoding scheme $E_1$. In the enlarged map showing the (4,4)-th SHD component, the fiber crossings in the pons region (see the 4th order DCSM map shown in Fig. 8) are not identified. (b) The regular SHD component maps of the coronal slice for the diffusion-weighted dataset acquired with the reoriented encoding scheme $E_2$. In the (4,4)-th SHD map, the fiber crossings in the corpus callosum and fornix are not identified.
component map exhibits a higher signal-to-noise ratio (SNR \( \approx 50 \)) than that of the 4th order DCSM map (SNR \( \approx 30 \)). Compared with the simulation results, the SNR difference between the odd- and even-degree SHD components (both with an even order) for the in vivo datasets is less significant.

Discussion

We have demonstrated the differences between the component maps calculated from the regular SHD method and the proposed RI-SHD method, while the imaging objects experienced an equivalent rotation with respect to the diffusion encoding scheme. Due to its rotation-invariant property, the proposed RI-SHD method possesses an advantage over regular SHD method similar to DTT’s advantage over previous techniques in characterizing diffusion anisotropies. Although the rotation-variant drawback of the regular SHD method was depicted by the simulation examples in (Frank, 2002), it has not been explicitly stated that these SHD components cannot be used as quantitative maps to ensure fair comparisons between different brain regions or across subjects.

Some “energy” indices were usually used in the regular SHD studies (Goldberg-Zimring et al., 2004; Frank, 2002), by summing up the squared magnitude of the SHD components at different orders to provide a certain degree of rotation-invariant property. However, these indices were unable to provide the rotation-invariant maps for individual SHD components.

It is worth noting that the rotation-invariant property of the DCSM method and the proposed RI-SHD method are based on an equivalent manipulation of the ADC profile reorientation according to the diffusion tensor eigenvectors. In the DCSM method described in (Zhan et al., 2003), the circular harmonic decomposition is performed on the circle spanned by the major and medium eigenvectors of the diffusion tensor. The DCSM algorithm is equivalent to the following two calculation steps: (1) to rotate the ADC profile into a new local coordinate system where the reoriented major, medium and minor eigenvector of the reoriented diffusion tensor is parallel with the \( x \), \( y \) and \( z \)-axis, respectively; and (2) to perform the circular harmonic decomposition along the circle embedded in the \( x-y \) plane. For the proposed RI-SHD method, the first step of ADC profile reorientation is the same as that in the DCSM, whereas the second step is alternatively to perform a spherical harmonic decomposition. Therefore, the rotation-invariant property of both methods is introduced by the reorientation procedure according to the diffusion tensor, i.e., depending on the rotation-invariant property of the diffusion tensor. Due to the fact that the diffusion tensor model is only a 2nd order approximation to the actual diffusion process, it is generally less sensitive to noise or artifacts compared with higher order statistics, thus the estimated tensor eigenvectors provide a relatively noise-robust yet rotation-invariant basis for estimating higher order indices. Empirically, the proposed RI-SHD method exhibited good robustness against the decrease of the encoding directions (NE) and the SNR within a reasonable range. In the described experimental settings, the RI-SHD decomposition results of up to the 4th order kept a clear consistency when NE \( \geq 90 \) and SNR \( \geq 60 \) for the reference image. The observed robust components rose up to the 6th order while the NE and SNR were increased to 128 and 80, respectively.

The consistency between the (4,4)-th component map of the proposed RI-SHD method and the 4-th order DCSM map suggests that the (4,4)-th RI-SHD component map can be used to identify intravoxel orthogonal fiber crossings, since the 4-th order DCSM map has been validated to do so in our previous studies (Zhan et al., 2003, 2004). Similarly, the (0,0)-th and (2,2)-th RI-SHD maps can also be used for identifying isotropic and linear diffusion patterns, respectively. This can be explained by the similarity between their SH kernel functions and the corresponding ADC profiles of the isotropic, single fiber and orthogonal fiber crossing diffusion patterns. Compared with the 4th order DCSM map, the (4,4)-th RI-SHD component map exhibits a higher signal-to-noise ratio, especially for relatively noisier diffusion-weighted datasets. This observed SNR advantage of the rotation-invariant SHD method can be explained by the fact that the DCSM method uses only the ADC information along the decomposition circle, leading to fewer diffusion acquisition points (only those close to the circle) being considered in the circular spectrum estimation.

Since the proposed RI-SHD method is rotation-invariant, its individual components can be useful for mapping various diffusion patterns. Besides the abovementioned (0,0)-th, (2,2)-th and (4,4)-th SHD maps, other even-order maps might be used to identify specific ADC profiles according to the SH kernel functions as shown in Fig. 1. For example, the (4,0)-th component map might be used to identify the diffusion of a “planar plus a perpendicular linear” pattern. In Fig. 9, this (4,0)-th SHD component map highlights gray matter areas, suggesting that there might exist a linear diffusion compartment (single fiber) perpendicular to the cortical surface coexisting with a planar structure lying in the cortical surface in those voxels.

The odd-order components of the RI-SHD are supposed to reflect the noise and/or artifacts of the imaging system. In Figs. 4,
Fig. 9. The RI-SHD component maps of the coronal slice are illustrated in subfigures (a and b) for the diffusion-weighted datasets acquired with encoding scheme $E_1$ and $E_2$, respectively. The clear consistency between the corresponding maps shown in (a and b) indicates again that the proposed RI-SHD method is rotation-invariant. Compared with Fig. 8, the (0,0)-th, (2,2)-th and (4,4)-th SHD component maps exhibit a clear consistency with the 0th, 2nd and 4th order DCSM maps, respectively.
6, 7 and 9, however, these components are partially visible compared to the clean background. This is because (1) the background noises are masked out (filling zeros in the background areas) in these figures to give better localization impression in the maps; and (2) each SHD component image is displayed with an individual normalized grayscale. Otherwise, these odd-order maps would look very dark and close to noise background. Actually, there is a weak (SNR < 0.2) signal contribution (matching the tissue structures) detected in these odd-order SHD components. Non-uniform encoding scheme is one of the possible causes for the small yet observable residual signal in the odd-order components.

For in vivo experiment, the EPI sampling rate along the phase-encoding direction is much slower than that along the readout direction. The heterogeneous nature of EPI sampling makes it vulnerable to off-resonance effects that cause image artifacts including geometric distortions particularly along the phase-encoding direction. Lower bandwidth in EPI data acquisition generally results in longer readout waveform (and thus slower sampling rate in the phase-encoding direction), leading to stronger geometric distortions that may not be completely corrected by the phase map correction.

**Conclusion**

The present study proposed a rotation-invariant spherical harmonic decomposition method for mapping the intravoxel multiple fiber structures. The rotation-invariant property of the proposed method was obtained by rotating the measured ADC profile in each voxel according to the eigenvectors of the local diffusion tensor. The results of phantom simulations and in vivo experiments indicated that the regular SHD component maps changed significantly with the rotation of the diffusion encoding scheme, whereas the RI-SHD component maps kept unchanged. The (0,0)-th, (2,2)-th and (4,4)-th component map from the proposed RI-SHD method exhibited a good consistency with the 0th 2nd and 4th order DCSM map, respectively, with a better signal-to-noise ratio. Other RI-SHD component maps are also potentially useful in mapping complex diffusion patterns.

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**References**


