Processing High Angular Resolution Diffusion Data

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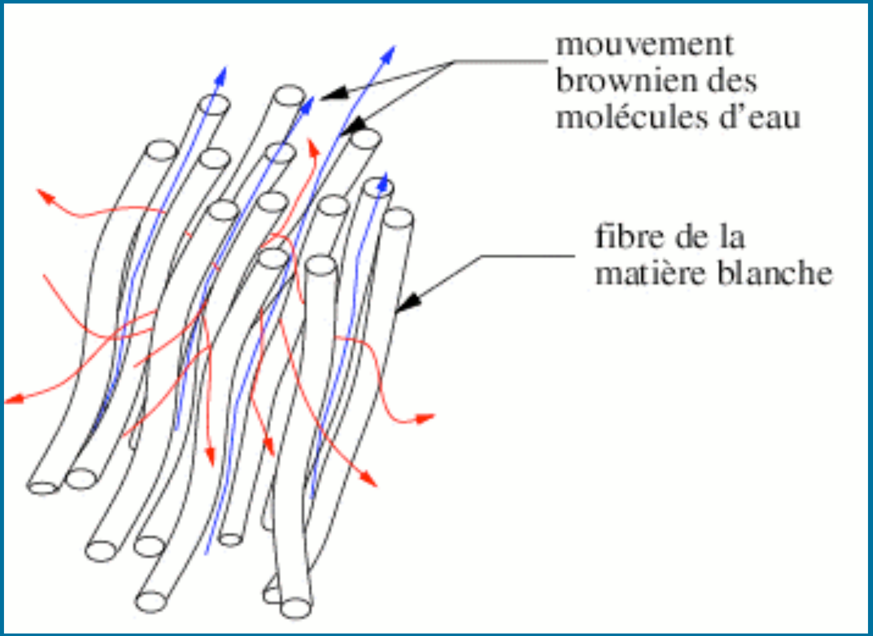
Diffusion MRI: recalling the basics

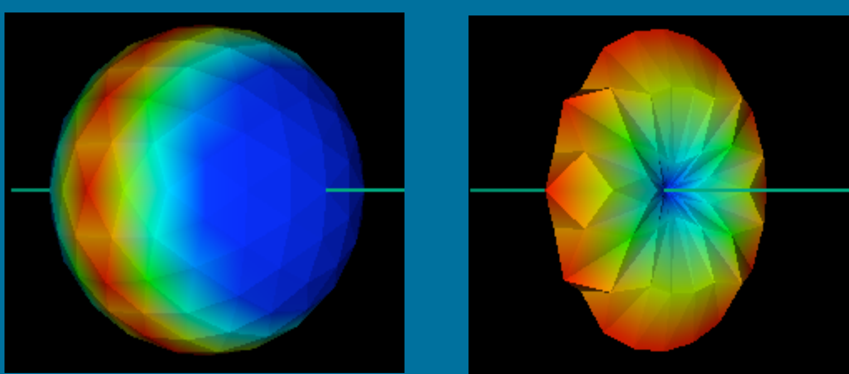
- Brownian motion of water molecules along white matter fibers
- Signal attenuation proportional to average diffusion in a voxel



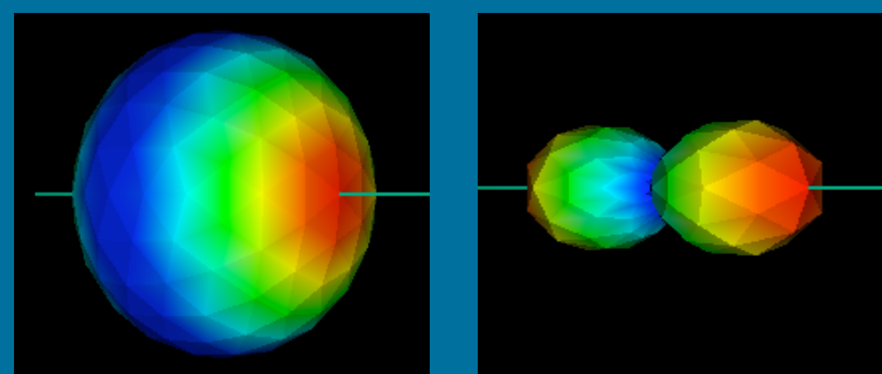
[Poupon, PhD thesis]

Apparent Diffusion Coefficient

$$D(q_i) = -\frac{1}{b} \log S(q_i)$$

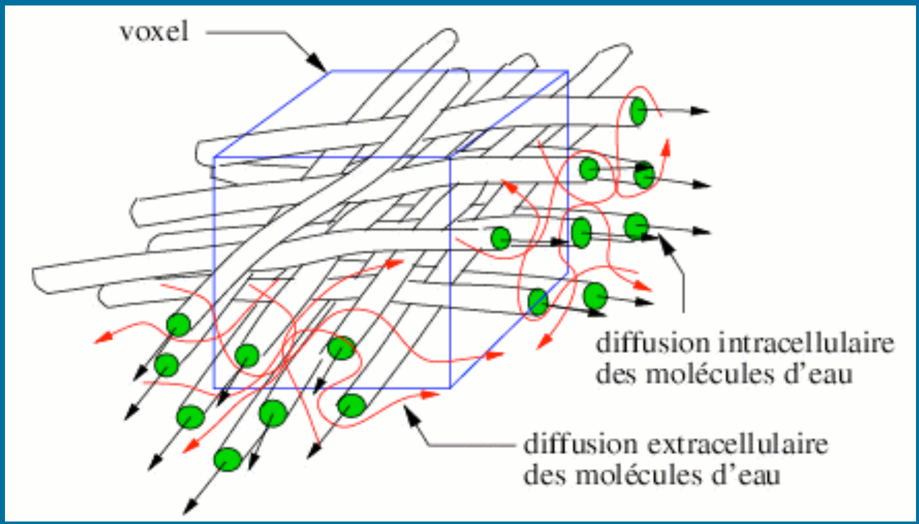


S(q) : diffusion MRI signal



D(q) : apparent diffusion coefficent (ADC)

Limitation of classical DTI



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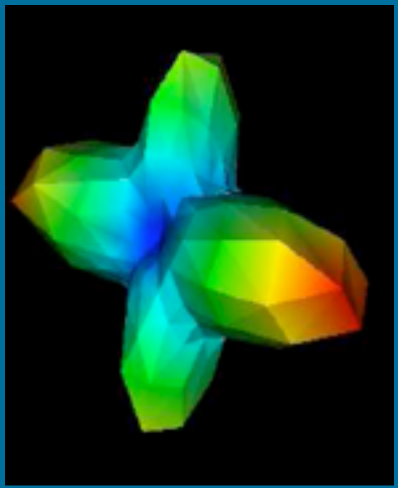
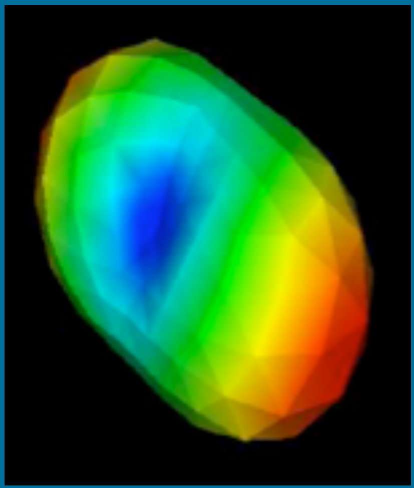


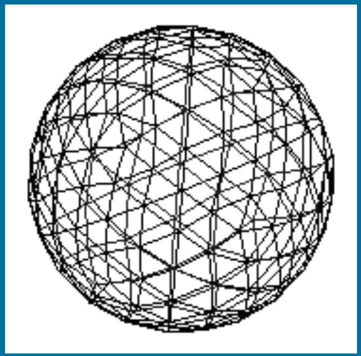
True ADC



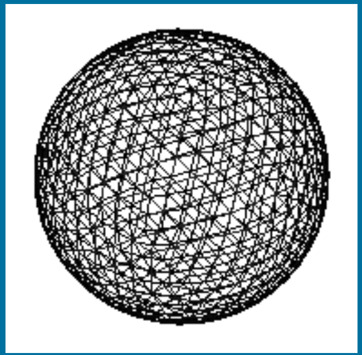
DTI estimate of the ADC

- DTI fails in the presence of many principal directions of different fiber bundles within the same voxel
- **Non-Gaussian** diffusion process

High Angular Resolution Diffusion Imaging (HARDI)



162 points



642 points

- N gradient directions
- We want to recover fiber crossings

Solution: Process all discrete noisy samplings on the sphere using **high order formulations**

Spherical harmonics formulation

- Orthonormal basis for complex functions on the sphere
- **Symmetric** when order ℓ is even
- We define a **real** and **symmetric** modified basis Y_j such that the ADC

$$D(q_i) \approx \sum_{j=1}^{n_j} c_j Y_j(\phi_i, \theta_i)$$

Spherical Harmonics (SH) coefficients

- Solving for coefficients

$$c_j = \int_\sigma D(q) Y_j(\phi, \theta)$$

- In matrix form,

  X : discrete HARDI data

  B : discrete SH, $Y_j(\phi,\theta)$

  C : SH coefficients

- Solve with least-square

$$C = (B^T B)^{-1} B^T X$$

[Alexander et al.]

Regularization with the Laplace-Beltrami Δ_b

- Squared error between spherical function F and its smooth version on the sphere $\Delta_b F$

$$E(F) = \int_\sigma (\triangle_b F)^2$$

- SH obey the PDE $\triangle_b F + l(l+1)F = 0$

- We have,

$$\begin{aligned} E(F) &= \int_\sigma \triangle_b \left(\sum_j c_j Y_j \right) \triangle_b \left(\sum_j c_j Y_j \right) \\ &= \sum_j c_j^2 l_j^2 (l_j + 1)^2 \\ &= C^T L C \end{aligned}$$

Minimization & λ regularization

- Minimize

$$(BC - X)^T(BC - X) + \lambda C^T LC$$

→

$$C = (B^T B + \lambda L)^{-1} B^T X$$

- Find best λ with L-curve method
- Intuitively, λ is a penalty for having higher order terms in the modified SH series

=> higher order terms only included when needed

High order diffusion tensor (HODT) formulation

- Equivalence relation between spherical harmonic series of order ℓ and rank-ℓ high order diffusion tensor [Ozarslan-Mareci 2003]

- Linear transformation M between coefficients C of SH series and HODT independent elements **D**

$$C = M * \mathbf{D} \quad \Rightarrow \quad \mathbf{D} = M^{-1} * C$$

Summary of algorithm

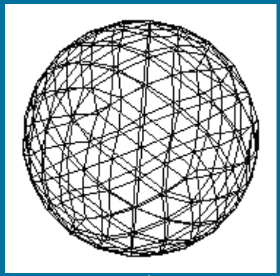
HARDI data on the sphere

λ-regularization & minimization with least-squares

Spherical harmonics coefficients

Linear transformation

High order diffusion tensor



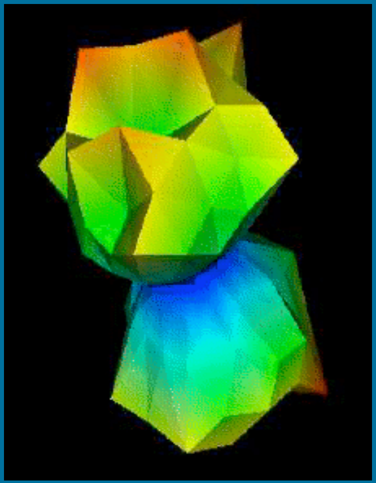
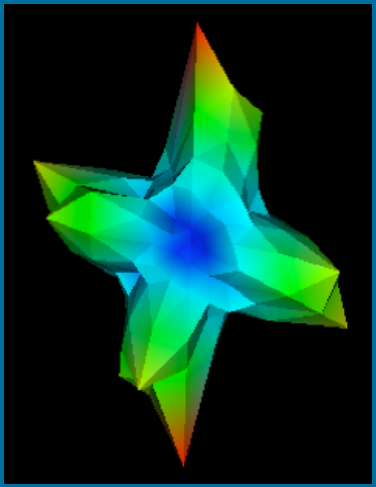
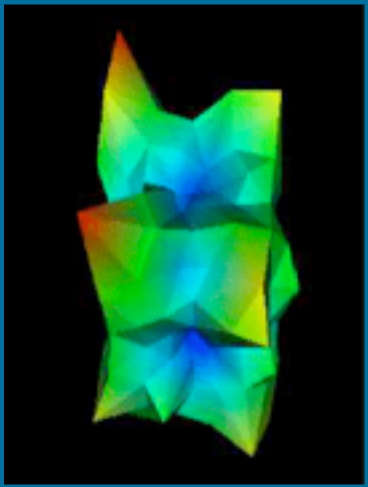
For $\ell = 4$,

$C = [c_1, c_2 , \ldots, c_{15}]$

$D = [D_{xxxx}\ D_{yyyy}\ D_{zzzz}\ D_{xxxy}\ D_{xxxz}\ D_{yzzz}\ D_{yyyz}\ D_{zzzx}\ D_{zzzy}\ D_{xyyy}\ D_{xzzz}\ D_{zyyy}\ D_{xxyy}\ D_{xxzz}\ D_{yyzz}\ ]$

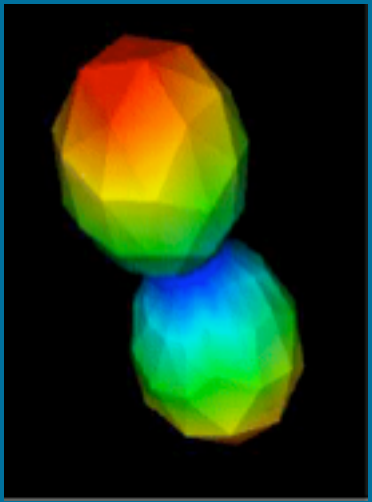
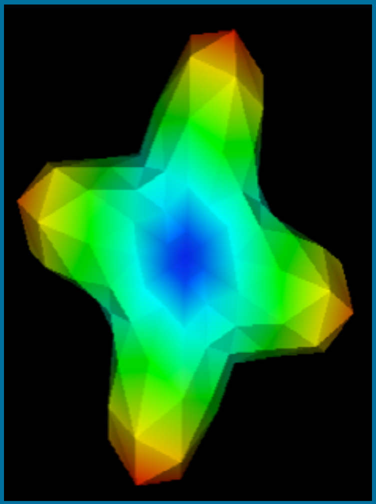
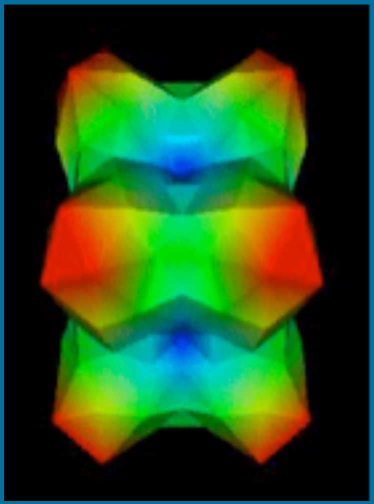
Apparent diffusion coefficient estimation

Noisy ADC

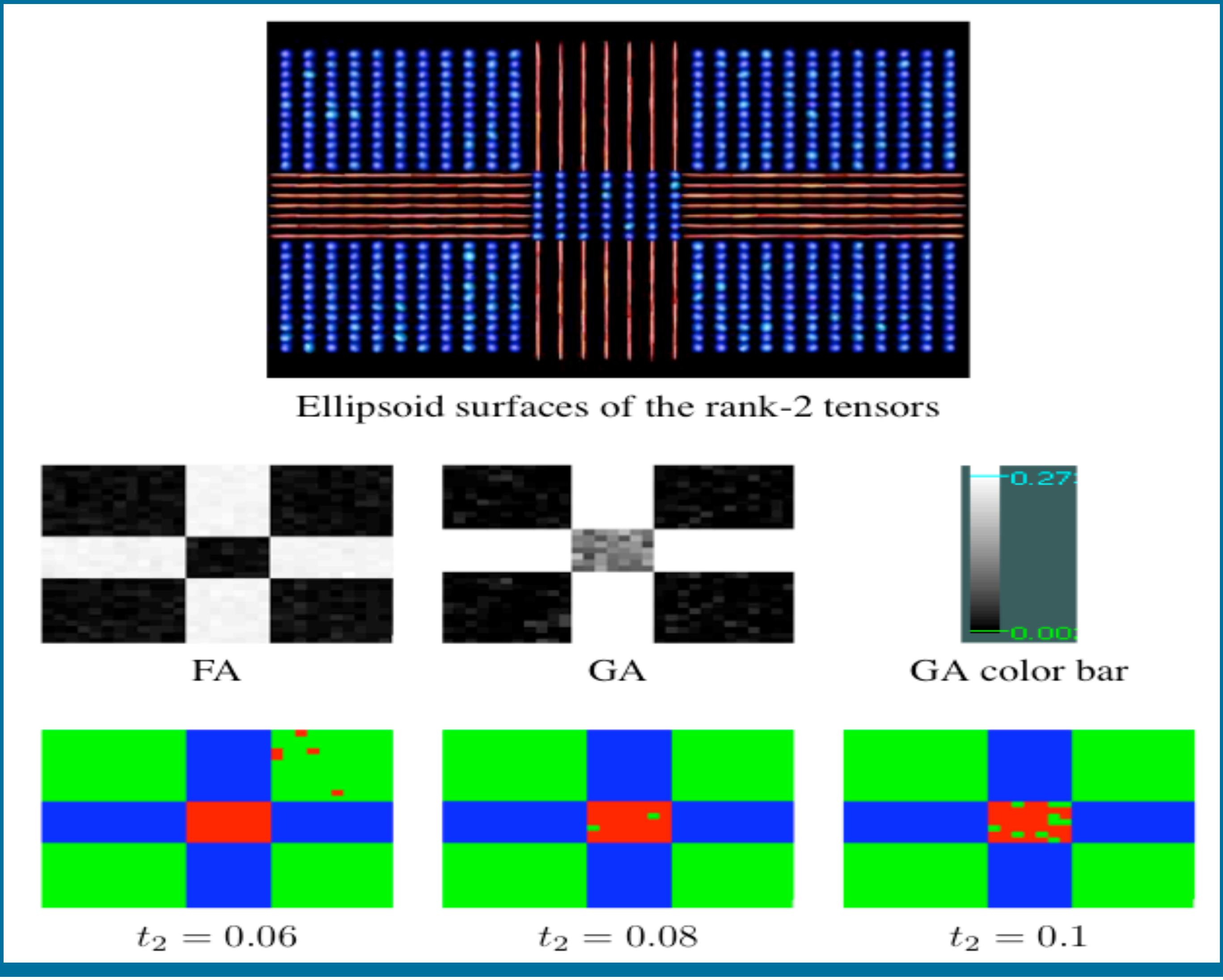




1 fiber **2 fibers** **3 fibers**

Estimated ADC
$\lambda = 0.006$

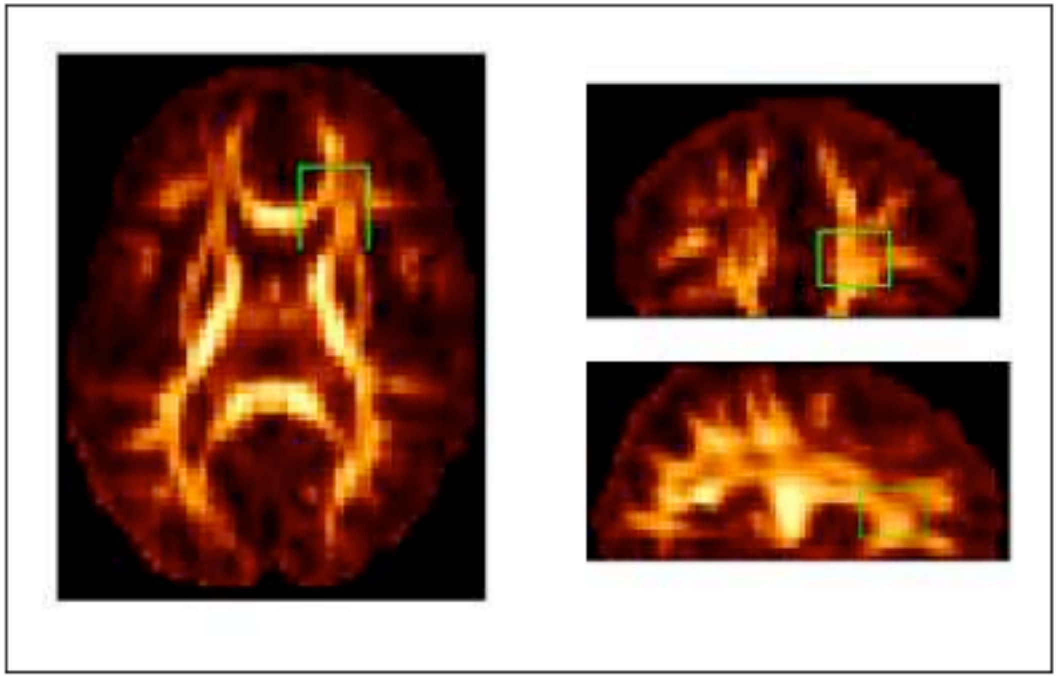
3 fiber crossing data and anisotropy measures



Ellipsoid surfaces of the rank-2 tensors

FA GA GA color bar

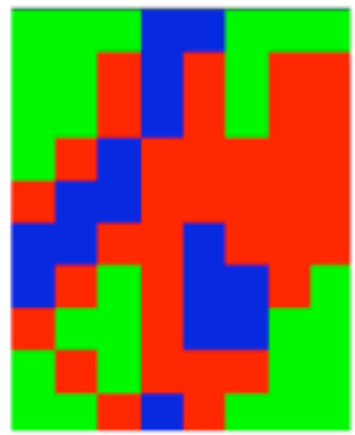
$t_2 = 0.06$ $t_2 = 0.08$ $t_2 = 0.1$



Transverse, coronal and sagittal FA slice of the region of interest

FA crop

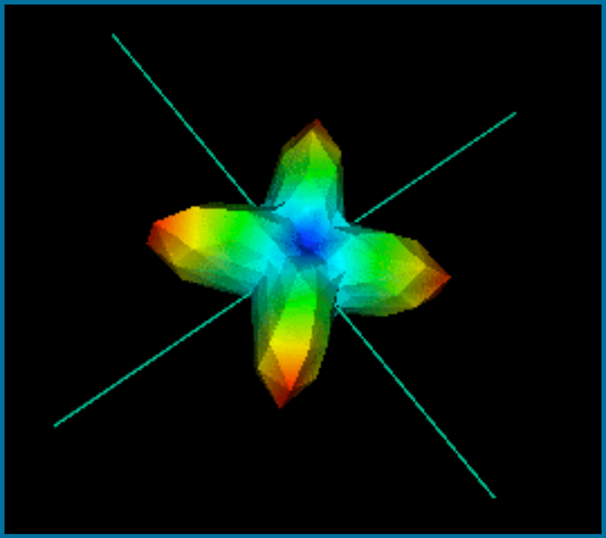
GA crop

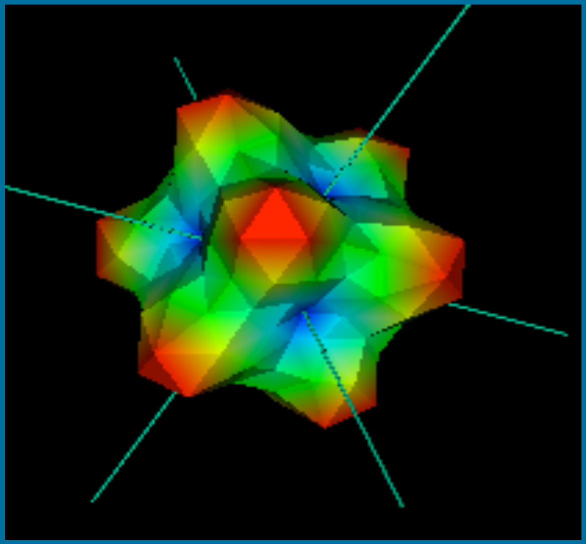


class crop

Finding fiber bundles orientations

- Maximas of the ADC **do not** correspond to fiber directions when we have multiple fiber distributions





Fiber orientations

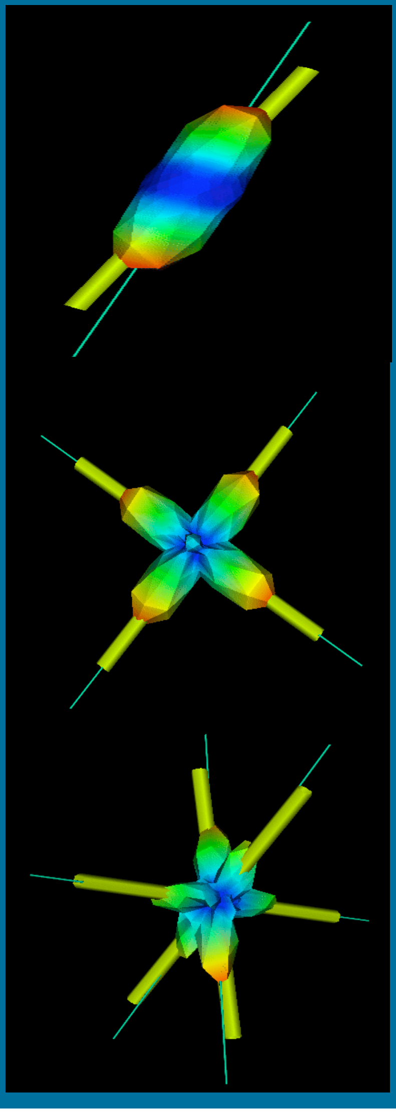
S(q) in Q-space

Funk-Radon transform
from spherical harmonics
coefficients

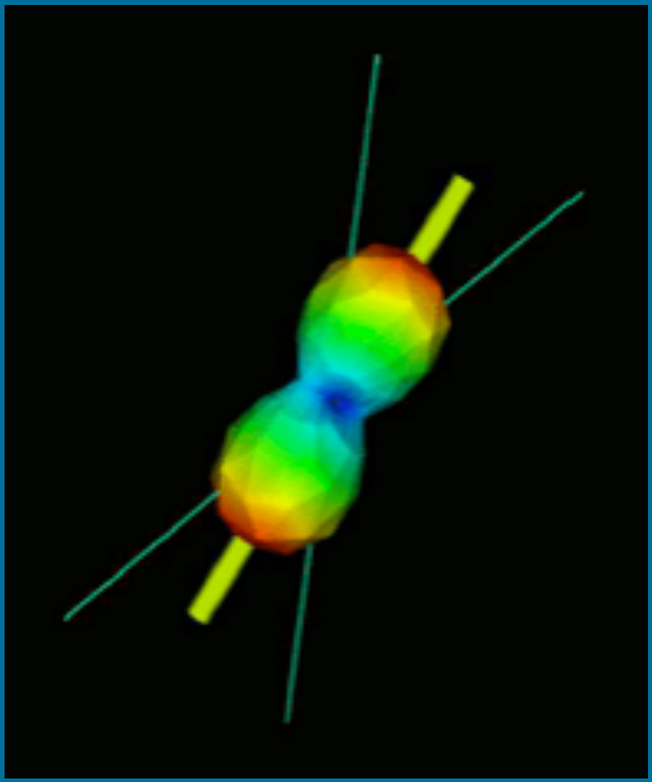
Orientation
Density Function
(ODF)

Sharpening &
maxima extraction

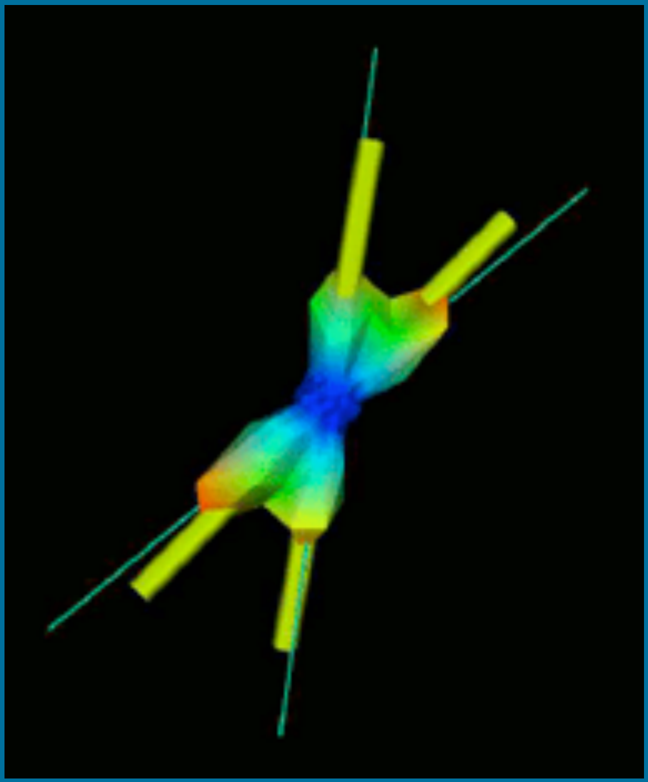
Fiber directions



Limitations of classical DTI



Classical DTI
rank-2 tensor



HARDI
high order formulation

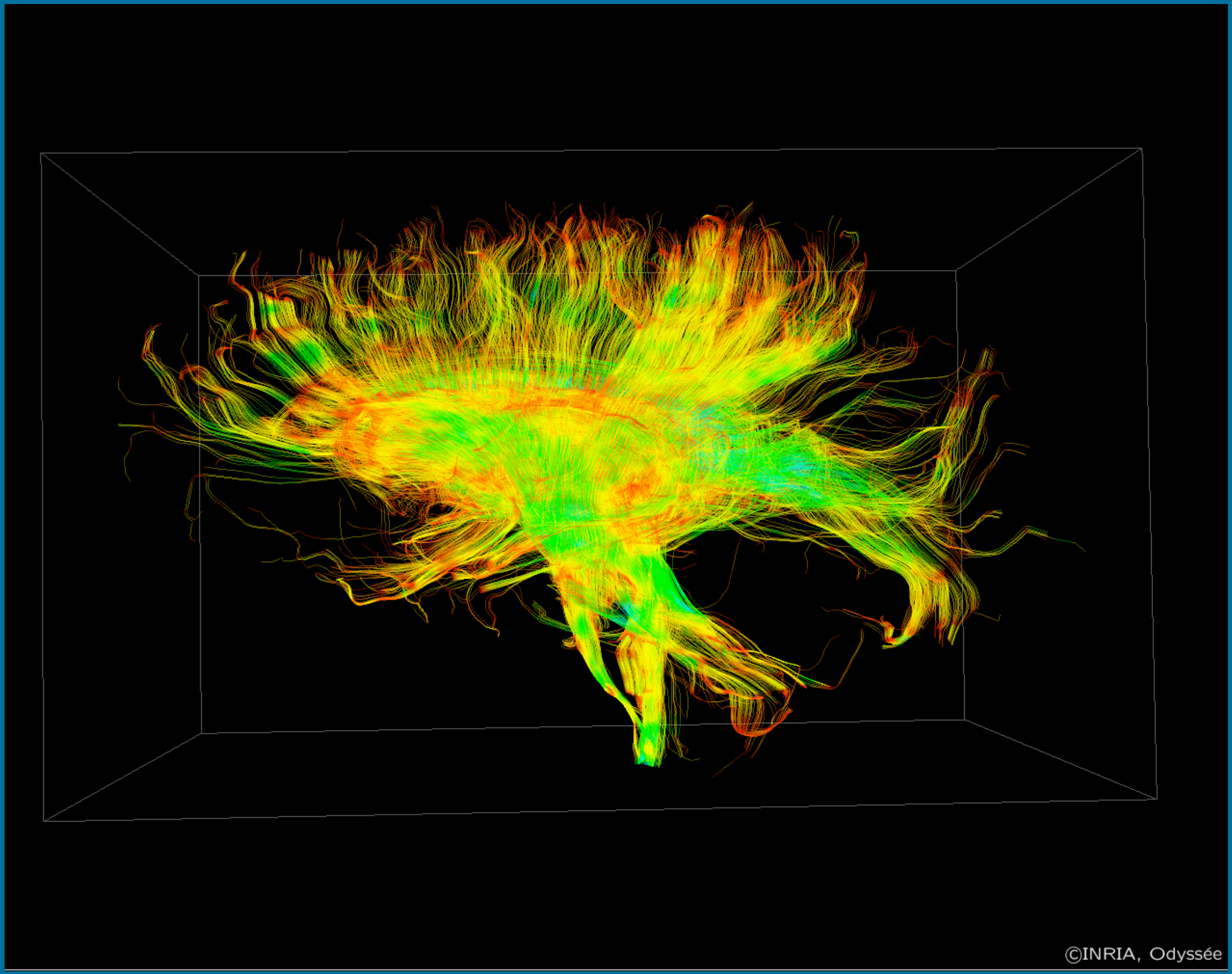
Validation on real data: a major challenge

- Most validation is done on synthetic data
  - Do not have a precise model for multiple fiber diffusion
  - Current models work under restrictive assumption
- Difficult to construct synthetic or real phantoms
  - Small plastic tubes in figure-8 configurations
  - Rat spinal cords configuration [Campbell et al. (MNI)]

Possible future collaborations with CIM + MNI

- HARDI processing
- Physics and acquisition of HARDI data
- Phantom construction with known ground truth to compare and validate our algorithms
- Closer interaction with neurosurgeons and biomedical engineers
- Collaboration between f-mri people and fiber tracking people to explore anato-functional connections in the brain

Thank you!



Spherical Harmonics

- SH $$Y_\ell^m(\theta,\phi) = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}P_\ell^m(\cos\theta)e^{im\theta}$$
- SH PDE $$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial F}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2 F}{\partial\phi^2} + \ell(\ell+1)F = 0$$
- Real $[(-1)^m Y_l^m + Y_l^{-m}]$ $i[(-1)^{m+1} Y_l^m + Y_l^{-m}]$
- Modified basis

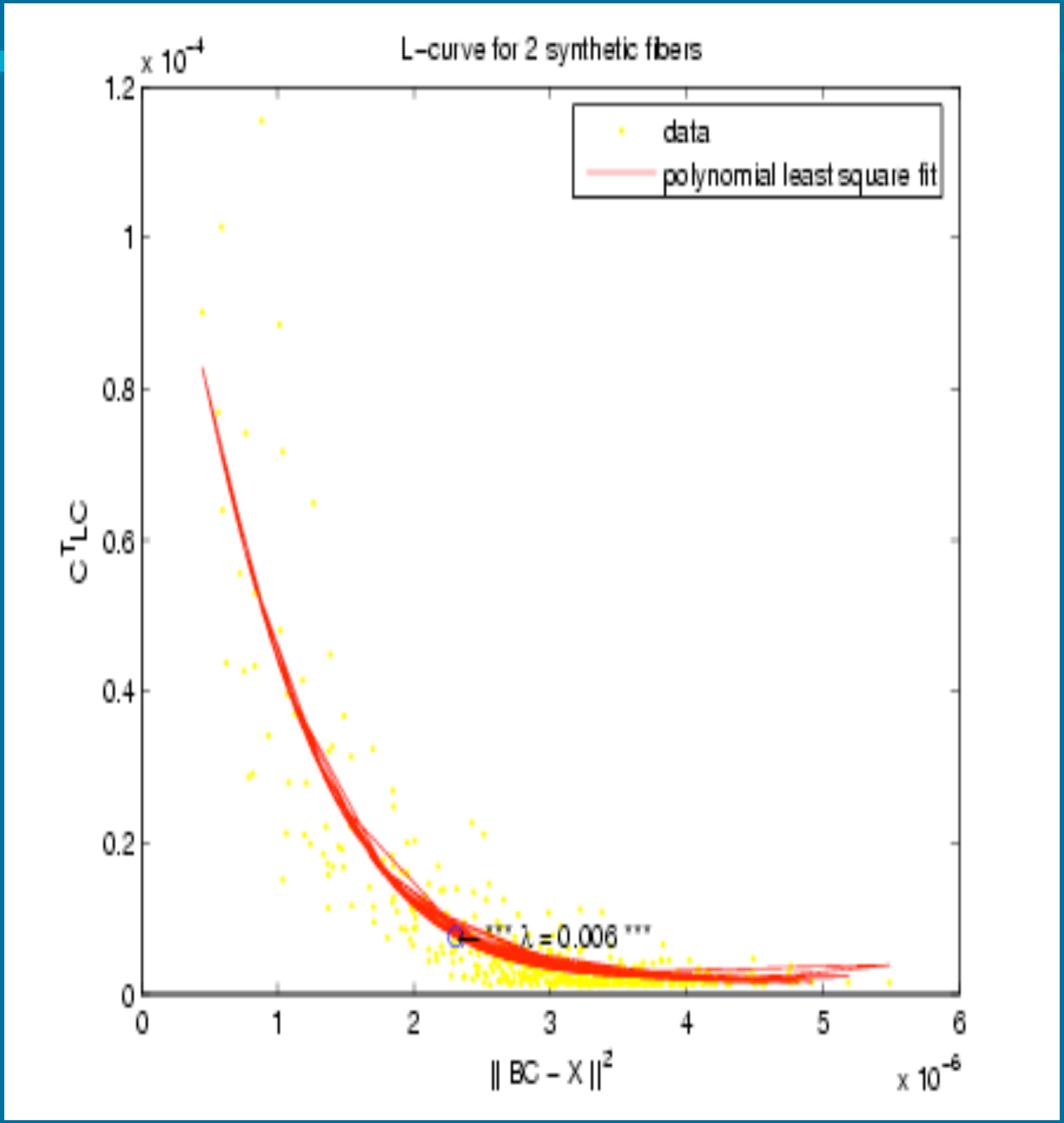
$$Y_l^0, \frac{\sqrt{2}}{2}\left[(-1)^m Y_l^m + Y_l^{-m}\right], \frac{\sqrt{2}}{2}\, i[(-1)^{m+1} Y_l^m + Y_l^{-m}]$$

SH to HODT

$$
\begin{aligned}
D(q) &= \textstyle\sum_{i_1}^{3}\sum_{i_2}^{3}\dots\sum_{i_l}^{3} D_{i_1 i_2 \dots i_l} q_{i_1} q_{i_2} \dots q_{i_l} \\
&= q_x^2 D_{xx} + q_y^2 D_{yy} + q_z^2 D_{zz} + q_x q_y D_{xy} + q_y q_x D_{yx} \\
&\quad + q_x q_z D_{xz} + q_z q_x D_{zx} + q_y q_z D_{yz} + q_z q_y D_{zy} \\
&= q_x^2 D_{xx} + q_y^2 D_{yy} + q_z^2 D_{zz} + 2 q_x q_y D_{xy} + +2 q_x q_z D_{xz} + 2 q_y q_z D_{yz} \\
&= \sum_{k=1}^{n_j} \mu_k D_k \prod_{p=1}^{l} q_{k(p)}
\end{aligned}
$$

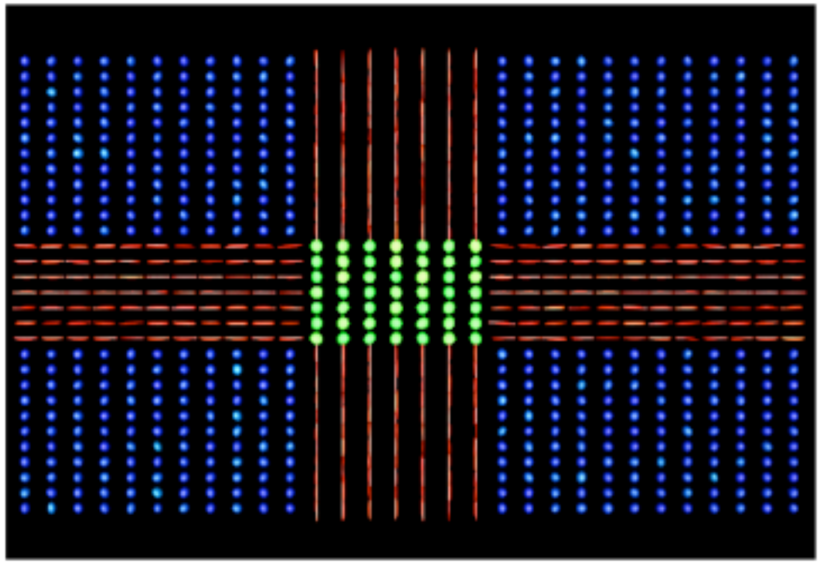
$$
c_j = \int_\sigma D(q) Y_j(\phi, \theta) \Longrightarrow c_j = \sum_{k=1}^{n_j} D_k \mu_k \int_\sigma \prod_{p=1}^{l} q_{k(p)} Y_j(q) dq
$$

L-curves

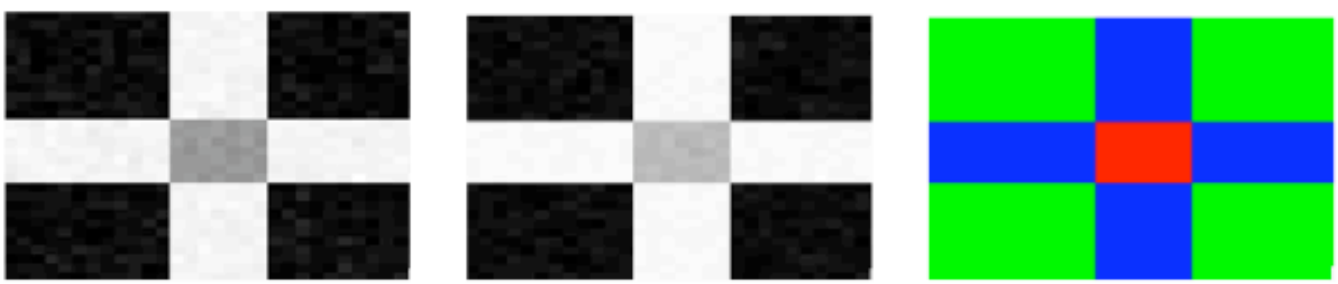


L-curve for 2 synthetic fibers
x 10^-4
1.2
1
0.8
0.6
0.4
0.2
0
C^T L C
data
polynomial least square fit
*** λ = 0.006 ***
0
1
2
3
4
5
6
|| BC − X ||^2
x 10^-6

Anisotropy measures (2 fibers)

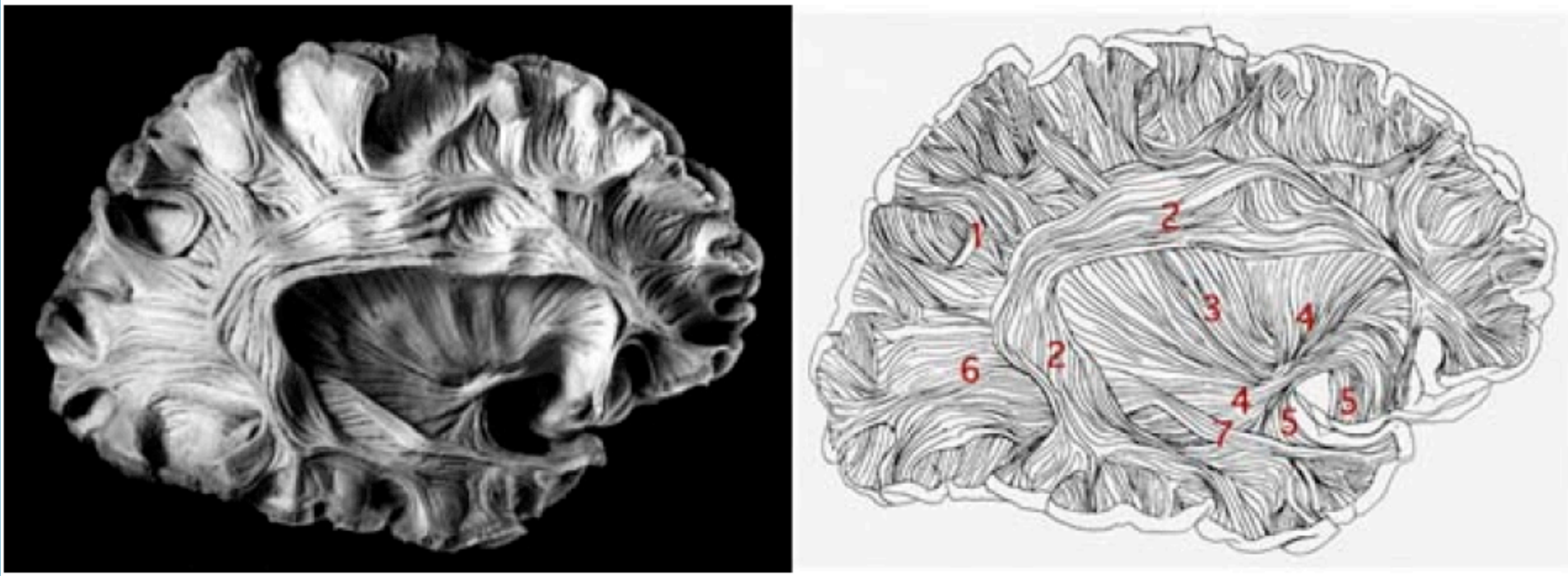


Ellipsoid surfaces of the rank-2 tensors



FA GA class

Anatomie cérébrale

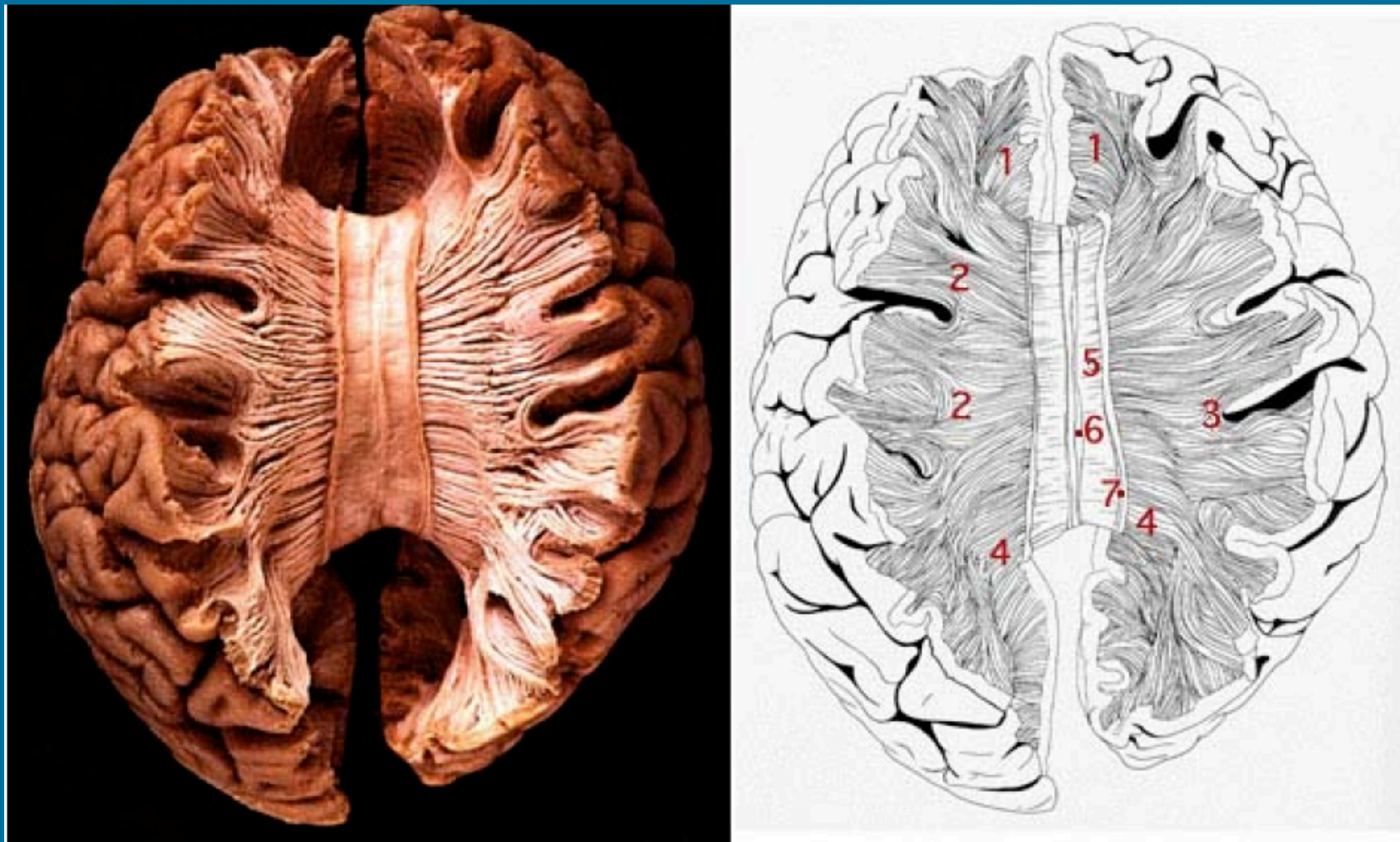


1. Short arcuate bundles. 2. Superior longitudinal fasciculus. 3. External capsule. 4. Inferior occipitofrontal fasciculus. 5. Uncinate fasciculus. 6. Sagittal stratum. 7. Inferior longitudinal fasciculus.

Faisceaux d'associations courtes et longues dans l'hémisphère droit

(images de [Williams-etal97])

Anatomie cérébrale



1. Frontal forceps 2. Corpus callosum commissural fibers 3. Short arcuate fibers 4. Occipital forceps 5. Indusium griseum 6. Medial longitudinal stria 7. Lateral longitudinal stria

Radiations du corps calleux (images de [Williams-etal97])

Applications

- Découvertes de nouvelles connections neuronales
- Comprendre les réseaux de régions corticales impliquées dans le fonctionnement cérébrale
- Étude de maladies neuro-dégénératrices
- Planification neuro-chirurgicale

Le tenseur de diffusion (DTI)

- The Stejskal-Tanner Imaging Sequence

$$S_k = S_0 \exp\left(-\gamma^2\delta^2\left(\Delta - \frac{\delta}{3}\right)|\mathbf{g}_k|^2\mathbf{D}\right)$$

S_k: Diffusion weighted images

γ : Spin gyromagnetic ratio

g_k: Diffusion gradient directions

δ : Gradient pulses duration

Δ : Time between 2 gradient pulses

$\mathbf{D}$: Apparent diffusion coefficient

$$D = 10^{-6} \cdot \begin{pmatrix} 1700 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{pmatrix}$$

Tenseur observé dans les régions anisotropes
de la matière blanche du cerveau humain

Mesures d’anisotropies

- Cartes d’anisotropies
- Arriver à identifier les régions
  - Isotrope
  - Anisotrope d’une fibre
  - Anisotrope avec plusieurs fibres
- Rapide et utile

Classification du processus de diffusion

	GA	Vemuri	Franck	Alexander
notre méthode	99.9%	96.0%	93.1%	94.8%
autre	99.3%	83.2%	85.1%	91.7%

- Succès de la classification du processus de diffusion à partir des mesures d'anisotropies
  - Diffusion isotrope
  - Diffusion anisotrope d'une fibre
  - Diffusion anisotrope de plusieurs fibres

adc = Apparent Diffusion Coefficient

LR = Linear Regression (Ozarslan et al)

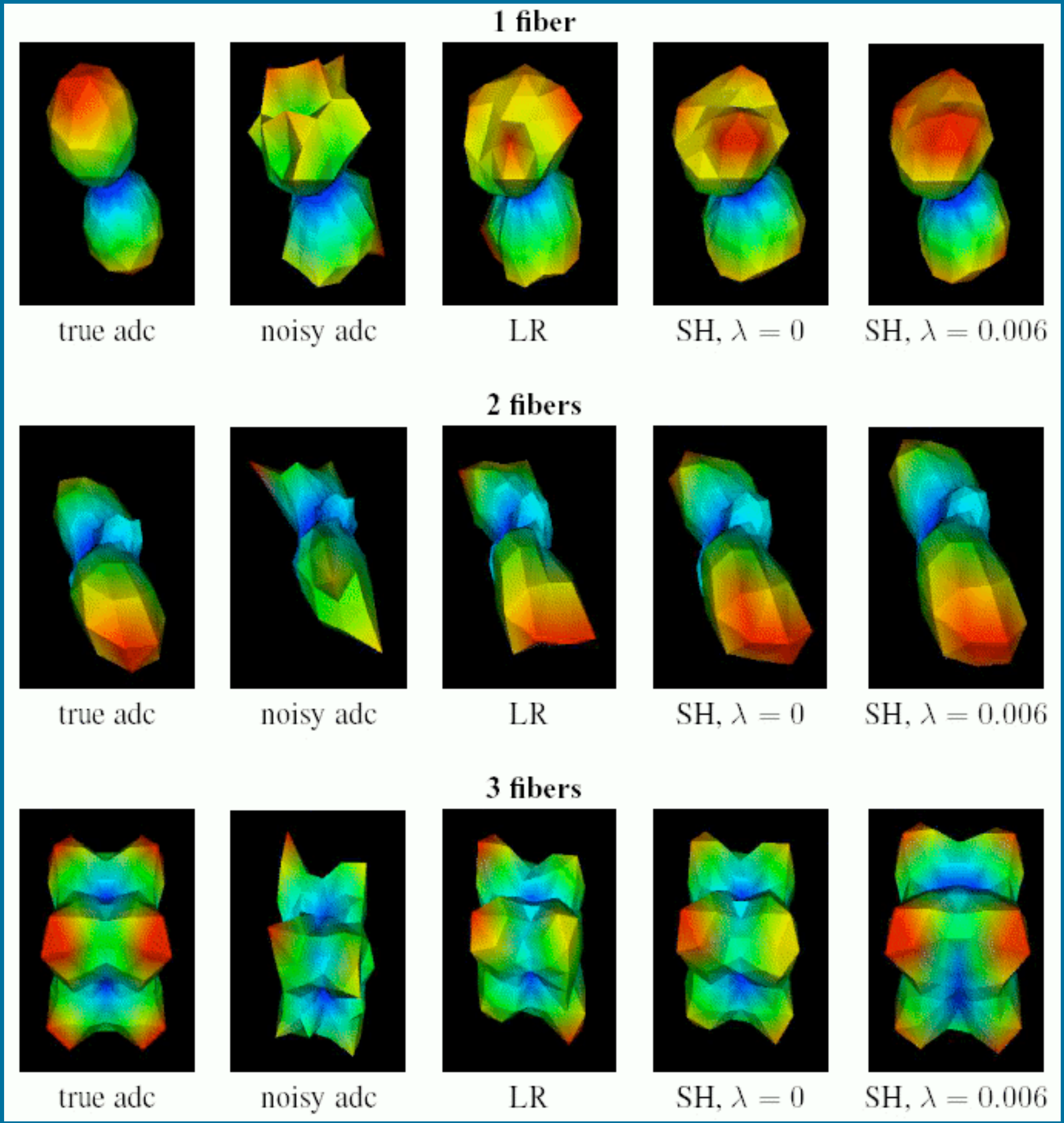
SH = Spherical Harmonics

$\lambda = 0$ (Frank, Alexander et al)

$\lambda = 0.006$ (notre approche)

Bruit Ricien, $\sigma = S_0 / 35$

Ordre de SH, $\ell = 8$



ADC estimation results

		1 fiber test with λ (x10^{-3})					
	LR	0	6	12	100	**300**	500
mean	0.083	0.083	0.071	0.068	0.052	**0.051**	0.054
std	0.064	0.063	0.051	0.046	0.036	**0.035**	0.037
		2 fiber test with λ (x10^{-3})					
	LR	0	3	**6**	9	12	15
mean	0.076	0.075	0.070	**0.069**	0.070	0.070	0.072
std	0.052	0.052	0.043	**0.041**	0.040	0.041	0.042
		3 fiber test with λ (x10^{-3})					
	LR	0	6	9	12	**15**	100
mean	0.092	0.092	0.049	0.040	0.034	**0.031**	0.057
std	0.037	0.037	0.028	0.026	0.025	**0.025**	0.026
		random fiber test with λ (x10^{-3})					
	LR	0	6	12	**100**	200	300
mean	0.078	0.078	0.068	0.065	**0.064**	0.067	0.070
std	0.054	0.055	0.045	0.043	**0.042**	0.044	0.045